WORKS OF

LIEUT.-COLONEL G. J. FIEBEGER

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CIVIL ENGINEERING

A TEXT-BOOK FOR A SHORT COURSE

BY

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PREFACE.

THIS text-book is designed primarily for the cadets of the U. S. Military Academy, who are being fitted for a profession in which the principles of civil engineering are of daily application.

In time of peace the officer in an isolated station finds himself called upon to act as engineer and constructor of buildings, roads, and bridges. If not the engineer charged with the construction of water-works and sewerage systems, he finds himself charged with their maintenance and repair. In time of war, a knowledge of the construction of buildings and bridges enables him to effect their destruction without loss of time or the aid of skilled workmen. The entire subject of military engineering, including fortification, sapping, mining, pontoneering, etc., is simply the application of the principles of civil engineering and tactics to military problems.

For the above reasons the Academic Board (Faculty) of the Military Academy has, since its origin, provided for a short course in civil engineering. The aim of the course is to teach the cadets the general principles governing the construction of engineering structures, without any attempt to make them specialists. The last must be effected in the postgraduate schools. In this course it has been necessary to avoid elaborate details and also to omit some of the branches of what is now classed as civil engineering, in contradistinction to electrical, mining, and mechanical engineering.

To fix the principles, problems are given under each head; in solving these problems use has been made of the Cambria handbook for 1904, to which references are made. Similar data may, however, be found in other handbooks. Additional problems, collected in a separate book, are employed in section-room work.

Descriptions and illustrations from current engineering publications are constantly used to supplement the text and explain the practice of engineering.

A short list of standard publications is appended to some of the chapters. This has been inserted for the use of the officer who finds himself in a position in which more exhaustive information on a subject is desirable. It is proposed to change these lists from time to time.

I desire to acknowledge my indebtedness to the authors whose works have been consulted, and to my assistants Captains James P. Jervey, Frederick W. Altstaetter, James A. Woodruff, Horton W. Stickle, and Lewis H. Rand, Corps of Engineers, U. S. Army, for their assistance and suggestions.

G. J. FIEBEGER.

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SYMBOLS EMPLOYED IN MECHANICS OF ENGINEERING.

A = area of cross-section.
b = breadth.
d = depth.
E, E_1, E_{11} = coefficients or moduli of longitudinal elasticity.
E_t = coefficient or modulus of lateral elasticity.
F = intensity of a force.
j' = coefficient of expansion.
g = acceleration due to force of gravity.
H_s = horizontal shear in a beam.
I = moment of inertia of the cross-section of a beam about its neutral axis.
I_p = moment of inertia of the cross-section of a shaft about the axis of the shaft.
$\frac{I}{y}$ = section modulus of a beam.
l, L = length.
M = bending moment.
M_m = maximum bending moment.
M_t = torsional moment.
r, R = radius of circle or radius of gyration.
R_1, R_2 = reactions at supports of a beam.
s = unit stress of flexure in surface fiber of beam.
s''' = " " " " at a unit distance from the neutral axis.
s_c = unit stress of compression.
s_e = " " " elongation or tension.
s_s = " " " shear.
s_t = " " " torsion in surface fiber of a shaft.
s_t''' = " " " " at a unit distance from the axis.
s'', s_c'', s_e'', s_s'', etc. = safe values of the unit stresses.
s', s_c', s_e', s_s', etc. = ultimate or breaking values of the unit stresses.
s_m = maximum value of s.
V_s = vertical shear in a beam.
$(V_s)_m$ = maximum vertical shear.
W = weight or load.
y' = distance of surface fiber of a beam from the neutral axis.
y_m = maximum deflection of a beam.
α, β, γ, and ϕ = angles.
δ = sign of differentiation.
$\int$ = sign of integration.
λ = unit elongation or shortening.
inch-pounds = work.
(inch-pounds) = moment of a force.

CIVIL ENGINEERING.

CHAPTER I.

CLASSIFICATION OF STRESSES AND LONGITUDINAL STRESS.

An engineering structure, as a building, bridge, dam, etc., is one designed on the hypothesis that its different elementary parts are to be in a state of rest. It differs in this respect from a machine, which is designed to admit relative motion between some of its parts.

Rest in a body is possible only when the system of forces acting on it is in equilibrium. In order that the structure as a whole shall be in a state of rest, the system of forces acting on the entire structure must be in equilibrium. In order that any elementary part of the structure shall be at rest, the system of forces acting directly on it, or transmitted to it through the other parts, must be in equilibrium.

The general conditions of equilibrium for any system of forces are:

1. *The algebraic sum of the components of all the forces in any direction must be equal to zero.* If this condition is fulfilled, the center of the body acted on can have no *linear motion* or *motion of translation.*

2. *The algebraic sum of the moments of the forces about any axis must be equal to zero.* If this condition is fulfilled, the body can have no *motion of rotation* about any axis.

In most engineering structures, it is assumed that the system of forces acting on each elementary part, and sometimes on the entire structure, is a *coplanar system;* that is, the action lines of all the forces of the system lie in the same plane. If the plane is vertical, the coplanar forces are usually resolved into vertical and horizontal components. The general conditions of equilibrium for a coplanar system of forces in a vertical plane are:

1. *The sum of the vertical components of the forces must be equal to zero.*

2. *The sum of the horizontal components of the forces must be equal to zero.*

3. *The sum of the moments of the forces, about any axis perpendicular to the plane of the forces, must be equal to zero.*

Operations in Structural Design.—The process of structural design may be divided into three operations:

1. The determination of the intensity and action line of every force that acts on the structure as a whole, or on any one of its elementary parts.

2. The determination of the behavior of the material of which the structure is made, under the application of force.

3. The designing of each elementary part, after the system of forces acting on it, and the behavior of its material under the application of force, are known.

First Operation.—The problems that arise under the first operation are usually those of coplanar forces and may be solved by the aid of the three equations of condition for equilibrium in a coplanar system. These are:

$$A \sin \alpha + B \sin \beta + C \sin \gamma + D \sin \delta + \text{etc.} = 0, \quad . \quad . \quad . \quad (1)$$

$$A \cos \alpha + B \cos \beta + C \cos \gamma + D \cos \delta + \text{etc.} = 0, \quad . \quad . \quad . \quad (2)$$

$$Al' \sin \alpha + Bl'' \sin \beta + Cl''' \sin \gamma + Dl^{\text{IV}} \sin \delta + \text{etc.} = 0, \quad (3)$$

in which A, B, C, D, etc. = the intensities of the different forces of the system;

$\alpha, \beta, \gamma, \delta$, etc. = the angles which the action lines of the forces make with the horizontal;

$l', l'', l''', l^{\text{iv}}$, etc. = the distances from the center of moments to the action lines of the different forces, measured on a horizontal line through the center of moments;

$l' \sin \alpha$, etc. = the lever-arms of the forces with respect to the center of moments.

In the above equations there are three quantities dependent on each force. Thus A is the *intensity* of the force A, the angle α fixes the *direction* of its action line in the plane, and the distance l' fixes the position of the action line in the plane or a *point of application* of the force A.

Since each intensity enters each of the three equations, we may by combination determine three unknown intensities if all the other quantities in the equations are known and the equations are independent.

Similarly we may determine three unknown directions or angles, if all the other quantities are given and the equations are independent. The combination of the equations is, however, difficult.

Since the quantities l', l'', l''', etc., enter but a single equation, (3), we can determine but a single unknown point of application when all the other quantities in this equation are given.

In combining the equations, components acting *upwards or to the right* are considered *positive*, and those acting in contrary directions *negative*. Moments which are *clockwise* are considered *positive*, and *counter-clockwise moments* are considered *negative*.

Non-concurrent and Non-parallel Forces.—If the action lines of the forces neither intersect at a common point nor are parallel, equations (1), (2), and (3) are wholly independent and we may determine any three unknown quantities in the system if all the others are given, provided that only one of the unknowns is a point of application.

Concurrent System.—If the action lines of the forces intersect at a common point, equation (3) is no longer independent of the others. This is most readily shown by resolving the forces into components parallel and perpendicular to the line joining the common point with the center of moments, instead of resolving them into vertical and horizontal components. The angles, α, β, γ, etc., will then be the angles which the action lines of the forces make with this new line, and the distances l', l'', l''', etc., will be measured along the line. Since the distances l', l'', l''', etc., will all be equal under this hypothesis, equation (3) becomes identical with equation (1), if we divide (3) through by the common value of l.

In a concurrent system we can therefore determine but two unknown quantities neither of which can be a point of application.

Parallel Forces.—If the action lines of the forces are parallel, equations (1) and (2) are no longer independent, since the angles α, β, γ, etc., are all equal to each other. By dividing equation (1) by the common sine, and equation (2) by the common cosine, the two equations become identical.

In a parallel system we can therefore determine but two unknown quantities, of which one only may be a point of application. In solving systems of parallel forces, the angles α, β, γ, etc., are usually assumed as 90°, and the distances l', l'', l''', etc., are measured on a line through the center of moments perpendicular to the action lines of the forces.

For a system of parallel forces the three equations then reduce to

$$A+B+C+D+\text{etc.}=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

$$Al'+Bl''+Cl'''+Dl^{\text{iv}}+\text{etc.}=0. \quad . \quad . \quad . \quad (3a)$$

Three Forces.—If three forces are in equilibrium or a body is at rest when acted on by three forces only, each force must be equal and opposite to the resultant of the other two; hence the three forces must form a parallel or a concurrent system. It follows that if in any system of forces in equilibrium all are known save two, these two must, with the resultant of the known forces, form a concurrent or a parallel system.

Weight.—One of the forces which must usually be considered as acting on an engineering structure is its own weight. It is therefore necessary to know the weights of the ordinary building materials. These are approximately as follows:

	Cubic Foot.
Wood	30 to 50 pounds
Brick masonry	100 " 140 "
Concrete masonry	125 " 140 "
Stone masonry	140 " 160 "
Cast iron	450 "
Wrought iron	480 "
Steel	490 "

Second Operation.—To determine the behavior of materials under the action of force, we must first consider the manner in which force may be applied.

If a solid body, as a beam, is firmly fixed, horizontally, in a wall (Fig. 1), it may be acted upon—

1. By a force whose action line coincides with the axis of the beam and whose direction may be either away from or towards the fixed end of the beam. Such a force is a *longitudinal force; tensile* if it acts away from the fixed end, and *compressive* if it acts towards the fixed end. Thus in Fig. 1, AB is a tensile force and AC is a compressive force.

A piece designed to resist a tensile force is called a *rod* or *tie;* a piece designed to resist a compressive force is called a *column* or *strut.*

2. By a force whose action line is in the plane of the axis of the beam and is perpendicular to the axis. This is a *bending-shearing force*, usually called simply a *bending force* or a *force of flexure.* Such a force becomes a *simple shearing force* when it acts in the plane adjacent to the fixed plane of cross-section. In Fig. 1 AD is a bending force; if the force AD were applied in the plane of cross-section adjacent to the fixed end of the beam, it would be a simple shearing force.

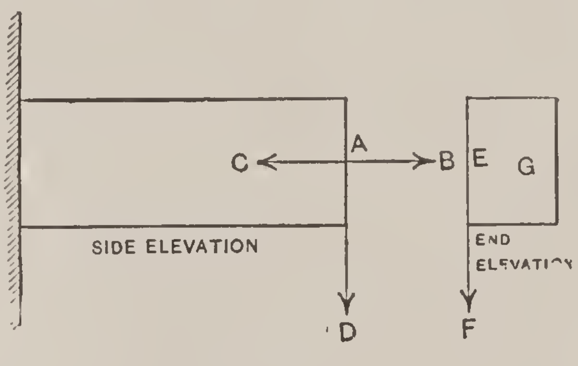

FIG. 1.

A piece designed to resist a bending force is called a *beam.*

3. By a force which acts in one of the planes of cross-section of the beam, but does not intersect the axis. This is a *torsional* or *twisting force*, and its moment with respect to the axis of the beam is called its *torsional moment.* In Fig. 1 the force *EF* acting in the plane of the end cross-section is a torsional force. A *torsional couple* is a couple formed of two equal torsional forces acting in contrary directions. A single torsional force produces bending as well as torsion; a torsional couple produces torsion only. If in Fig. 1 a parallel force equal to *EF* acts upwards in the same cross-section, the two together will form a torsional couple.

A piece designed to resist a torsional force is called a *shaft.*

Any system of forces acting on an elementary piece of a structure may be resolved into these elementary forces. This resolution is usually necessary before the effect of the system can be determined.

If a force intersects the axis of the beam in Fig. 1, but is oblique to that axis, it may be resolved into a bending force and a tensile or compressive force. If a force is perpendicular to the axis but does not intersect it, it may be resolved into a bending force and a torsional couple. Thus the force *EF* in Fig. 1 may be resolved into or replaced by an equal bending force acting downwards through *G*, the center of gravity of the cross-section, and a couple consisting of the original force *EF* and an equal force acting upwards through *G*. A force which is oblique to the axis which it does not intersect may be resolved into a bending force, a torsional couple, and a longitudinal force.

Classification of Stresses.—The effect of a force on a body in a state of rest is to move its molecules with respect to each other. This disturbs the equilibrium of the molecular forces, and results in the establishment of new conditions of equilibrium in which the molecular forces developed by the change in the relative positions of the molecules are in equilibrium with the force which has caused the change.

To distinguish the force which is applied to the body from the molecular forces which hold the applied force in equilibrium, the former is often called an *applied* or *extraneous force*, and the latter are called *stresses* or *internal stresses*.*

1. Under the action of the force AB or the force AC (Fig. 1) every molecule in the beam will move parallel to the axis of the beam. The resulting stress is called a *longitudinal stress; tensile* if the molecules are pulled apart, and *compressive* if pushed closer together. In Fig. 1 the force AB will develop tensile stress, and the force AC compressive stress.

2. Under the action of the force AD or of the force EF (Fig. 1), every molecule of the beam will move in its plane of cross-section. The resulting stress is called a *lateral stress; shearing* if the molecule moves in a right line, and *torsional* if the molecule moves in a circular path. In Fig. 1 the force AD will develop shearing stress, and the force EF torsional stress.

The space passed over by a molecule in longitudinal stress is called its *strain*,* and that passed over by a molecule in lateral stress is called its *distortion*.*

The path of any molecule of the beam under the action of any applied force may be resolved into its component strains and distortions.

The determination of the behavior of material under each of the elementary forces is usually effected in a testing laboratory. For each kind of force it is necessary to determine the *law governing the distribution of the stress developed by it over the plane of cross-section.* This law is general and applies to all materials. It is also necessary to determine *for each kind of force the elastic properties developed by it in each kind of material,* and the *ultimate and allowable strength of each kind of material.* These results are usually tabulated for use in designing the elementary parts of a structure.

Tensile Stress.—If a steel rod is suspended from a ceiling and to the lower end of its axis is attached a weight, the rod will be subjected to a tensile force. Laboratory experiments show—

* By some engineers the term *stress* is applied to any force which acts on a body in a state of rest; the term *strain* to an internal stress; and the term *distortion* to a strain.

1. That all the fibers between the ceiling and weight are elongated.

2. Every cross-section normal to the axis of the rod before the weight is applied remains normal to the axis after the weight is applied.

3. That these effects are observed whatever be the form of cross-section of the rod.

It follows, therefore, since the material is homogeneous and all fibers have the same resistance, that the stress on the cross-section must be uniform and hence on each unit of area the same.

Let F = total tensile force in pounds applied to, or the total tensile stress developed by it in, any cross-section;
A = area of cross-section considered, in square inches;
s_e = tensile force or tensile stress in pounds per square inch, called the *unit tensile force* or *unit tensile stress*. Then

$$s_e = \frac{F}{A}. \qquad \qquad (4)$$

In the English system the unit tensile force and unit tensile stress are generally expressed in pounds per square inch.

The law governing the distribution of the stress over the area of cross-section, and expressed in equation (4), is general for all materials under tensile stress, whatever be the form of cross-section, providing the force is applied along the axis of the rod. Unless great accuracy is desired, it is also assumed to be true for non-axial forces.

General Laws of Elasticity.—The general laws governing the elasticity developed in rods and columns by longitudinal forces as derived from experiments are:

1. *All bodies are elastic.*

2. *Within a limited field all bodies are perfectly elastic.*

3. *Within the field of perfect elasticity, the strain varies directly with the intensity of the applied force.*

4. *Beyond the field of perfect elasticity, the strain increases*

more rapidly than the applied force until rupture takes place.

Limit of Tensile Elasticity.—Conceive a rod of structural steel *one square inch in cross-section,* hung from the ceiling of a room, and supporting at its lower extremity a scale-pan for weights. If this pan is loaded with a gradually increasing load, the bar will stretch proportionally to its load until the aggregate of the suspended weights reaches about 35,000 pounds. During the loading, if the weights are, at any time, temporarily removed, the bar will at once return to its original dimensions. If the weights are increased above 35,000 pounds, the ratio of the elongation to the increase in the weights will no longer be constant, nor will the bar return to its original dimensions when the weights are temporarily removed. This indicates that a unit stress of 35,000 pounds limits the field of perfect elasticity of structural steel. It is therefore called the *limit of tensile elasticity* of that metal.

If, when the weights exceed 35,000 pounds, they are temporarily removed, it will be observed that the bar has received a slight permanent elongation; this elongation is called a *permanent set.*

The limit of tensile elasticity of any material is the greatest unit tensile force which may be applied to, or the greatest unit tensile stress which may be developed in, a rod of the material without producing a permanent set.

Coefficient, or Modulus, of Tensile Elasticity.—Let

F = tensile force in pounds applied to, or the tensile stress developed in, any area of cross-section of a rod;
A = area of cross-section in square inches;
L = original length of the rod in inches;
l = elongation of the rod in inches.

Then $\dfrac{F}{A}$ is the *unit force* applied to, or *unit stress* developed in, the area of cross-section, and $\dfrac{l}{L}$ is the *unit elongation* or the elongation per unit of length; it is usually represented by λ.

By the third general law of elasticity, the unit elongation must, within the field of perfect elasticity, vary directly with the unit force or unit stress; hence

$$\frac{\frac{F}{A}}{\frac{l}{L}}=E, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

in which E is a constant.

If as before we make $\frac{F}{A}=s_e$, and $\frac{l}{L}=\lambda$, equation (5) becomes

$$E=\frac{s_e}{\lambda}, \quad \text{or} \quad s_e=E\lambda, \quad \text{or} \quad \lambda=\frac{s_e}{E}. \quad . \quad . \quad . \quad (6)$$

Since the second form of equation (6) is of the same form as $x=ay$, the equation of a right line, in which a is a constant coefficient, E is called the *coefficient of elasticity.* From the third form of equation (6) it is evident that for any unit stress or force, within the field of perfect elasticity, we may determine the corresponding elongation for any material whose value of E is known; it is therefore also called the *modulus of elasticity.*

From the first form of equation (6) it is seen that since λ is merely an abstract fraction, E must be expressed in the same units as s_e, or in pounds per square inch.

The coefficient, or modulus, of tensile elasticity of any material is the force or stress obtained by dividing any unit tensile force or unit stress, not exceeding its limit of tensile elasticity, by the corresponding unit elongation.

Stress-strain Curve.—If from any origin of rectangular coordinates we lay off on the axis of ordinates the values of the successive tensile forces applied to a steel rod, and on the axis of abscissas the corresponding elongations, the resulting curve will be a *stress-strain curve.*

If the curve is plotted automatically by a testing-machine, it will be of the general form shown in Fig. 2. Within the field of perfect elasticity it will be a right line, since its equation is

$F=al$, in which a is constant. Beyond the field of perfect elasticity a becomes variable and the equation is no longer that of a right line.

The ordinates of the curve will increase until rupture takes place. The point of rupture of a ductile metal, like structural steel, is marked by the running of the metal and a decrease of cross-section at the point of rupture. This is indicated on the stress-strain curve plotted automatically by a testing-machine, by a dropping of the curve towards the axis of elongations, and by the final tearing apart of the decreased section by a force less than the maximum applied force. If the material is not ductile, the points of rupture and final separation are coincident.

If the plotted ordinates are the successive applied forces divided by the original area of cross-section, or $\frac{F}{A}$, and the original length of the rod is assumed as 100 units, and the plotted abscissas are $\frac{l}{L}$, the resulting curve is that shown in Fig. 2. The

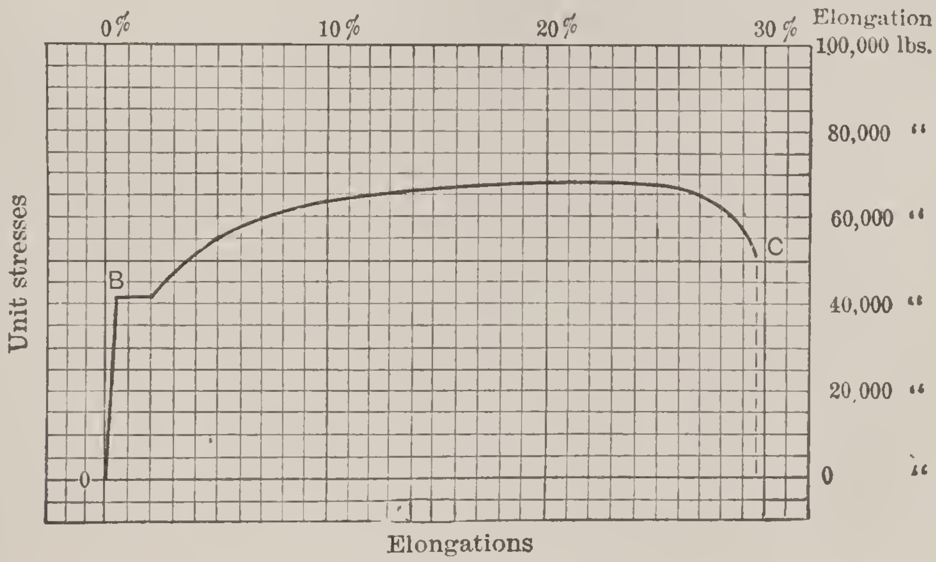

FIG. 2.

ordinates are unit forces or stresses, from the origin to the point of separation, and the abscissas are unit elongations or strains.

The point B of the curve where the elongations suddenly increase is called the *yield-point*. The ordinate of the point just below the yield-point, where the straight portion of the stress-strain curve ceases, is the limit of elasticity.

Modulus of Tenacity.—If the load on the one-inch rod of structural steel, page 9, is increased until it reaches about 70,000 pounds, the rod will tear apart and the weight will drop to the floor. This indicates that a unit stress of 70,000 pounds measures the tensile strength of this metal; it is therefore called the *modulus of tenacity, or the ultimate strength of the material in tension.* It will be represented by s_e'.

The modulus of tenacity of any material is the unit tensile force applied to, or the unit tensile stress developed in, a rod of the material at the moment of rupture. In the stress-strain curve shown in Fig. 2 it is the ordinate of the highest point of the curve.

Compressive Stress.—If a solid steel column whose cross-section is a circle or a regular polygon, and whose length does not exceed ten times its least dimension of cross-section, is placed in a vertical position and loaded, it will be observed that—

1. All the fibers between the support and load are shortened.
2. Every cross-section normal to the axis of the column before the weight is applied remains normal to the axis after the weight is applied.
3. That these effects are observed whatever be the regular form of the cross-section.

It follows, therefore, that the stress on each cross-section must be uniform and hence on each unit of area the same.

Let F = total applied compressive force, or total compressive stress in pounds, on any cross-section of a short column;

A = area of cross-section in square inches;

s_c = compressive force or stress in pounds per square inch. Then

$$s_c = \frac{F}{A} \quad \ldots \ldots \ldots \quad (7)$$

for the unit stress of compression. The law of distribution of compressive stress in short columns as expressed in equation (7) applies to all materials.

Limit of Compressive Elasticity.—*The limit of compressive elasticity* may be determined in a manner similar to that of tensile elasticity and is *the greatest unit compressive force which may be applied to, or the greatest unit compressive stress which may be developed in, a short column of the material without producing a permanent set.*

Coefficient, or Modulus, of Compressive Elasticity.—Let

F = the compressive force applied to, or the compressive stress developed in, the cross-section of a short column, expressed in pounds;

A = area of cross-section in square inches;

L = original length of the column in inches;

l = the amount of shortening in inches.

Then, as under the action of a tensile force, we shall have

$\frac{F}{A} = s_c$, and $\frac{l}{L} = \lambda$,

$$E = \frac{s_c}{\lambda}, \quad s_c = E\lambda, \quad \text{and} \quad \lambda = \frac{s_c}{E}, \quad . \quad . \quad . \quad . \quad (8)$$

in which E is the coefficient or modulus of compressive elasticity.

This coefficient may be defined as *the force or stress obtained by dividing any unit compressive force or stress, not exceeding the limit of elasticity of the material considered, by the corresponding unit shortening of the column to which it is applied.*

Stress-strain Curve.—A stress-strain curve may be constructed for compression in the same manner as that for tension. When both curves for the same material are plotted on the same diagram, it is usual to lay off compressive forces from the origin downwards and shortenings to the left. The tensile curve is then in the first, and the compressive curve in the third, quadrant.

Limit and Coefficient of Longitudinal Elasticity.—It is found by experiment that the limits and coefficients of elasticity for tension and compression of ordinary structural materials differ so little from each other that common values of these constants may be employed in all practical problems. These common values for any material are called its *limit* and its *coefficient of longitudinal elasticity.*

Modulus of Crushing.—The *modulus of crushing* or *ultimate strength in compression* is determined in a manner similar to that employed in determining the modulus of tenacity and is *the unit stress which is developed in a column of material at the moment of rupture, or the unit force which produces rupture.* The column must be so short that the material is crushed without bending its fibers. If of steel, a cylindrical column will give way by crushing if its diameter is at least one-tenth its length; a wooden column must have a diameter at least one-fifth its length. The modulus of crushing will be represented by s_c'.

Allowable or Safe Unit Stress.—*The allowable or safe unit tensile stress* is the greatest unit stress to which it is deemed advisable to subject a material. For all materials it must be less than the modulus of tenacity, and for ductile metals it is usually less than the limit of longitudinal elasticity. In compression the allowable or safe unit stress must be less than the modulus of crushing, and for metals it is usually less than the limit of longitudinal elasticity. The safe unit stress in tension or elongation will be represented by s_e'' and in compression by s_c''.

The following table gives general values of the constants above described, in pounds per square inch, for ordinary building materials. They vary, however, considerably for different varieties of the same material.

	Wood.	Stone.	Cast Iron.	Wrought Iron.	Steel.
Limit of longitudinal elasticity.	3,000	....	6,000	25,000	35,000
Coefficient of longitudinal elasticity.	1,500,000	1,000,000	15,000,000	27,000,000	30,000,000
Modulus of tenacity.	10,000	500	20,000	50,000	75,000
Safe unit stress in tension.	1,000	50	3,000	10,000	12,000
Modulus of crushing.	6,000	5,000	80,000	50,000	50,000
Safe unit stress in compression.	800	600	10,000	10,000	12,000

Third Operation.—The third operation in structural design consists in determining the dimensions of the elementary pieces of a structure when the forces which it is required to resist are fully determined, and the constants of the material are known.

If the piece is a rod which must resist a tensile force, or a short column which must resist a compressive force, the operation is usually that of finding the minimum area of cross-section which will safely resist the applied force.

Let F = the applied tensile or compressive force, in pounds;
A = the required area of cross-section in square inches;
s_e'' = safe unit stress in tension;
s_c'' = safe unit stress in compression. Then

$$A = \frac{F}{s_e''} \quad \text{or} \quad \frac{F}{s_c''}. \qquad (9)$$

From these equations we may determine any one of the quantities when all the others are given.

It is also sometimes necessary to know the amount of elongation or of shortening which will occur in a rod or column under the action of a force which will produce a unit stress not exceeding the limit of elasticity.

Let F = applied force in pounds;
A = area of cross-section in square inches;
L = length of rod or pillar in inches;
l = elongation or shortening in inches;
E = coefficient of elasticity. Then

$$l = \frac{FL}{EA}.$$

From this equation we may determine any one of the five quantities when the others are known.

The values of the constants to be used in solving problems are those given in the tables. The weight of the piece itself will be omitted unless specifically stated otherwise.

PROBLEMS.

1. Determine the diameter of a standard* circular wrought-iron rod which will safely support a weight of 100,000 pounds.
Ans. $3\frac{5}{8}$ inches.

2. What load may be safely borne by a standard* wrought-iron rod $\frac{7}{8}$ inch in diameter? *Ans.* 6013 pounds.

3. How much will a hundred-foot steel tape, $\frac{1}{2}$ inch wide and $\frac{1}{50}$ inch thick, stretch under a pull of 50 pounds? *Ans.* 0.2 inch.

4. Find the coefficient of elasticity of wrought iron of which a rod 1 inch in diameter† and 11 feet long will stretch $\frac{1}{8}$ inch under a tensile force of 20,000 pounds. *Ans.* 26,890,756.

Work Performed by a Longitudinal Force.—The force applied to elongate a rod or compress a column may be either a *gradually* or a *suddenly applied force.* A gradually applied force is one whose intensity gradually increases from zero to a maximum; a suddenly applied force is one whose intensity is constant.

Work of Gradually Applied Force.—The expression for the work done by a force of variable intensity, as a gradually applied force, is

$$\text{work} = \int F\partial l, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

in which F = variable intensity of the force;
l = path of the force.

If F is a longitudinal force, and l is its strain, equation (10) gives the work of elongation or compression. If a stress-strain curve is constructed from corresponding values of F and l, the expression $\int F\partial l$ will measure the area included between the curve itself, the axis of elongations or abscissas, and the ordinate of the point corresponding to the maximum strain considered. Thus in Fig. 2 the area included between the stress-strain curve, the axis of elongations, and the ordinate of the highest point of the curve measures the work done in rupturing a rod of steel of a unit cross-section. The work neces-

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* Cambria Handbook, pp. 383–389.
† *Ibid.*, p. 407, areas of circles.

sary to tear the rod apart is measured by the area included between the curve, the axis of abscissas, and the ordinate of the point C. If L in Fig. 2 is assumed as 100 inches, each small rectangle will represent 5000 inch-pounds. It will be observed that the work required to stretch the rod to its elastic limit is but a small fraction of the work of rupture, although the modulus of rupture is only double the elastic limit.

Suddenly Applied Force.—The expression for the work done by a force of constant intensity, as a suddenly applied force, is

$$\text{work} = F\int \partial l, \qquad \qquad (11)$$

in which F = intensity of force;
l = path of force.

If the force is a longitudinal one, equation (11) gives the total work of elongation or compression.

In Fig. 2 the work performed by a suddenly applied force is represented by the rectangle whose height is the intensity of the unit force and whose length is the temporary elongation produced by it.

Relation between Work, Strains, and Stresses of Suddenly and Gradually Applied Forces.—If a pound weight is placed in a scale-pan of a spring balance and the pan finally comes to rest one inch below its original position, the *final work* performed will be one inch-pound; the *temporary work* will be one pound multiplied by the total distance the pan temporarily descends. If a pound of sand is poured into the same scale-pan grain by grain, the final and temporary work will be the same, and each equal to one inch-pound. Therefore the *final work* of a suddenly applied and a gradually applied force of the same intensity will be equal; but the *temporary work* of a suddenly applied force will be greater than that of a gradually applied one of equal intensity.

From equation (10) we may determine the work performed by a gradually applied force by substituting for F its value in terms of l from equation (5), and integrating between proper limits; or

$$\left.\begin{array}{l}\text{temporary work}\\ \text{final work}\end{array}\right\} = \int_0^{l'} F\delta l = \frac{EA}{L}\int_0^{l'} l\delta l = \frac{EAl'^2}{2L} = \frac{F'l'}{2}. \qquad (12)$$

$\frac{F'l'}{2}$ is the temporary and also the final work performed by the gradually applied force whose maximum intensity is F', in producing a strain whose maximum value is l'.

From equation (11) we may determine the *temporary work* performed by an equal suddenly applied force F', in producing the temporary strain l', by making $F=F'$ and $l=l'$; thus,

$$\text{temporary work} = F'l'. \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The *final work* performed by the suddenly applied force F' must be equal to that of the gradually applied force, or

$$\text{final work} = \frac{F'l'}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

From equations (12) and (13) it follows that—

1. *The temporary work performed by a suddenly applied longitudinal force, within the field of perfect elasticity, is twice that performed by a gradually applied load of the same maximum intensity.*

If the work of a gradually applied force is equal to that of a suddenly applied one, this can result only by doubling either the intensity of the force or its strain. From equations (12) and (13) we have

$$\text{temporary work} = \frac{(2F')l'}{2} = \frac{F'(2l')}{2} = F'l'. \quad . \quad . \quad (15)$$

Hence—

2. *A suddenly applied load will, within the field of perfect elasticity, produce a temporary strain as great as the permanent strain produced by a gradually applied force of double its intensity.*

3. *Since within the field of perfect elasticity the unit stress varies directly with the unit strain, it follows that if the unit stress is not to exceed the limit of longitudinal elasticity or a permanent*

set is not allowable, the unit intensity of a suddenly applied force must not exceed one-half the limit of elasticity. This results from the definition of the limit of longitudinal elasticity.

This last conclusion is important in determining the safe unit stress on bridge members which must be designed to resist suddenly applied loads.

In equation (12) the work of a gradually applied load is expressed in terms of l', E, A, and L; in some problems it is convenient to have it in terms of F', E, A, and L; this may be done by substituting the value of l' in terms of F' from equation (5), or

$$\text{work of gradually applied force} = \frac{F'^2 L}{2EA}. \quad . \quad . \quad (16)$$

PROBLEMS.

5. The work expended by a gradually applied force in elongating a 1-inch square wrought-iron rod 25 feet long is 100 foot-pounds. What is the intensity of the force applied and the elongation produced? *Ans.* Force 14,697 pounds.
Strain 0.167 inch.

6. A standard circular steel rod is required to support a suddenly applied load of 20,000 pounds; what is the minimum diameter of such a rod if a permanent set is to be avoided?
Ans. $1\frac{1}{4}$ inches.

Resilience.—The work expended by a longitudinal force in straining a rod or column within the field of perfect elasticity is equal to the potential energy of elasticity thus stored in it. This potential energy of elasticity is called its *longitudinal resilience.*

It may be derived from the equation

$$\text{resilience} = \text{work} = \int F\delta l = \frac{EA\int l\delta l}{L}. \quad . \quad . \quad . \quad (17)$$

by integrating between the limits of zero and any value l' less than that corresponding to the limit of elasticity; or

$$\text{resilience} = \frac{EAl'^2}{2L}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Since s_c or $s_e=\frac{F'}{A}=\frac{El'}{L}$, we have

$$s_e^2=\frac{E^2l'^2}{L^2}. \qquad (19)$$

Multiplying the expression $\frac{EAl'^2}{2L}$ by $\frac{EL}{EL}$, and substituting in the resulting equation, s_e^2 for $\frac{E^2l'^2}{L^2}$, we have

$$\text{resilience}=\frac{s_e^2AL}{2E}, \qquad (20)$$

or $$\frac{\text{resilience}}{AL}=\text{resilience in a unit of volume}=\frac{s_e^2}{2E}. \qquad (21)$$

The expression $\frac{s_e^2}{2E}$ becomes the *modulus of longitudinal elastic resilience* when s_e is increased until it becomes equal to the limit of longitudinal elasticity.

The modulus of longitudinal elastic resilience of any material measures the capacity of a rod or column of the material, in comparison with similar rods or columns of other materials, to store the work expended on it, within the field of perfect elasticity, by a longitudinal force. It is obtained by dividing the square of the limit of elasticity of the material by twice its coefficient of longitudinal elasticity.

PROBLEMS.

7. What potential energy in foot-pounds is stored in a steel rod 3 inches in diameter and 10 feet long by increasing its unit tensile stress from 10,000 to 20,000 pounds?

Ans. 353.6 foot-pounds.

8. In increasing the unit tensile stress in a wrought-iron rod from 10,000 to 20,000 pounds, 325 foot-pounds of work are expended. If the diameter of the rod is 4 inches, what is its length?

Ans. 4.6 feet.

Stress Due to Impact Force.—An *impact force* is the striking force of a moving body. If a weight W falls through a distance h before it begins to strain a rod or column, the total amount of work performed before it ceases its motion downwards will be

$$\text{work} = W(h+l'), \quad \ldots \ldots \quad (22)$$

in which W = weight of W in pounds;
h = distance of fall in inches;
l' = amount of strain in inches.

If the strain is within the field of perfect elasticity, the work of the weight will be equal to the resilience of the rod or column, or

$$\text{work} = W(h+l') = \frac{F'l'}{2}, \quad \ldots \ldots \quad (23)$$

in which F' is the intensity of a gradually applied force which produces a permanent strain, l', which is equal to the temporary strain, l', produced by the falling weight.

If we substitute for F' its values $\frac{EAl'}{L}$ and s_eA, in which s_e = unit tensile stress corresponding to F' and l', A = area of cross-section, we shall have

$$W(h+l') = \frac{F'l'}{2} = \frac{EAl'^2}{2L} = \frac{s_eAl'}{2}. \quad \ldots \ldots \quad (24)$$

From these equations we can determine any one of the quantities when all the others are known.

If we transform the expression $\frac{EAl'^2}{2L}$ as in the discussion in resilience, we shall have

$$W(h+l') = \frac{s_e^2AL}{2E}. \quad \ldots \ldots \quad (25)$$

If in this expression we make $\frac{s_e^2}{2E}$ equal to the modulus of longitudinal elastic resilience, l' equal to the strain corresponding to the limit of elasticity, and give W, A, and L their values, we may determine the greatest height, h, from which W may be dropped without producing a permanent set in the rod.

Ratio of Stress and Strain of Impact and Gradually Applied Forces.—From equation (23) we have, making $W=F$,

$$\frac{F'}{W}=\frac{F'}{F}=\frac{2(h+l')}{l'}. \qquad (26)$$

If we represent by l the permanent strain produced by W or F acting as a gradually applied force, we shall have

$$F':F::l':l, \quad \text{or} \quad l'=\frac{F'l}{F}. \qquad (27)$$

This follows from the fact that within the field of perfect elasticity the strains are proportional to the gradually applied forces or stresses.

Substituting this value of l' in equation (26) we have

$$\frac{F'}{F}=\frac{2h+\frac{2F'l}{F}}{\frac{F'l}{F}}=\frac{2Fh+2F'l}{F'l}, \qquad (28)$$

$$F'^2l=2F^2h+2F'Fl,$$

$$F'^2-2F'F=\frac{2F^2h}{l},$$

$$F'^2-2F'F+F^2=\frac{2F^2h}{l}+F^2, \qquad (29)$$

$$F'=F\left(1\pm\sqrt{\frac{2h}{l}+1}\right),$$

$$\frac{F'}{F}=1\pm\sqrt{\frac{2h}{l}+1}, \qquad (30)$$

in which $\frac{F'}{F}$ is the ratio of the *stress* produced by the weight W falling through the height h, to the stress produced by the weight

W acting as a gradually applied load. From equation (27) it is seen that this is also the ratio of the *strains* produced by the impact force W and the gradually applied force W.

PROBLEMS.

9. A 1000-pound weight falls 2 inches before commencing to stretch the steel rod by which it is supported. This rod is 3 inches in diameter and 10 feet long. Find its elongation due to the falling weight. *Ans.* 0.048 inch.

10. A steel rod 10 feet long and 1 inch square is to support a weight of 400 pounds which falls through a height h before it acts on the rod. Find the maximum value of h which will produce no permanent set in the rod. *Ans.* 5.98 inches.

Application of the Principles of Longitudinal Stress.

Elongation of a Rod of Uniform Cross-section.—In Fig. 3 let the origin of coordinates be at B, the axis of Y vertical, and let D be any section at a distance y from B. Let it be required to determine the elongation of the rod due to its own weight, within the field of perfect elasticity.

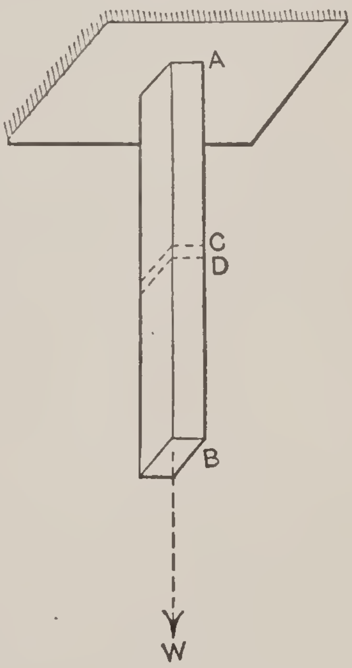

Fig. 3.

Let $DC = \partial y$ = length of an elementary portion of the rod, as CD;

w = weight of a cubic inch of the rod in pounds;

A = area of cross-section in square inches;

L = original length of the rod in inches;

l = elongation of the rod in inches;

F = tensile force in pounds on the area of cross-section at D. Then

Ay = volume of rod between B and D,

Awy = weight of rod between B and D,

$F = Awy$ = force applied to cross-section D.

Substituting in equation (5), transformed,

$$l = \frac{FL}{EA}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

for F its value above given, Awy; and for L, δy; we have for the elongation of the elementary portion of the rod at D

$$\frac{Awy\delta y}{EA}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

To find the elongation of the entire rod due to its own weight we must integrate this expression between the limits $y=0$ and $y=L$. This gives for the elongation of the entire rod

$$l = \frac{\frac{1}{2}wL^2A}{AE} = \frac{wL^2}{2E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

If a weight wAL, the weight of the entire rod, is applied to the bottom of the rod, and the weight of the rod itself is neglected, we shall have for every cross-section $F=wAL$. Substituting, in the equation $l=\frac{FL}{EA}$, for F its value wAL, we have for the elongation of the entire rod within the field of perfect elasticity

$$l = \frac{wL^2}{E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Comparing equations (33) and 34), we see that *the elongation of a rod of uniform cross-section due to its own weight is just one-half the elongation of the same rod caused by suspending from its lower end a weight equal to the weight of the rod, and neglecting the weight of the rod itself.*

The above discussion is equally applicable to the shortening of a vertical column under its own and a superimposed weight, within the field of perfect elasticity.

PROBLEM.

11. Determine the elongation of a vertical steel rod 1 inch in diameter and 50 feet long, under its own weight and a weight of 20,000 pounds suspended from its lower extremity.

Ans. 0.51 inch.

Determination of Form of a Rod of Uniform Strength.—By a rod of uniform strength is meant one which *has the same unit stress at every cross-section.*

Let s_e = unit tensile stress at any cross-section of rod, in pounds;
F = applied tensile force in pounds;
A = area of cross-section of rod in square inches.

Then in the general expression

$$s_e = \frac{F}{A}, \qquad (35)$$

s_e must be constant, and A must vary directly with F.

Illustration.—*A vertical rod is firmly fastened at its upper end and supports a weight at its lower end; let it be required to find the area of cross-section at any point of the rod if the weight of the rod is considered.*

Let the axis of the rod shown in Fig. 4 be the axis of Y and the origin of coordinates at bottom.

Let W = weight suspended from bottom of rod in pounds;
w = weight of cubic inch of rod in pounds;
L = length of rod in inches;
y = distance in inches of any cross-section, as C, from bottom;
A = area of the cross-section C in square inches;
A' = area of the bottom cross-section B in square inches;
A'' = area of top cross-section A in square inches;

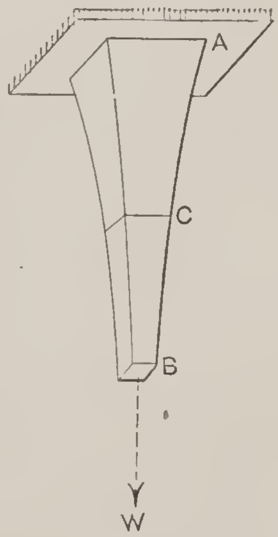

FIG. 4.

$W + w\int A\,\partial y$ = total tensile stress on the cross-section C in pounds;
s_e'' = allowable unit stress in pounds on any cross-section.

Then

$$W+w\int A\,\partial y=s_e''A. \quad \ldots\ldots \quad (36)$$

In this equation A and y are the only variables; if we find an expression for A in terms of y, by assuming values of y between zero and L, we may determine the corresponding values of A and thus determine the dimensions of the rod.

Differentiating the equation

$$W+w\int A\,\partial y=s_e''A \quad \ldots\ldots \quad (37)$$

with respect to A, we have

$$wA\,\partial y=s_e''\partial A. \quad \ldots\ldots \quad (38)$$

Transposing we have

$$\frac{w\partial y}{s_e''}=\frac{\partial A}{A}. \quad \ldots\ldots \quad (39)$$

Integrating we have

$$\frac{wy}{s_e''}=\text{Nap. log } A+C. \quad \ldots\ldots \quad (40)$$

Making $y=0$, in which case A becomes equal to A', we have

$$C=-\text{Nap. log } A'. \quad \ldots\ldots \quad (41)$$

Substituting the value of C in (40) we have

$$\frac{wy}{s_e''}=\text{Nap. log } A-\text{Nap. log } A'=\text{Nap. log }\frac{A}{A'}. \quad \ldots \quad (42)$$

Passing to equivalent numbers we have

$$A=A'\mathrm{e}^{\frac{wy}{s_e''}}, \quad \ldots\ldots \quad (43)$$

in which e is the base of the Naperian system, or 2.718+.

Since the only force acting in the cross-section at the bottom is W, we must have

$$s_e''=\frac{W}{A'}, \quad \text{or} \quad A'=\frac{W}{s_e''}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (44)$$

Substituting this value of A' in (43) we have

$$A=\frac{W e^{\frac{wy}{s_e''}}}{s_e''}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (45)$$

in which A is given in terms of y and known quantities.

Making $y=L$, in which case A becomes equal to A'', we have

$$A''=\frac{W e^{\frac{wL}{s_e''}}}{s_e''}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (46)$$

Since the tensile force acting at the base is $W=s_e''A'$ and the tensile force acting at the top is

$$W+\text{weight of rod}=s_e''A'', \quad . \quad . \quad . \quad . \quad . \qquad (47)$$

the weight of the rod must be $s_e''(A''-A')$.

Form of Rod of Circular Cross-section.—No particular form has been assigned to the cross-section of the rod. Assume it to be circular and let

r = radius of a section at a distance y from the bottom;
r' = radius of bottom section;
r'' = radius of top section.

Then

$$A=\pi r^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (48)$$

$$A'=\pi r'^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (49)$$

Substituting these values in equation (43) we have

$$\pi r^2=\pi r'^2 e^{\frac{wy}{s_e''}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (50)$$

Cancelling π and taking the logarithm of both members,

$$\text{Nap. log } r = \text{Nap. log } r' + \frac{wy}{2s_e''}. \quad . \quad . \quad . \quad . \quad (51)$$

Making $r=r''$ and $y=L$ we have

$$\text{Nap. log } r'' = \text{Nap. log } r' + \frac{wL}{2s_e''}. \quad . \quad . \quad . \quad . \quad (52)$$

In these equations we may substitute the common logarithm of r, r', and r'' by applying the formula $\dfrac{\log a}{0.43429} = \text{Nap. log } a$.

Equation (51) is the equation of a line whose rectangular coordinates are the values of r and y. It is the line cut out of the surface of the rod by a plane through its axis.

The discussion above given applies equally well to a column of uniform strength to resist its own weight and a superimposed load.

PROBLEM.

12. A wrought-iron circular rod of uniform strength, 300 feet long, suspended vertically, supports its own weight and a weight of 50,000 pounds attached to its lower extremity. Determine the dimensions of the upper and lower cross-sections of the rod, and its weight.

Ans. $A' = 5$ square inches; $A'' = 5.524$ square inches.
Weight = 5240 pounds.

Determination of Thickness of Thin Cylinders and Spheres to Resist Internal Pressure.—The internal pressure of steam and water in pipes and spheres is usually considered as uniformly distributed over the surface, and the stress developed by the pressure to be uniformly distributed over the area of the section of rupture.

Pipes.—If we consider a cylindrical water-pipe to be divided into two equal parts by a plane through its axis, it is evident that the resultant water pressure on one half will be equal to and directly opposed to the resultant pressure in the other half. The tendency of these equal internal pressures is to separate the pipe into two equal parts, by tearing it along two rectilinear elements at opposite extremities of the same diameter.

Since, in each half-cylinder, the components of the normal pressure which are parallel to the plane of separation neutralize each other, the resultant pressure normal to the plane of separation is equal to the unit pressure on the surface, multiplied by the area of the plane included between the rectilinear elements of rupture, or the plane whose width is the interior diameter of the pipe and whose length is the length of the pipe.

Hence if p = internal pressure on a square inch of the pipe in pounds,

l = length of the pipe in inches,

d = interior diameter of the pipe in inches,

the two opposing resultant pressures normal to the plane of separation are each pdl.

The tendency of the pipe to rupture is resisted by the tensile strength of the material in the plane of separation.

If t = thickness of pipe in inches,

$2tl$ = area of metal in the plane of separation in square inches,

$2s_e tl$ = tensile stress on this area in pounds.

For safety the total allowable stress in the metal must be equal to or greater than the normal pressure, or

$$2s_e''tl = \text{or} > pdl, \qquad (53)$$

in which s_e'' = allowable or safe unit tensile stress of the metal.

To find the minimum allowable thickness of metal, we have, therefore,

$$t = \frac{pd}{2s_e''} \qquad (54)$$

The allowable stress in water-pipes must be small because of the shocks, or *water-hammer*, due to the sudden stoppage of the flow of water in pipes when valves have been opened and are suddenly closed.

Spheres.—A hollow sphere subjected to internal pressure tends to rupture along the circumference of a great circle.

If p = internal pressure in pounds on the square inch of surface,
r = radius of sphere in inches,
the resultant pressure in each hemisphere normal to the plane of separation is $p\pi r^2$.

The resistance of the metal in the plane of separation is the resistance of the ring of metal included between two concentric circles whose radii are r and $r+t$. The stress in the metal is therefore $$s_e\pi[(r+t)^2-r^2]=s_e\pi(2rt+t^2),$$
in which s_e is the unit stress.

The least allowable thickness of metal is the value derived from the solution of the equation

$$s_e''\pi(2rt+t^2)=p\pi r^2. \qquad (55)$$

If t is very small in comparison with r, the second term in the parenthesis may be omitted. Hence

$$2s_e''t=pr. \qquad (56)$$

The error thus committed is on the side of safety.

In the discussions above, it is assumed that the stress due to interior pressure is uniform over the section of metal made by the plane of separation. This is not true when the metal is very thick, but is sensibly true for ordinary steam- and water-pipes.

PROBLEMS.

13. What should be the thickness of metal of an 8-inch cast-iron water-main to resist a water pressure of 300 pounds?

Ans. 0.5 inch.

14. A force of 500 pounds is applied to the piston-head of a forcing-pump which communicates its pressure to a hollow sphere 10 inches in diameter. If the diameter of the piston-head is 1 inch, what should be the thickness of the metal of the sphere?

Ans. 0.4 inch.

Longitudinal Stresses Due to Changes of Temperature.—All materials employed in engineering practice expand and contract with changes of temperature. Within the field of perfect elasticity, the elongations and contractions are directly proportional to the number of degrees of change. For each material, therefore, there is a constant, j', which represents its change per unit of length for each degree Fahrenheit from absolute zero temperature. If the ends of a rod are firmly fixed and it is then exposed to a change of temperature, it will be subjected to a longitudinal stress corresponding to the change of length it would have undergone had the ends been free to move. Representing the length of the bar by L, its coefficient of linear expansion per degree Fahrenheit by j', the number of degrees change by n, and the total elongation due to n degrees by l, then

$$l=j'nL, \text{ and the unit elongation is } \frac{l}{L}=j'n. \quad . \quad . \quad . \qquad (57)$$

Since the unit stress is equal to the modulus of elasticity multiplied by the unit elongation, there results

$$s_e \text{ or } s_c=\frac{El}{L}=Ej'n, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (58)$$

and the total stress in any cross-section

$$s_eA \text{ or } s_cA=AEj'n, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (59)$$

in which A is expressed in square inches. The values of j' for ordinary building materials are:

Stone	0.0000055
Cast iron	0.0000062
Wrought iron	0.0000067
Steel	0.0000067

PROBLEM.

15. A wrought-iron bar, 2 square inches in cross-section, has its ends fixed immovably between two blocks when the temperature is at 60° F. What pressure will be exerted on the blocks when the temperature is at 100° F.? *Ans.* 14,472 pounds.

CHAPTER II.

SHEARING AND TORSIONAL STRESS.

Simple Shearing Force.—If two thin flat bars of steel are laid one upon the other and thus fastened by rivets, any force applied to the bars to slide one along the other will be a simple shearing force as applied to the rivets.

It is assumed from experiments—

1. That all the fibers of each rivet suffer the same lateral distortion.

2. That the cross-sections of the rivets which lie in the planes of the adjacent faces of the two plates move laterally in their own planes.

3. That these effects are observed whatever be the form of the cross-section.

It follows, therefore, since the material is homogeneous and all fibers offer the same resistance, that the stress on the cross-section is uniformly distributed.

If F = total force in pounds applied to distort the cross-section,
A = area of cross-section in square inches,
s_s = shearing stress per square inch,

then

$$s_s = \frac{F}{A}, \quad \ldots \ldots \ldots \quad (60)$$

in which s_s is the *unit stress of shear* and equation (60) expresses the law of distribution of the stress over the plane of cross-section.

The limit of shearing elasticity of a material is the greatest unit shearing force which may be applied to, or the greatest unit stress which may be developed in, the material without producing a permanent set. Unlike the limit of longitudinal elasticity, it cannot be directly measured, because of the impossibility of accurately measuring the distortions.

The coefficient of shearing elasticity of any material is the force or stress obtained by dividing any unit force or unit stress, not exceeding the limit of shearing elasticity of the material considered, by the unit distortion produced by it. As the distortion cannot be accurately measured, the coefficient, like the limit, cannot be accurately determined. It is usually assumed to be equal to the coefficient of torsional elasticity.

The modulus of shearing, or the ultimate strength in shearing, is the unit shearing force or unit shearing stress at the moment of rupture. This may be obtained from equation (60) by increasing F until rupture takes place. The value of s_s at the moment of rupture is the modulus of shearing. It will be represented by s_s'.

The allowable or safe unit stress in simple shear is obtained by dividing the modulus of shearing by a suitable factor of safety. It will be represented by s_s''.

Constants in Pounds.	Wood with Grain.	Wood across Grain.	Cast Iron.	Wrought Iron.	Steel.
Modulus of shearing, s_s'. . . .	600	3,000	20,000	40,000	50,000
Safe unit stress, s_s''.	150	1,000	3,000	9,000	12,000

Designing.—The ordinary problem of design is, as under tension and compression, to determine the area of cross-section of a piece to resist a given shearing force.

Let F = the applied shearing force in pounds;
A = required area of cross-section in square inches;
s_s'' = safe unit stress in simple shear. Then

$$A = \frac{F}{s_s''} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

From this equation any one of the quantities can be determined when all the others are known.

PROBLEMS.

16. What must be the least diameter of a steel bolt which is to resist a simple shearing force of 30,000 pounds?

Ans. 1.78 inches.

17. What pressure is required to punch a hole $\frac{3}{4}$ inch in diameter in a wrought-iron plate $\frac{1}{4}$ inch thick? *Ans.* 29,452 pounds.

18. What is the safe shearing load of a wrought-iron rivet $\frac{1}{2}$ inch in diameter? *Ans.* 1768 pounds.

Shear in a Rod or Column.—If a rod, as in Fig. 5, is suspended from a ceiling and supports at its bottom a force F, this force will produce tension in every horizontal cross-section of the rod. If, however, we assume any oblique section, as that shown in the figure, the force acting on this section may be divided into two components; one is perpendicular to the section and tends to tear the rod apart along the section, and the other is parallel to the section and tends to shear it off at the section.

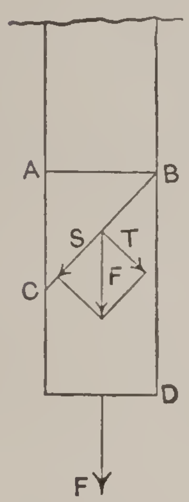

FIG. 5.

Let A = area of horizontal cross-section, AB;
A' = area of oblique cross-section, BC;
ϕ = angle ACB;
F = force applied to end of rod;
T = component of F perpendicular to BC;
S = component of F parallel to BC;
$\frac{T}{A'}$ = unit tensile force perpendicular to BC;
$\frac{S}{A'}$ = unit shearing force parallel to BC.

The weight of the rod itself is not considered.

It is evident that the rod will have the greatest tendency to tear apart along the plane on which $\frac{T}{A'}$ is greatest, and the greatest tendency to shear off along the plane on which $\frac{S}{A'}$ is greatest.

From the figure $T = F \sin \phi$, and $A' = \frac{A}{\sin \phi}$; hence $\frac{T}{A'} = \frac{F}{A} \sin^2 \phi$. This is a maximum when $\sin \phi$ is a maximum, or ϕ is 90°. The unit tensile stress is, therefore, greater on a horizontal plane of cross-section than on any oblique plane. From the figure $S = F \cos \phi$ and $A' = \frac{A}{\sin \phi}$; hence $\frac{S}{A'} = \frac{F \sin \phi \cos \phi}{A}$. S will therefore be a maximum when $\cos \phi = \sin \phi$, or $\phi = 45°$.

The unit shearing stress on oblique planes is therefore greatest on the oblique plane, making an angle of 45° with the axis.

Since the sine of 90° is unity, and the sine and cosine of 45° are both $\sqrt{0.5}$, the maximum unit stress of tension on the horizontal cross-section is $\frac{F}{A}$, and the maximum shearing stress on the oblique cross-section is $0.5\frac{F}{A}$. The rod will therefore rupture by tension along a horizontal cross-section unless the modulus of shearing is less than half the modulus of tension. If the grain of a wooden rod is not parallel to its axis, it may rupture along an oblique section.

Shearing Stress in Plane Perpendicular to Plane of Cross-section.—Hitherto we have considered only the shearing stress in the plane of cross-section. If we study the deformation produced by a shearing force, it will be seen that there is also a shearing stress developed in planes normal to the first and normal to the action line of the shearing force.

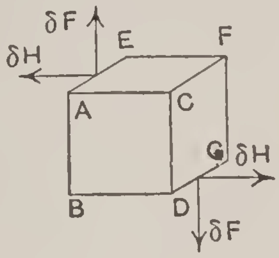

FIG. 6.

In Fig. 6 let $ABCDEFG$ be an elementary molecule in the interior of the rivet subjected to simple shear. The faces BAE and $DCFG$ are parallel to the plates united by the rivet and hence parallel to the applied shearing force. Let

$\delta d = AB$, the depth of the molecule;
$\delta b = AE$, " width " "
$\delta l = AC$, " length " "

δH and δH = elementary shearing force developed on faces $AEFC$ and BDG;

δF and δF = elementary shearing force applied to the face $CDGF$ and the resistance developed by it in the face ABE attached to its adjacent molecule;

$(s_s)_1$ = unit stress on each face $\delta d\delta b$;

$(s_s)_1\delta d\delta b$ = total stress on each face $\delta d\delta b$.

Hence

$$\delta F = (s_s)_1\delta d\delta b.$$

The effect of the force δF is to move the face on which it acts, in its own plane, and to increase the angles of the elementary molecule at B and C, and decrease those at A and D. This will cause the planes $AEFC$ and BDG to slide on the corresponding faces of the molecules above and below and thus produce a shear in the upper and lower faces of the elementary molecule. Since the molecule is at rest and the vertical shearing forces form a couple, the horizontal shearing forces must also form a couple.

Let $(s_s)_2$ = unit shear on the upper face.

Then $(s_s)_2 \partial b \partial l$ = total shear on the upper face,

and $$\partial H = (s_s)_2 \partial b \partial l.$$

The same expression represents the equal shear on the lower face which has the same area.

There will be no shear on the faces $ABCD$ and EFG, since the molecules on either side are each subjected to a couple equal to ∂F, ∂F.

The molecule $ABCDEFG$ being in a state of rest under the action of the two couples ∂F and ∂F, ∂H and ∂H, the moments of these couples must be numerically equal. Hence

$$\partial F \times \partial l = \partial H \times \partial d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

Substituting for ∂F and ∂H their values we have

$$(s_s)_1 \partial b \partial d \partial l = (s_s)_2 \partial b \partial l \partial d. \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

Hence $(s_s)_1 = (s_s)_2$, or the *unit shearing stresses on the vertical and horizontal faces of the elementary molecule are equal.*

Torsion.

Torsional Moment.—Any force whose action line lies in a plane normal to the axis of a shaft and does not intersect that axis, will produce torsional stress in the shaft, unless the shaft is absolutely free to rotate on its axis. The moment of the torsional force with respect to the axis is called the *torsional moment*, and is positive if it tends to rotate its plane clockwise, and negative if it tends to rotate its plane counter-clockwise.

If a shaft, as in Fig. 7, is fixed at one end, and at the other is subjected to a torsional bending force F whose moment with respect to the axis ab is positive, it is evident that this torsional moment in the end cross-section will be transmitted from cross-section to cross-section until it reaches the fixed end. Hence every cross-section between the applied force and the fixed end will be subjected to the same torsional moment. If a second torsional force F is applied at the middle point of the shaft so as to produce a positive torsional moment equal to the first, it is evident that the resultant torsional moment between the fixed section and the middle point will be doubled; the moment between the forces will, however, remain unchanged. If the second force is applied so as to produce a negative torsional moment, the resultant torsional moment between the second force and the fixed section will be reduced to zero, while that between the forces again remains unchanged. Hence

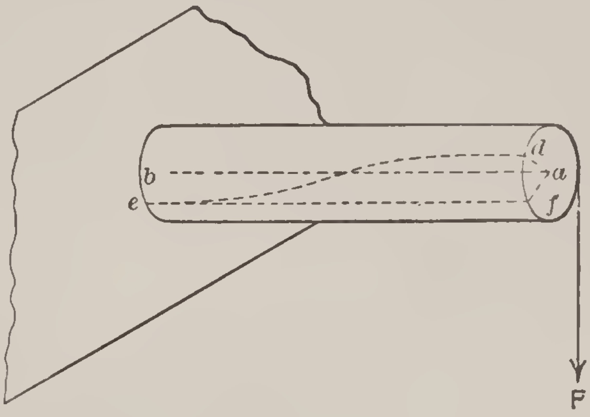

FIG. 7.

The torsional moment at any cross-section of a shaft acted upon by torsional forces is equal to the algebraic sum of the torsional moments of all the forces acting on one side of the section.

In the above discussion the bending effect of the force F has not been considered, since it is independent of the torsional effect. The bending effect will be considered in the chapters following.

Torsional Stress.—The theory of torsion is based on the following hypotheses, derived from observation:

1. The effect of a torsional force is to rotate each cross-section, between the point of application and the fixed section, about the axis of the shaft.

2. Each cross-section remains a plane surface during its rotation.

3. The amount of rotation of any section will vary with its distance from the fixed section.

The effect of the torsional moment of F, Fig. 7, is therefore to turn the end section and all the other cross-sections with it around the axis until the resultant moment of resistance developed in the fixed section is equal to the torsional moment; this prevents any further motion in its adjacent section. The resultant moment of resistance developed in this adjacent section will, when it becomes equal to the torsional moment, prevent further rotation of the section to its right. This effect will be transmitted from section to section until it reaches the plane of the force F. The fiber ef, which was originally straight, will become a spiral ed. The angle fad is called the *angle of torsion*, and the arc df the *arc of torsion*. The amount of distortion of the molecules of the fiber ef will depend upon their distances from the fixed section.

Law of Distribution.—If we examine the end section, it is seen that the fiber at a is not distorted, and the fiber at f is distorted through the arc df. The distortion of any other fiber along the radius af is distorted an amount dependent on its distance from a. Since within the elastic limit the stress is proportional to the distortion, *the stress on any fiber of the cross-section will vary with its distance from a, the axis of rotation.*

Unit Stress.—Let R = radius of shaft in inches;

r = variable distance in inches measured along the radius;

$\partial y \partial z$ = elementary area of cross-section;

s_t''' = unit torsional stress in pounds in a fiber at a unit distance from the axis;

s_t = unit torsional stress in pounds in the surface fiber. Then

$s_t'''r$ = unit torsional stress in a fiber at a distance r from axis;

$s_t'''\partial y \partial z r$ = total stress in an elementary area at a distance r from the axis;

$s_t'''\partial y \partial z r^2$ = moment of $s'''\partial y \partial z r$ about the axis;

$s_t'''\int\int \partial y \partial z r^2$ = resultant moment of all the stresses about the axis, or the *torsional moment of resistance of the cross-section.*

As stated above, the rotation of any section will cease when the torsional moment of resistance developed in the section becomes equal to the torsional moment of the twisting force at the section. Whence

$$s_t'''\int\int \partial y \partial z r^2 = M_t = Fr, \qquad (63)$$

in which M_t = torsional moment.

The expression $\int\int \partial y \partial z r^2$ occurs in mechanics as the moment of inertia of a plane area about an axis through its center and normal to its plane, or the *polar moment of inertia of the cross-section.* In this case it is the moment of inertia of the cross-section of the shaft about the axis of the shaft.

Representing this moment by I_p and substituting in equation (63) we have

$$s_t'''I_p = M_t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

in which $s_t'''I_p$ = moment of resistance, from which

$$s_t''' = \frac{M_t}{I_p} = \text{unit stress at a unit distance from the axis.}$$

Multiplying both members by r we have

$$s_t'''r = \frac{M_t r}{I_p} = \text{unit stress at a distance } r \text{ from the axis.} \qquad (65)$$

Since the stress is not uniformly distributed over the area of cross-section, there can be no unit area, 1 square inch, over which the stress is uniform, as in tension, compression, and shearing. The unit stress at a distance r from the axis is, therefore, the stress on a hypothetical area 1 square inch in cross-section, each element of which is subjected to the same stress as the elementary area $\partial y \partial z$ at a distance r from the axis.

Limit of Torsional Elasticity.—If we substitute in equation (65) R for r, we shall have for the surface fiber

$$s_t'''R = s_t = \frac{M_t R}{I_p}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

If the force F in Fig. 7 is gradually increased from zero, we shall eventually reach a value which will produce a permanent set in the surface fiber. This limiting value of s_t is the *limit of elasticity of the material in torsion* or the *limit of torsional elasticity of the material.*

The limit of torsional elasticity of a material is the greatest unit torsional stress which can be developed in the surface fiber of a shaft of the material without producing a permanent set.

Coefficient of Torsional Elasticity.—Within the field of perfect elasticity the unit stress must be proportional to the unit distortion.

If E_t = coefficient of torsional or lateral elasticity,
l = distortion of surface fiber = fd in Fig. 7,
l' = distortion of fiber at unit distance from axis,
L = length of every fiber = ef in Fig. 7, then

$$s_t = \frac{E_t l}{L} = \frac{E_t l' R}{L},$$

and

$$E_t = \frac{s_t}{\frac{l}{L}}. \qquad (67)$$

In this expression l is expressed in linear units; it may be expressed in degrees by multiplying $\frac{l}{R}$ by $\frac{180}{\pi}$. *The coefficient of lateral elasticity of any material is the quantity obtained by dividing the unit torsional stress on the surface fiber of a shaft, by the unit distortion of that fiber, within the field of perfect elasticity.*

Modulus of Torsion.—If, in equation (66), M_t be increased until the shaft breaks, the corresponding value of s_t will be the *modulus of torsion* of the material of the shaft or its *ultimate strength in torsion.* It will be represented by s_t'. *The modulus of torsion is the unit torsional stress on the extreme fiber of a shaft of the material, at the instant of rupture.*

Safe Unit Stress.—The greatest unit stress to which it is deemed advisable to subject the surface fiber is called the *safe or allowable unit stress in torsion.* It will be represented by s_t''.

The values of the constants of torsion for ordinary building materials are given approximately in the following table:

Constants.	Wood.	Cast Iron.	Steel.	Wrought Iron.
Coefficient of elasticity, E_t...	......	6,000,000	12,000,000	10,000,000
Modulus of torsion, s_t'.......	1,500	25,000	60,000	50,000
Safe unit stress, s_t''.	375	4,000	10,000	9,000

Designing.—The problem of designing a piece to resist torsional stress is usually that of determining the diameter of a shaft to resist a given torsional moment, or conversely, to determine the torsional moment which can be resisted by a given shaft.

The shaft is usually of circular cross-section, though the law of distribution is approximately true for any cross-section whose form is that of a regular polygon, as a square, a regular hexagon, etc.

If, in equation (66), we substitute for s_t its safe value s_t'', we shall have

$$s_t''=\frac{M_tR}{I_p}, \quad \ldots \ldots \quad (68)$$

or

$$R=\frac{s_t''I_p}{M_t}, \quad \ldots \ldots \quad (69)$$

in which M_t = torsional moment at the cross-section in (inch-pounds);

I_p = polar moment of inertia of the cross-section expressed in terms of R;

R = radius in inches of the circular cross-section, or the radius in inches of the circumscribed circle of the regular cross-section;

s_t'' = safe unit stress in pounds of the material in torsion.

From these equations we can determine the value of any one of the quantities when all the others are given.

PROBLEMS.

19. A circular steel shaft is subjected to a torsional moment of 10,000 (foot-pounds). What should be its diameter? $I_p=\frac{1}{2}\pi R^4$.

Ans. Diameter 3.94 inches.

20. What torsional moment in (foot-pounds) may be safely borne by a steel shaft six inches in diameter?

Ans. 35,343 (foot-pounds).

21. A circular steel shaft is 6 inches in diameter and 20 feet long; through what angle may it be safely twisted?

Ans. 3.8 degrees.

Dangerous Section.—From the equation

$$s_t=\frac{M_tR}{I_p} \quad \ldots \ldots \quad (70)$$

it is seen that for any shaft of uniform cross-section s_t must vary directly with M_t, since R and I_p are constant. Hence the greatest fiber stress in the surface fiber will take place at the section of greatest torsional moment. As this is the section where the shaft of uniform diameter is most liable to break, it may be called the *dangerous section.*

Shaft of Uniform Strength.—In equation

$$s_t'' = \frac{M_t R}{I_p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

the safe unit stress s_t'' is a constant. If the condition is imposed that the maximum stress or the stress in the surface fiber in each cross-section shall be s_t'', the shaft becomes one of uniform strength. In such a shaft the value of the ratio of M_t to $\frac{R}{I_p}$ must be constant.

If the cross-section is circular, $I_p = \frac{1}{2}\pi R^4$; hence the ratio $\frac{I_p}{R} = \frac{1}{2}\pi R^3$. In a circular shaft of uniform strength, therefore, R must vary directly with $\sqrt[3]{\frac{2M_t}{\pi}}$.

Elastic Torsional Resilience.—The elastic torsional resilience is the elastic energy stored in a shaft by twisting it within its field of perfect elasticity. It is equal to the work done by the twisting force. Conceive a scale-pan suspended from a tape wrapped around the free end of a horizontal shaft which is fixed at the other end and supported throughout to prevent bending. By gradually introducing weights into the pan the shaft will be brought under stress and twisted, until for a particular weight, W, the stress in the surface fiber is s_t, and the angle of torsion n degrees. During the process of loading, the scale-pan will descend a distance equal to the arc of torsion at the circumference, or $\frac{n\pi R}{180}$. Representing $\frac{n\pi R}{180}$ by l and remembering that the work of a gradually applied force is equal to one-half its intensity into its path, we have for the work of the force

$$\text{work} = \frac{Wl}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

In this expression if we substitute for l its value Rl', in which l' is the length of the arc of torsion at a unit distance from the axis, we shall have

$$\text{work} = \text{resilience} = \frac{WRl'}{2} = \frac{M_t l'}{2}. \quad . \quad . \quad . \quad . \quad . \quad (73)$$

To obtain an expression for the resilience in terms of the stress we must substitute for M_t and l' their value deduced from the expressions

$$M_t = \frac{s_t I_p}{R} \quad \text{and} \quad l' = \frac{s_t L}{E_t R} \quad . \quad . \quad . \quad . \quad . \quad (74)$$

Making these substitutions we have

$$\text{work} = \text{resilience} = \frac{1}{2} \frac{s_t I_p}{R} \cdot \frac{s_t L}{E_t R} = \frac{s_t^2}{2E_t} \cdot \frac{I_p L}{R^2}. \quad . \quad . \quad (75)$$

Substituting for I_p its value Ar_p^2, in which A = area and r_p = its radius of gyration,

$$\text{work} = \text{resilience} = \frac{s_t^2}{2E_t} \cdot \frac{r_p^2}{R^2} \cdot AL. \quad . \quad . \quad . \quad . \quad . \quad (76)$$

This expression becomes the maximum elastic torsional resilience when s_t becomes the limit of torsional elasticity, and $\frac{s_t^2}{2E_t}$ becomes the modulus of elastic torsional resilience.

The modulus of torsional resilience of any material is the quantity obtained by dividing the square of its limit of torsional elasticity by twice its coefficient of lateral elasticity. It measures the capacity of that material in comparison with other materials to store up the work expended on similar shafts by the same torsional force, within the field of perfect elasticity.

Power Transmitted by Shafts.—Conceive a circular shaft with a wheel at each end for a driving-belt; the belt at the end A being connected with the engine, and that at B with a machine. It is assumed that neither belt slips.

Let the uniform pull on belt A in pounds be represented by F, and the radius of the wheel in inches by r. At each revolution of the wheel the path of the force F is $2\pi r$, and in n revolutions it will be $2\pi rn$. The work performed by the force F in making

n revolutions will be $2\pi rnF$. If we assume that n revolutions are performed in one minute, this work per minute may be reduced to horse-power by dividing by $33,000 \times 12$, a horse-power being 33,000 foot-pounds per minute. Hence

$$\text{Horse-power in inch-pounds} = \frac{2\pi rnF}{33,000 \times 12}. \quad . \quad . \quad (77)$$

In this equation there are four variable quantities, the horse-power, n, F, and r. This enables us to make assumptions on any three and deduce the corresponding value of the fourth.

Since

$$Fr = M_t = \frac{s_t I_p}{R},$$

we may, by substituting, obtain an expression for the horse-power in terms of s_t and $\frac{I_p}{R}$, or

$$\text{Horse-power} = \frac{2\pi n s_t I_p}{33,000 \times 12 \times R}; \quad . \quad . \quad . \quad . \quad (78)$$

or substituting for I_p its value in terms of R, $\frac{1}{2}\pi R^4$,

$$\text{Horse-power} = \frac{s_t \pi^2 R^3 n}{396,000} = \frac{9.87 s_t R^3 n}{396,000} = \frac{s_t R^3 n}{40,000}, \text{ approximately. } (79)$$

In this equation there are also four variable quantities, the horse-power, the unit stress in the extreme fiber, the radius of the shaft, and the number of revolutions. This enables us to make assumptions on any three and deduce the corresponding value of the fourth.

PROBLEMS.

22. Find the diameter of a steel shaft which will safely transmit 100 H.P. when making 225 revolutions per minute.
Ans. 2.43 inches.

23. Find the horse-power which can be safely transmitted by a hollow steel shaft ($R = 10$ inches, $r = 5$ inches) when making 100 revolutions per minute. $I_p = \frac{1}{2}\pi(R^4 - r^4)$. *Ans.* 7438 H.P.

CHAPTER III.

FLEXURE OR BENDING.

Bending Moment.—A simple bending force is a force whose action line is perpendicular to and intersects the axis of a beam. Its effect is to bend a beam upon which it acts.

The *bending moment* of such a force with respect to any cross-section of a beam is the product of the intensity of the force, by its lever-arm or the distance between the action line of the force and the plane of cross-section. It is usually expressed in inch-pounds. To distinguish it from inch-pounds of work it will be represented (inch-pounds).

In all our discussions we have assumed clockwise moments positive and counter-clockwise moments negative. Bending moments are, however, considered positive when they produce compression in the upper fibers of the beam and tension in the lower fibers. The two systems of notation will agree if the center of moments is to the right of the forces; if the center of moments is to the left of the forces, we must give the positive sign to counter-clockwise moments.

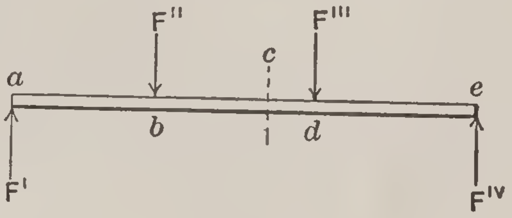

Fig. 8.

Let Fig. 8 represent a beam in a state of rest acted upon by four bending forces F', F'', F''', F^{IV}. Let c be a cross-section of the beam. Then will

$F' \times ac$ = positive moment and positive bending moment;
$F'' \times bc$ = negative " " negative " "
$F''' \times dc$ = positive " " " " "
$F^{IV} \times ec$ = negative " " positive " "

The bending moment at c in Fig. 8 is

$$M = +F' \times ac - F'' \times bc = -F''' \times dc + F^{IV} \times ec. \quad . \quad . \quad (80)$$

Hence

The bending moment at any section of a beam is the bending moment of the resultant of all the bending forces acting on either side of the section, or the algebraic sum of the bending moments of all the bending forces acting on either side of the section.

The determination of the bending moment at any section of a beam therefore consists in—

1. Finding the intensities and action lines of all the bending forces which act on one side of the section. This may be done by employing the equations of equilibrium, as modified for vertical parallel forces.

2. Finding the resultant moment of such forces with respect to the section considered.

It is evident from the explanations above given that it will usually be better to consider the forces on the left of the section, since the moment of each force and its bending moment will then have the same sign; if, however, merely a numerical value of the moment is desired, or the forces on the right can be more readily determined, the bending moment may be deduced from the forces on the right, if proper attention is paid to the signs.

Vertical or Transverse Shear.—A second effect of bending force is to move the consecutive planes of cross-section along each other and thus develop shearing stress. The total effect at any cross-section is called the *vertical or transverse shear*, or simply the *shear* at that cross-section. As beams are usually horizontal, the term vertical shear will be used, and it will be represented by V_s.

If we consider the cross-section c in the beam, Fig. 8, to be at rest, all the vertical forces must be transmitted to it through the internal resistances of the beam itself.

Thus the force F', acting in the end section, will tend to shear off that section, but it will be opposed by the resistance to shearing of the fibers connecting it with the next section. The effect of F' will therefore be transmitted from section to section, from a to c. In a similar manner the effect of F'', acting in a contrary direction, will be transmitted from section to section, from b to c.

The resultant shear, or force transmitted to c, will be their difference.

Similarly on the right of c, the force transmitted to c, or the shear at c, is the difference between F''' and F^{IV}. Since the section c is at rest, the force transmitted to it from the left must be equal in intensity but contrary direction to that transmitted to it from the right.

The shear at any section is therefore equal to the resultant of all the bending forces acting on either side of the section.

Since the bending moment is the resultant of all the bending forces acting on one side of the section, multiplied by its distance from that section, we have

$$M = V_s x, \quad \ldots \ldots \ldots \quad (81)$$

and

$$\frac{\delta M}{\delta x} = V_s, \quad \ldots \ldots \ldots \quad (82)$$

in which M = the bending moment at any section;
V_s = the vertical shear at same section, or resultant of vertical forces on one side of the section;
x = lever-arm of the resultant of vertical forces on one side of the section with respect to that section.

The vertical shear therefore measures the rate of change of the bending moment.

The shear at every section in that part of the beam in which the bending moment is an increasing algebraic function of x is considered *positive;* the shear at every section in that part of the beam in which the bending moment is a decreasing function of x is considered *negative.*

If the shear at any section is determined by finding the resultant of the forces on the left of the section, the shear will be positive if this resultant acts upwards, and negative if this resultant acts downwards. If only the numerical value of the shear at the section is desired, its sign is immaterial.

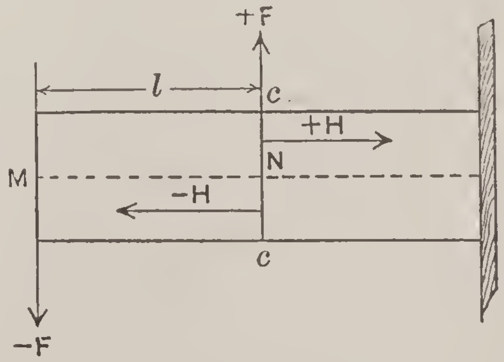

FIG. 9.

Theory of Bending or Flexure.—Let Fig. 9 represent a cantilever beam fixed in a horizontal position and acted upon by a single bending force F at its outward extremity. Let it be required to determine the law of distribution of the stress over any cross-section, as that at c.

Laws of Distribution.—The common theory of flexure is based upon the following hypotheses derived from experiments on the effect of bending forces on a beam. The forces are assumed as acting in the vertical plane through the axis of the beam.

1. *The fibers on the convex side are lengthened, those on the concave side are shortened; the stresses on the separate fibers are therefore longitudinal and either tensile or compressive.*

2. *Between the lengthened and shortened fibers there is a neutral surface whose fibers are neither lengthened nor shortened. The line cut out of this surface by the vertical plane through the axis is called the mean or neutral fiber, or the elastic curve.*

3. *The planes of cross-section normal to the fibers before bending will remain planes and be normal to the fibers after bending.*

4. *The planes of cross-section have no motion of translation along the neutral fiber, but simply rotate about the line cut out of the neutral surface by the plane of cross-section. This line is called the neutral axis.*

In Fig. 9, MN is the mean fiber, and the horizontal line, perpendicular to it at N, is the neutral axis of the section cc.

Under the action of the force F, therefore, the cross-section c will rotate about its neutral axis N, and the fibers above the axis will be lengthened and those below will be shortened. *The strain or stress on any fiber will therefore vary directly with its distance from the neutral axis.*

Unit Stress.—In Fig. 9 let

F = intensity of bending force in pounds;

l = lever-arm of F with respect to section c, in inches;

$M = -Fl$ = bending moment at section cc in (inch-pounds);

$+H$ = resultant tensile stress in fibers above the neutral axis;

$-H$ = resultant compressive stress in fibers below the neutral axis; (these resultants must be equal to each other, since by hypothesis the section has no motion of translation along the axis of the beam.)

d_1 = vertical distance between the resultant tensile and compressive stresses or the lever-arm of the couple HH.

If the beam is separated at the section cc, and longitudinal stresses in the fibers at the section replaced by the forces $+H$ and $-H$, and the vertical shear by the force $+F$ acting upwards, the segment between the section and the free end will be in a state of rest, and the four forces acting on it will be in equilibrium, or

$$Hd_1=Fl=M.$$

To determine an expression for Hd_1 in terms of the unit stress, let

s''' = unit longitudinal stress at a unit distance from the neutral axis;

$s'''y$ = unit longitudinal stress at a distance y from the neutral axis;

$\partial y \partial z$ = elementary area of cross-section at c;

$s'''y\partial y \partial z$ = stress on the elementary area;

$s'''y^2\partial y \partial z$ = moment of stress on the elementary area with respect to the neutral axis;

$s'''\int\int y^2\partial y \partial z$ = moment of stress on the entire area of cross-section with respect to the neutral axis, if integrated between proper limits. Hence

$$s'''\int\int y^2\partial y \partial z=Hd_1. \quad . \quad . \quad . \quad . \quad . \quad (83)$$

The expression $\int\int y^2\partial y \partial z$ occurs in mechanics as the *moment of inertia of a plane area about an axis in its own plane;* in this discussion it is the moment of inertia of the area of the cross-section cc about the neutral axis. If we represent this moment by I, we have

$$s'''I=Hd_1=Fl=M. \quad . \quad . \quad . \quad . \quad . \quad (84)$$

In this equation the moment Hd_1 or its equivalent $s'''I$ is called the *moment of resistance* of the fibers.

From equation (84) we have

$s''' = \frac{M}{I}$, for the unit longitudinal stress at a unit distance from the neutral axis;

$s'''y = \frac{My}{I}$, for unit longitudinal stress at a distance y from the neutral axis.

Position of the Neutral Axis.—Let .

$s'''y$ = unit longitudinal stress at a distance y from the neutral axis,

$\partial y \partial z$ = elementary area of the cross-section cc,

$s'''y\partial y \partial z$ = stress on the elementary area at a distance y from the neutral axis,

$s'''\int\int y\partial y \partial z$ = total longitudinal stress on the area of cross-section cc, if integrated between proper limits.

The expression $\int\int y\partial y \partial z$ is called the *static moment of an area about its axis of rotation*, and is equal to the total area multiplied by the distance of its center of gravity from the axis of rotation, or

$$\int\int y\partial y \partial z = Ad_2, \quad . \; . \; . \; . \; . \; . \; . \quad (85)$$

in which A = area of cross-section in square inches;

d_2 = distance of its center of gravity from the axis of rotation in inches.

In our discussion A is the area of the cross-section cc, and d_2 is the distance of the center of gravity of the cross-section from the neutral axis.

Since $+H$, the resultant stress in the fibers above the neutral axis, is equal to $-H$, the resultant stress in the fibers below the neutral axis, but acts in a contrary direction, the resultant horizontal stress in the cross-section must be zero.

Hence in the section cc

$$s'''\int\int y\partial y \partial z = s'''Ad_2 = 0. \quad . \; . \; . \; . \; . \; . \quad (86)$$

Since neither s''' nor A is equal to zero, d_2 must be equal to zero, or *the neutral axis must pass through the center of gravity of the cross-section.*

Maximum Unit Stress in Cross-section.—From the equation

$$s'''y = \frac{My}{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

we may determine the maximum unit stress in the cross-section whose resultant bending moment is M by substituting for $s'''y$, s, the unit stress in the fiber farthest from the neutral axis, and for y, y', the distance of the extreme fiber from the neutral axis; or

$$s = \frac{My'}{I} = \frac{M}{\frac{I}{y'}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

or

$$s\left(\frac{I}{y'}\right) = M, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (89)$$

in which $\frac{I}{y'}$ is the *section modulus.*

The stress in the surface fiber of a beam at any cross-section is equal to bending moment at the section divided by the section modulus of the section.

Modulus of Flexure.—If M in the second member of equation (88) is increased without changing the section modulus, s will be increased, and finally the extreme fiber will rupture. The value of s at the moment of rupture, represented by s', is called the *modulus of flexure, or the ultimate strength in flexure.*

The modulus of flexure is therefore the unit longitudinal stress on the extreme fiber of a beam at the moment of rupture by a bending force.

Allowable or Safe Unit Stress.—*The allowable or safe unit stress, s'', is the greatest unit longitudinal stress to which it is advisable to expose the fiber farthest from the neutral axis.*

For safety we must have

$$s'' = \text{or} > \frac{My'}{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

at every cross-section of the beam.

The general values of these constants for ordinary building materials in pounds per square inch are:

Constants.	Wood.	Cast Iron.	Steel.	Wrt. Iron.
Modulus of flexure..........	6, 000	36,000	60,000	48,000
Safe unit stress............	1,000	6,000	15,000	12,000

Factor of Safety.—The factor of safety of any material is the ratio of the ultimate to the safe unit stress under the assumption that this ratio is the same for tension, compression, shearing, torsion, and bending. It may be employed to determine the safe unit stress when the tables give only the ultimate or breaking unit stress.

The factor of safety employed depends on the uniformity of structure of the material and the character of the force which must be resisted.

The following table gives safe values of such factors for ordinary building materials:

	Quiescent Force, Buildings.	Varying Force, Bridges.	Shocks, Machines.
Stone and brick................	15	23	30
Timber and cast iron............	8	12	16
Steel and wrought iron..........	4	6	8

Section Modulus.—The section modulus $\frac{I}{y'}$ is the quotient resulting from dividing the moment of inertia of the cross-section about the neutral axis, by the distance of the extreme surface fiber from that axis.

The value of I for any form of area of cross-section is determined by integrating the expression $\int\int y^2 \partial y \partial z$ between the proper limits of y and z. In this expression z is the coordinate of any elementary area measured parallel to the neutral axis, and y is the coordinate of the same area measured in a direction perpendicular to that axis.

The value of y' is the greatest distance of any fiber from the neutral axis.

The neutral axis, it will be remembered, always passes through the center of gravity of the cross-section.

The values of I, y', and $\frac{I}{y'}$ for ordinary forms of cross-section are tabulated in engineering manuals.[1] Some of these are given in the following table:

Forms of Cross-section.	Dimensions.	I	y'	Section Modulus, $\frac{I}{y'}$
Rectangle.	{ breadth b depth d	$\frac{bd^3}{12}$	$\frac{d}{2}$	$\frac{bd^2}{6}$
Square with a vertical diagonal.	{ breadth d depth d	$\frac{d^4}{12}$	$d\sqrt{\frac{1}{2}}$	$\frac{d^3}{8.5}$
Triangle with horizontal base.	{ base b altitude d	$\frac{bd^3}{36}$	$\frac{2}{3}d$	$\frac{bd^2}{24}$
Circle.	diameter d	$\frac{\pi d^4}{64}$	$\frac{d}{2}$	$\frac{d^3}{10+}$
Ellipse	{ axes b and d b = neutral axis	$\frac{\pi bd^3}{64}$	$\frac{d}{2}$	$\frac{bd^2}{10+}$
I beam, cut from rectangle $b \times d$, by removing two areas, each $\frac{d' \times b'}{2}$.	. . . .	$\frac{(bd^3 - b'd'^3)}{12}$	$\frac{d}{2}$	$\frac{(bd^3 - b'd'^3)}{6d}$

It will be observed that as a general rule $\frac{I}{y'}$ is the function of the two dimensions b and d; in the circle and square it is a function of a single dimension d or r.

Design.—The problems in design are either to find the dimensions of a beam which will support a given load, or to find the load which will be supported by a beam of given dimensions.

Beams may be divided into two general classes: *beams of uniform cross-section* and *beams of uniform strength.*

The first is the ordinary form of beam, as it is a form easily sawed out of wood or rolled out of steel or wrought iron.

The second, or an approximation to it, is employed when it is desired to reduce the weight of the material in a beam to a

Cambria handbook, pp. 144–151.

minimum. Built-up beams of wood or metal plate are often made of this form; formerly when cast-iron beams were used they were also of this form.

Beams of Uniform Cross-section.—In beams of uniform cross-section the section modulus is the same at every cross-section.

Therefore, in the general equation

$$\frac{sI}{y'}=M, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

s, or the unit stress in the surface fiber, varies directly with M and is numerically greatest in the cross-section where the bending moment is greatest. As the beam is more liable to break at this section than at any other, it is called the *dangerous section*. If

M_m = the resultant bending moment at the dangerous section in (inch-pounds),

$\frac{I}{y'}$ = section modulus,

s'' = safe unit stress in pounds,

then the equation of condition for safety in a beam of uniform cross-section is

$$\frac{s''I}{y'}= \text{ or } > M_m. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

If the problem is to determine the dimensions of a beam to carry a given load, the value of s'' is taken from the table; M_m is computed from the loading, as will be explained hereafter; and $\frac{I}{y'}$ is expressed in terms of b and d. As all the quantities in the equation, except b and d, are known, we may assume one of the dimensions and compute the other.

Let it be required to find the minimum depth of a wooden beam of uniform cross-section, whose width is 6 inches, to resist a bending moment of 100,000 (inch-pounds) at the dangerous section.

From the table

$$s''=1000,$$

$$\frac{I}{y'}=\frac{bd^2}{6}=d^2;$$

hence
$$1000d^2 = 100{,}000,$$
$$d^2 = 100,$$
$$d = 10 \text{ inches.}$$

We may therefore always determine the dimensions of the cross-section of a beam of uniform cross-section when we know the section modulus in terms of b and d, and the bending moment at the dangerous section.

Beams of Uniform Strength.—In beams of uniform strength the unit stress in the surface fiber at every cross-section is the same; or in the general equation

$$\frac{sI}{y'} = M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (93)$$

s is the same for every cross-section and $\frac{I}{y'}$ must vary directly with M.

For safety s should be equal to or less than s'', the safe unit stress, or

$$s'' = \text{or} > \frac{M}{\frac{I}{y'}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (94)$$

If we substitute for $\frac{I}{y'}$ its value in terms of b and d, as, for instance, in a rectangular cross-section, we have

$$s'' = \frac{6M}{bd^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (95)$$

In this equation we may assume a value for one of the dimensions, as d, and we shall have

$$\frac{s''d^2}{6} = \frac{M}{b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (96)$$

In this equation the first term is constant, hence at every section of the beam the value of b must be such that the ratio $\frac{M}{b}$ is constant and equal to the first member of the equation.

Since the bending moment varies from point to point of the axis of a beam, it follows that the dimensions of the cross-section of a beam of uniform strength can only be determined when we know the law governing this variation.

The Law of Variation of the Bending Moment and the Bending Moment at the Dangerous Section.—As the resultant bending moment at any section of a beam is the resultant moment

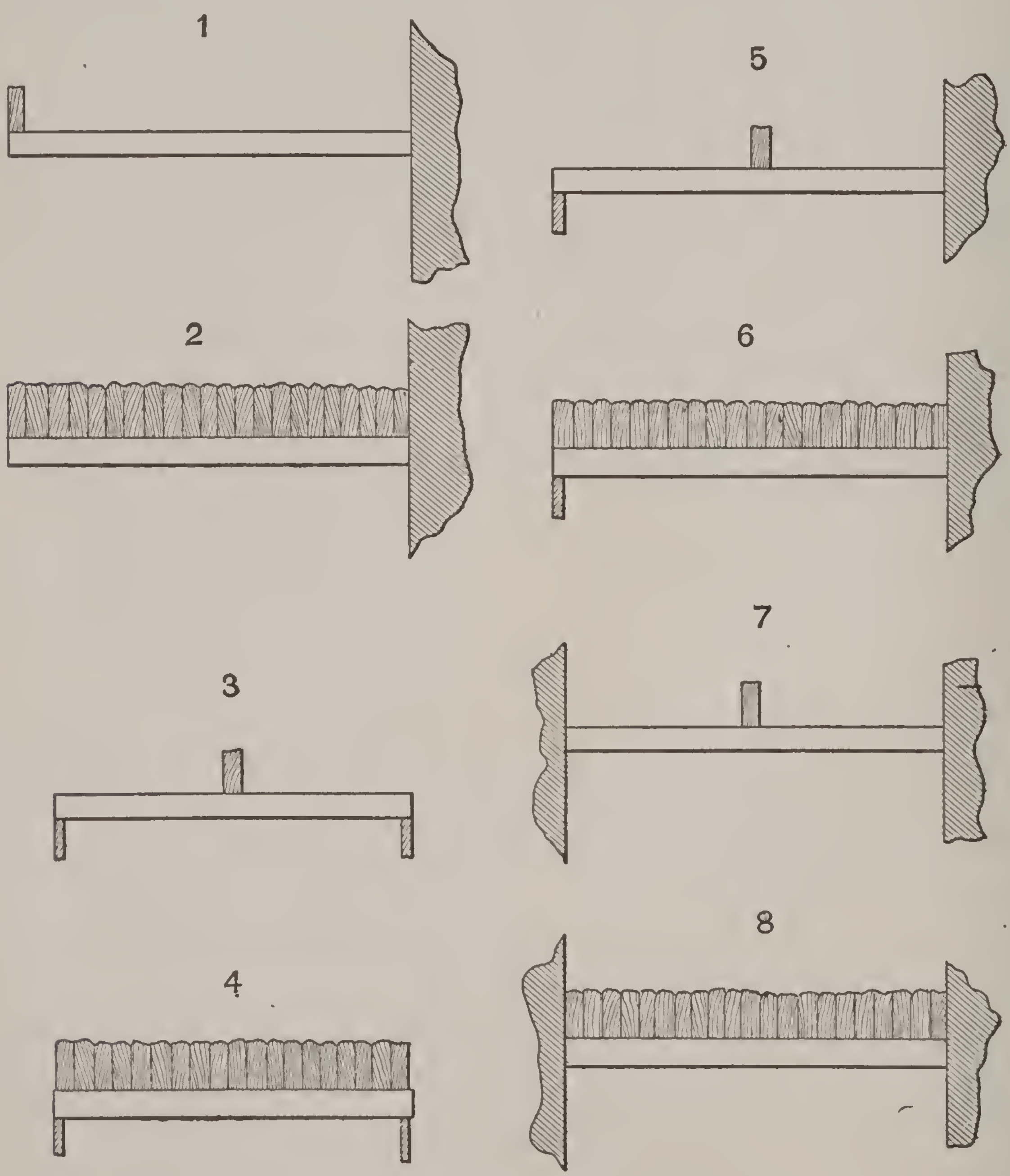

FIG. 10.

of all the bending forces acting on one side of the section, it is dependent on the intensities and action lines of the applied forces, or on the method of loading and supporting the beam, and is independent of the form and dimensions of cross-section.

According to the method of loading and supporting we have, in Fig. 10, the following classes of beams: cantilevers, 1 and 2;

beams resting on two end supports, 3 and 4; beams fixed at one end and supported at the other end, 5 and 6; beams fastened at both ends, 7 and 8.

To prevent confusion, in deducing the law of variation of the bending moment, only the axis of the beam will be represented in the figures.

1. Cantilever without Weight Supporting a Load at its Extremity.—In Fig. 11 let

O = origin of coordinates;
OX = axis of beam and axis of x;
OM = axis of moments;
OV_s = axis of shears;
l = length of beam in inches;
W = load concentrated at end in pounds;
c = any section of the beam between O and X;
M = bending moment at c;
M_m = bending moment at dangerous section.

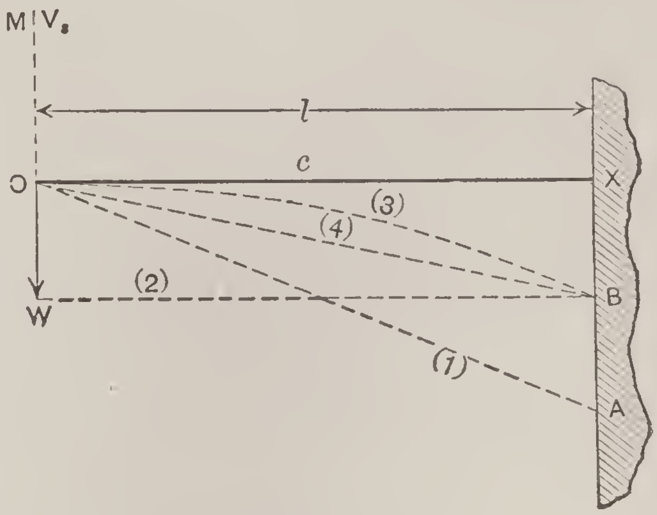

FIG. 11.

Considering the forces to the left of c, we have from definition the bending moment at c,

$$M = -Wx. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

This equation, called the *equation of bending moments*, expresses the law of variation of the value of the bending moment from section to section. Since M is a numerically increasing function of x, it is greatest numerically at X, where x is greatest and equal to l; and least numerically at O, where $x = 0$. The dangerous section is therefore at X, and

$$M_m = -Wl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

If we lay off $XA = Wl$, the line (1) will be the line whose equation is $M = -Wx$. This line is called the *line or curve of bending moments.* From it by inspection we may determine the position of the dangerous section.

Shear.—By definition, the vertical shear at any section of the beam, as c, is equal to $-W$. It is negative because the force is to the left of the section and acts downwards. Hence

$$V_s = -W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (99)$$

is the equation of shear for the cantilever Fig. 11.

If we lay off $XB = -W$, the line (2) will be the line whose equation is $V_s = -W$. This is called the *line or curve of shear*. The shear is the same at every section.

2. **A Cantilever Uniformly Loaded.**—If, in Fig. 11, the weight W is removed, and the beam is one of uniform cross-section whose weight is considered, it becomes a uniformly loaded cantilever.

Employing the same momenclature as before, and representing by w the weight of the beam per lineal inch, we have for the bending moment at any section, as c,

$$M = -wx \cdot \frac{x}{2} = -\frac{wx^2}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

This is the equation of bending moments. The line of bending moments represented by this equation is a parabola, which may be constructed by points. Since M is negative, it will lie below the axis of X. If $wl = W$, the line of bending moments will be line (3) in Fig. 11.

As M is a numerically increasing function of x, it will have its greatest value when $x = l$. *The dangerous section is therefore at X*, and

$$M_m = -\frac{wl^2}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (101$$

By definition the shear at the section c is $-wx$, hence

$$V_s = -wx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (102)$$

is the equation of shear. The line of shear is a right line. The abscissa of the point where it crosses the axis of x may be found by making $V_s = 0$ and solving for x; and the point where it crosses the axis of V_s by making $x = 0$, and solving for V_s; if these points are the same, a second point may be determined by giving x some other value and solving for V_s.

If $W=wl$, line (4) in Fig. 11 will be the line of shear. As V_s is a numerically increasing function of x, it will have its greatest value at X, where

$$(V_s)_m=-wl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (103)$$

3. A Beam without Weight Resting on End Supports and Supporting a Load at the Middle Point.—Let OX, Fig. 12, represent the axis of the beam and let the nomenclature be as in the preceding case.

The first step is to determine the value of the reactions R_1 and R_2. Since the load is at the middle point, they must be equal to each other, and each equal to $\frac{W}{2}$.

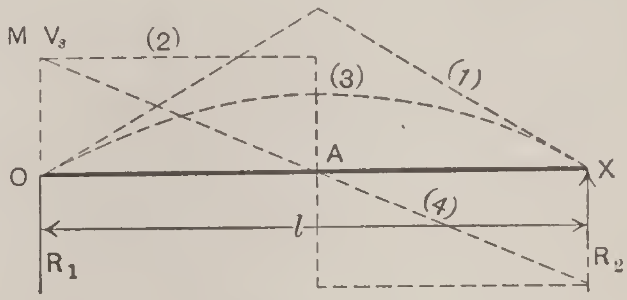

FIG. 12.

The bending moment at any section between O and A, taking the forces to the left of the section, is

$$M=\frac{Wx}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (104)$$

The line of bending moments between O and A is therefore a right line which passes through O, where $x=0$; at A, where $x=\frac{l}{2}$, it has a positive ordinate of $\frac{Wl}{4}$.

The bending moment at any section between A and X, taking the forces from the left, is

$$M=\frac{Wx}{2}-W\left(x-\frac{l}{2}\right)=-\frac{Wx}{2}+\frac{Wl}{2}. \quad . \quad . \qquad (105)$$

The line of bending moments between A and X is a right line which passes through X, where $x=l$; at A, where $x=\frac{l}{2}$, it has a positive ordinate of $\frac{Wl}{4}$. It is the line (1) in Fig. 12.

The dangerous section is evidently at A, and

$$M_m = \frac{Wl}{4}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (106)$$

The shear at any section between O and A is $+\frac{W}{2}$, and that at any point between A and X is

$$\frac{W}{2} - W = -\frac{W}{2}.$$

Hence

$$V_s = \frac{W}{2} \text{ from } O \text{ to } A, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (108)$$

and

$$V_s = -\frac{W}{2} \text{ from } A \text{ to } X. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (109)$$

The line of shear is therefore represented by line (2), Fig. 12.

4. A Beam Resting on End Supports and Uniformly Loaded.— Let OX, Fig. 12, be the axis of such a beam, and w be the uniform load per inch of length. Let the nomenclature be as before.

The reactions will then each be equal to $\frac{wl}{2}$.

The bending moment at any section between O and X will be

$$M = \frac{wlx}{2} - \frac{wx^2}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (110)$$

This is the equation of a parabola, which passes through O, where $x=0$, and through X, where $x=l$. Its maximum ordinate is at A, where $x=\frac{l}{2}$. The dangerous section is therefore at A, and

$$M_m = \frac{wl^2}{8}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (111)$$

If we make $W=wl$, the line of bending moments will be represented by line (3), Fig. 12.

The shear at any section will be

$$V_s = \frac{wl}{2} - wx. \quad \ldots \ldots \quad (112)$$

The line of shear is therefore a right line which passes through A, where $x = \frac{l}{2}$, and crosses the axis of V_s at a point $\frac{wl}{2}$ above O; if $W = wl$, it is line (4), Fig. 12.

The shear is greatest at the sections O and X, where

$$(V_s)_m = \frac{wl}{2} \quad \text{and} \quad -\frac{wl}{2}. \quad \ldots \quad (113)$$

Since $V_s = \frac{\delta M}{\delta X}$, we know from calculus that if M has a maximum or minimum state, it will be found where $\frac{\delta M}{\delta X}$ or $V_s = 0$. To determine whether it is a maximum or minimum state we must find the value of x from the equation $\frac{\delta M}{\delta X} = 0$, and substitute it in the second differential coefficient. If it makes the second differential coefficient negative, it is a maximum state, and if it makes the second differential coefficient positive, it is a minimum state.

Since the dangerous section is at the section where the bending moment has its greatest *numerical* value, it is usually immaterial whether it is a maximum or minimum state of the ordinate of the curve of bending moments.

It will be observed also that the section of mathematical maximum or minimum bending moment need not necessarily be the dangerous section, since the bending moment may be numerically greater at some other section of the beam.

In plotting the curves of shear or bending moments approximately, it should be remembered that a curve is convex upwards when the second differential coefficient of its ordinate with respect to x is negative, and concave, when the second differential coefficient is positive.

Modification of Cases 1 to 4.—If the cantilever in Case 1 has one or more additional forces applied between its extremity and the fixed end, the equations and lines of bending moments and shear may be deduced for each segment into which the beam is divided by the applied forces, by the application of the method above described.

If the beam resting on end supports, in Case 3, has one or more forces applied between the middle point and supports, the reactions will not usually be equal to each other. The intensities of the reactions may, however, always be determined by applying the equations of equilibrium for parallel vertical forces.

Having determined the reactions, the equations and lines of bending moments and shear may be deduced by applying the methods above described.

Application.—A beam AB, Fig. 13, 20 feet long, rests on two supports 16 feet apart, one of which is under the right end of the beam. The beam supports a uniform load of 20 pounds

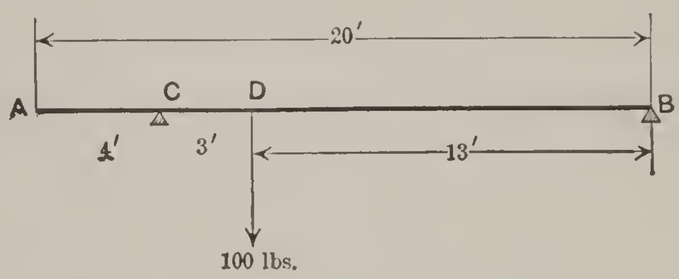

FIG. 13.

per lineal foot and also a weight of 100 pounds, 13 feet from its right end. Deduce the equations of bending moments and shear.

To Find the Reactions.—Let R_1 = reaction at left support;
R_2 = " " right "

Then

$$+R_1+R_2-100-(20\times 20)=0. \quad . \quad . \quad . \quad . \quad (a)$$

Since the resultant of the uniform load acts at the middle point of the beam, we have for the moments about B

$$-(100\times 13)-(400\times 10)+R_1\times 16=0, \quad . \quad . \quad . \quad (b)$$

$$-1300-4000+R_1\times 16=0,$$

$$R_1=\frac{5300}{16},$$

$$=331\tfrac{1}{4}.$$

Substituting this value of R_1 in equation (a) we have

$$R_2 = 500 - 331\tfrac{1}{4} = 168\tfrac{3}{4}.$$

Bending Moment.—Mark the left support C, and the point of application of the weight D, and assume the origin at A.

$$\begin{array}{ll} \text{From } A \text{ to } C & M = -\dfrac{20x^2}{2} = -10x^2; \\ \text{“ } C \text{ “ } D & M = -10x^2 + 331\tfrac{1}{4}(x-4); \\ \text{“ } D \text{ “ } B & M = -10x^2 + 331\tfrac{1}{4}(x-4) - 100(x-7). \end{array}$$

The curve of the bending moments in each segment of the beam is therefore a parabola.

By substituting proper values for x in this equation, the bending moment at any section of the beam may be readily ascertained, and by plotting the curves the position of the dangerous section can be found.

Shear.

$$\begin{array}{ll} \text{From } A \text{ to } C & V_s = -20x; \\ \text{“ } C \text{ “ } D & V_s = -20x + 331\tfrac{1}{4}; \\ \text{“ } D \text{ “ } B & V_s = -20x + 331\tfrac{1}{4} - 100. \end{array}$$

The line of shear in each segment is therefore a straight line. By substituting proper values for x, the shear at any section of the beam can be readily ascertained and the line of shear can be constructed.

Dimensions of Beams of Uniform Section for Simple Loading.—If, in the equation

$$\frac{s''I}{y'} = M_m,$$

we substitute the value of M_m deduced for cantilevers and beams resting on end supports, we shall have

$$\frac{s''I}{y'} = Wl \text{ for a cantilever with a load } W \text{ at its end, } \quad . \quad . \quad . \quad (114)$$

$$\frac{s''I}{y'} = \frac{wl^2}{2} \text{ “ “ “ “ “ uniform load of } w, \quad . \quad . \quad (115)$$

$\frac{s''I}{y'}=\frac{Wl}{4}$ " " beam resting on end supports with a load W at its middle point, ... (116)

$\frac{s''I}{y'}=\frac{wl^2}{8}$ for a beam resting on end supports with a uniform load of w. . . . . (117)

In which s''=safe unit stress in pounds;

$\frac{I}{y'}$=section modulus;
W=weight in pounds;
l =length of beam in inches;
w=weight of uniform load per inch.

By substituting for s'' and the section modulus their proper values from the tables, we can determine any one of the five quantities b, d, l, W, and w, when all the others are known.

PROBLEMS.

24. A rectangular wooden cantilever 10 feet long and 6 inches deep must support a weight of 200 pounds at its outer extremity; what should be its width? The weight of the cantilever is not considered. *Ans.* 4 inches.

25. A wooden beam 4 inches square, resting on end supports, must support a uniform load of 36 pounds per lineal foot, including its own weight; what is the maximum safe distance between the supports? *Ans.* 14 feet.

26. What load may be safely hung from the middle point of a wooden beam of circular cross-section which rests on end supports 10 feet apart? The radius of the beam is 4 inches and the weight of the beam is not considered. $\frac{I}{y'}=\frac{d^3}{10}$. *Ans.* 1707 pounds.

27. A wooden beam, 10 feet long, the cross-section of which is an isosceles triangle whose horizontal base is 6 inches, rests on end supports. If it carries a uniform load, including its weight, of 120 pounds per lineal foot, what must be the altitude of its cross-section? *Ans.* 8.48 inches.

To Find the Strongest Rectangular Beam of Uniform Cross-section which can be Cut Out of a Circular Log.

Let D = diameter of the log in inches;
b = breadth of beam in inches;
d = depth of beam in inches.

The expression for the stress in the surface fiber at the dangerous section of a beam is

$$s=\frac{M_m}{\frac{I}{y'}}=\frac{M_m}{\frac{bd^2}{6}}=\frac{6M_m}{bd^2}. \quad . \quad . \quad . \quad . \quad . \quad (118)$$

The strongest beam is the one which has the least stress at the surface fiber at the dangerous section. It is therefore the beam in which the relation between b and d is governed by the condition that bd^2 in equation (118) shall be a maximum.

Since the log is circular,

$$b^2+d^2=D^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (119)$$

From equation (119) we have

$$d^2=D^2-b^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (120)$$

Multiplying through by b we have

$$bd^2=bD^2-b^3. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (121)$$

Differentiating with respect to b we have

$$\frac{\delta(bd^2)}{\delta b}=D^2-3b^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (122)$$

and

$$\frac{\delta^2(bd^2)}{(\delta b)^2}=-6b. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (123)$$

To determine the value of b which will make bd^2 a maximum, we must place the first differential coefficient equal to zero and solve with respect to b:

$$D^2 - 3b^2 = 0; \quad \dots \dots \dots \quad (124)$$

$$b = D\sqrt{\tfrac{1}{3}}. \quad \dots \dots \dots \quad (125$$

This substitution in the second differential coefficient gives a negative value and therefore corresponds to a maximum.

Substituting this value in equation (120) and solving for d we have

$$d = D\sqrt{\tfrac{2}{3}}. \quad \dots \dots \dots \quad (126)$$

To construct the cross-section, draw a circle with a diameter D. From either end of the diameter, lay off on it a distance equal to $\frac{D}{3}$, and at this point erect a perpendicular to the diameter. From the point where the perpendicular intersects the circumference draw chords to the ends of the diameter. These chords will be the sides of the required inscribed rectangle.

Dimensions of Beams of Uniform Strength.—If, in the equation $\frac{s''I}{y'} = M$, we substitute for M its values for the different beams above considered, we shall have

$$\frac{s''I}{y'} = -Wx, \quad \dots \dots \dots \quad (127)$$

$$\frac{s''I}{y'} = -\frac{wx^2}{2}, \quad \dots \dots \dots \quad (128)$$

$$\frac{s''I}{y'} = \frac{Wx}{2}, \quad \dots \dots \dots \quad (129)$$

$$\frac{s''I}{y'} = \frac{wlx}{2} - \frac{wx^2}{2}. \quad \dots \dots \dots \quad (130)$$

If, in equation (127), we substitute, from the table, for s'' and

$\frac{I}{y'}$ their proper values, for a wooden beam of rectangular cross-section we shall have

$$\frac{1000 \times bd^2}{6} = -Wx. \quad . \quad . \quad . \quad . \quad . \quad (131)$$

If we assume a value for b and W, and solve with respect to d^2, we have

$$d^2 = -\frac{6Wx}{1000b}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (132)$$

in which d and x are the only variables.

If we assume values of x, and deduce corresponding values of d, they will be the varying depths of cross-section of a wooden cantilever of uniform strength, since $\frac{My'}{I}$ will be constant and equal to s'' at every section of the beam.

Equation (132) is the equation of a parabola. If the values of $\frac{d}{2}$ are laid off above and below the axis, the beam will be of the form shown in Fig. 14, page 68.

If we assume values for d and W, and make b vary with x, the equation for a wooden cantilever loaded at the end, (127), becomes

$$b = -\frac{6Wx}{1000d^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (133)$$

This is the equation of a right line, and the beam is of the form shown in Fig. 15, if the value of $\frac{b}{2}$ is laid off on either side of the axis.

The cross-section of the beams Figs. 14 and 15 must be enlarged near their free end, as shown in Fig. 15, to meet the requirement that the shearing strength of the sections near the end shall be sufficient to support the weight, or

$$s_s''bd = \text{or} > W.$$

In the discussion above given we may substitute any other material for wood by substituting for s'' its proper value; we

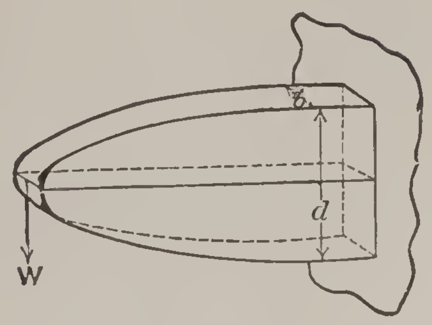

FIG. 14.

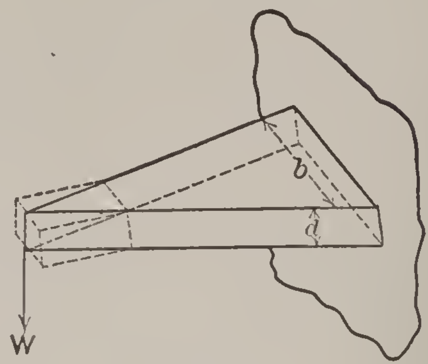

FIG. 15.

may substitute any other form of cross-section for the rectangular, by substituting for $\frac{I}{y'}$ its proper value; and we may change the system of loading by substituting for M its proper value in terms of l and W or w.

PROBLEMS.

28. A wooden beam, resting on end supports 10 feet apart, must support a load of 1000 pounds at its middle point. The beam is of circular cross-section and of uniform strength to resist bending except at its end, where it is to be cylindrical in shape and of sufficient cross-section to resist shearing. Find diameter of cross-section at the end, at the middle point, and at the quarter points. $\frac{I}{y'} = \frac{d^3}{10}$.

Ans. At end diameter 0.8 inch; at middle point 6.69 inches; at quarter points 5.31 inches.

29. Determine the equation of the curve of meridian section of a beam of uniform strength which is of circular cross-section, rests on end supports, and is uniformly loaded.

Ans. $d^3 = \left(\frac{5w}{s''}\right)(lx - x^2)$.

CHAPTER IV.

BEAMS FIXED AT THE ENDS, AND CURVE OF MEAN FIBER.

A beam is *fixed in position* at any point by imposing the condition that the direction of its axis or its mean fiber at that point shall remain fixed.

To determine the reactions of the supports, it is necessary to combine this equation of condition with the equations of equilibrium already given.

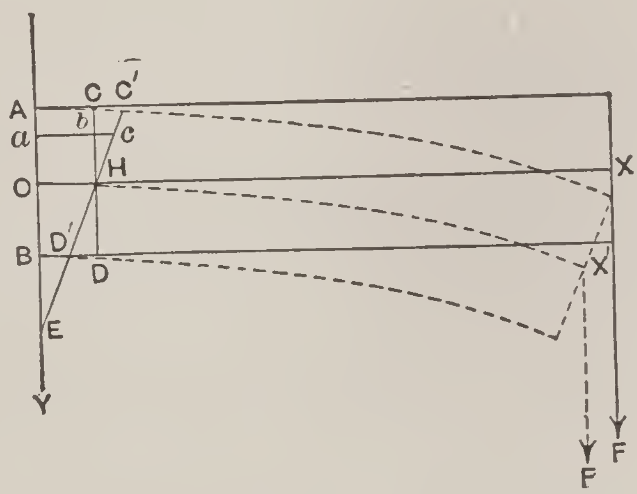

FIG. 16.

To deduce the equation of the mean fiber or the axis of a beam after deflection, let the cantilever shown in Fig. 16 be acted upon by the force F.

Let AB = fixed section;

CD = adjacent section;

$OH = \delta x = L$ = length of fiber ab before deflection;

$l = bc$ = elongation of fiber ab;

y = ordinate of fiber ab referred to neutral axis H;

E = coefficient of longitudinal elasticity;

$\rho = HE$ = radius of curvature of mean fiber after deflection;

$s'''y$ = unit stress in fiber ab.

Within the field of perfect elasticity we must then have

$$s'''y=\frac{El}{L}. \qquad (134)$$

From similar triangles we have

$$bc:bH::OH:OE,$$

or

$$l:y::\delta x:\rho;$$

hence

$$l=\frac{y\delta x}{\rho}. \qquad (135)$$

Substitute for δx its value L.

$$l=\frac{yL}{\rho}, \quad \text{or} \quad \frac{l}{L}=\frac{y}{\rho}. \qquad (136)$$

Substituting this value of $\frac{l}{L}$ in equation (134) we have

$$s'''y=\frac{Ey}{\rho}, \quad \text{or} \quad s'''=\frac{E}{\rho}. \qquad (137)$$

From the theory of flexure or bending we have

$$s'''=\frac{M}{I}; \qquad (138)$$

hence

$$\frac{M}{I}=\frac{E}{\rho}, \qquad (139)$$

or

$$M=\frac{EI}{\rho}. \qquad (140)$$

From calculus we have

$$\rho=\frac{\delta l^3}{\delta x \delta^2 y}, \qquad (141)$$

in which δl = length of elementary part of a curve;
δx = its projection on the axis of X;
δy = " " " " " " Y;
x and y = coordinates of points of the curve.

If the deflection of the beam is very small, as it must be within the field of perfect elasticity, δl is sensibly equal to δx and we have

$$\rho = \frac{\delta x^2}{\delta^2 y}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (142)$$

Substituting this value of ρ in equation (140) we have

$$M = \frac{EI\delta^2 y}{\delta x^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (143)$$

in which y and x are coordinates of points of the curve.

If, in this expression, the value of M can be expressed in terms of x, we can determine, by double integration, an expression for y in terms of x, which will be the equation of the mean fiber.

From this equation we can plot the curve of mean fiber and determine the maximum deflection of the axis of the beam under its load.

Applications.

I. Cantilever without Weight, Fixed Horizontally, Supporting a Load at its Extremity.—In Fig. 11, page 57, let

OX = axis of beam before deflection = axis of X;
OM = axis of Y;
l = length of beam;
y_m = maximum deflection of beam;
c = any section of beam between O and X;
W = weight suspended from extremity;
M = bending moment at c.

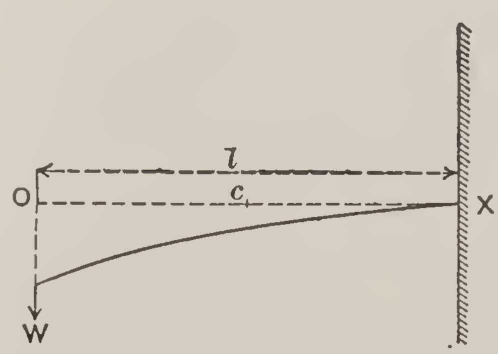

FIG. 17.

From the deduction, page 57, we have

$$M = -Wx;$$

hence

$$\frac{EI\partial^2 y}{\partial x^2} = -Wx. \quad \ldots \ldots \quad (144)$$

Integrating with respect to x we have

$$\frac{EI\partial y}{\partial x} = -\frac{Wx^2}{2} + C, \quad \ldots \ldots \quad (145)$$

in which $\frac{\partial y}{\partial x}$ is the tangent of the angle made by the mean fiber with the axis of X, and C is the constant of integration.

By hypothesis, at X, where $x = l$, $\frac{\partial y}{\partial x} = 0$; hence $C = \frac{Wl^2}{2}$.

Substituting this value in (145) and integrating,

$$EIy = -\frac{Wx^3}{6} + \frac{Wl^2x}{2} + C', \quad \ldots \ldots \quad (146)$$

where $x = l$, $y = 0$; hence

$$C' = \frac{Wl^3}{6} - \frac{Wl^3}{2} = -\frac{Wl^3}{3}$$

and

$$y = -\frac{W}{6EI}(x^3 - 3l^2x + 2l^3). \quad \ldots \ldots \quad (147)$$

Since y is a numerically decreasing function of x, it will have its greatest numerical value, within the limits of the beam, where $x = 0$ or

$$y_m = -\frac{Wl^3}{3EI}.$$

It will be observed that the expression for the bending moment at any section is the second differential coefficient with respect to x of the ordinate of curve of mean fiber at the same section, multiplied by the constant EI.

Therefore where the bending moment of a curve passes through zero, changing its sign, there will be a point of inflection in the curve of mean fiber.

The shear at any section, $\frac{\partial M}{\partial x}$, is the third differential coefficient of the ordinate of the mean fiber with respect to x, multiplied by the same constant.

The form of the curve of mean fiber is that approximately shown in Fig. 17; it is convex upwards, since for any value of x less than l, M, the second differential coefficient, is negative.

II. Cantilever Fixed Horizontally and Loaded Uniformly.— Let w=load in pounds per unit of length.

From the deduction, page 58, Fig. 11, we have

$$M=\frac{EI\partial^2 y}{\partial x^2}=-\frac{wx^2}{2}. \qquad (148)$$

Integrating,

$$\frac{EI\partial y}{\partial x}=-\frac{wx^3}{6}+C. \qquad (149)$$

By hypothesis, where $x=l$, $\frac{\partial y}{\partial x}=0$; hence $C=\frac{wl^3}{6}$.

Integrating again,

$$EIy=-\frac{wx^4}{24}+\frac{wl^3x}{6}+C', \qquad (150)$$

where $x=l$, $y=0$; hence $C'=\frac{wl^4}{24}-\frac{wl^4}{6}=-\frac{wl^4}{8}$,

and

$$y=-\frac{w}{24EI}(x^4-4l^3x+3l^4). \qquad (151)$$

As in the preceding case, the curve is convex upwards, and of the form shown in Fig. 17. Its greatest ordinate is where $x=0$, or

$$y_m=-\frac{wl^4}{8EI}. \qquad (152)$$

III. Beam without Weight Resting on End Supports, with a Load W at Middle Point.—From previous deduction we have at c, Fig. 18, any section in left half of beam,

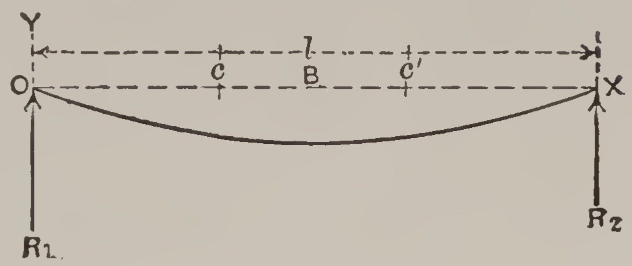

FIG. 18.

$$M=\frac{EI\delta^2y}{\delta x^2}=\frac{Wx}{2}. \quad \ldots \ldots \quad (153)$$

Integrating,

$$\frac{EI\delta y}{\delta x}=\frac{Wx^2}{4}+C. \quad \ldots \ldots \quad (154)$$

Because of symmetrical loading where $x=\frac{l}{2}$, $\frac{\delta y}{\delta x}=0$; hence $C=-\frac{Wl^2}{16}$.

Substituting,

$$\frac{EI\delta y}{\delta x}=\frac{Wx^2}{4}-\frac{Wl^2}{16}. \quad \ldots \ldots \quad (155)$$

Integrating,

$$EIy=\frac{Wx^3}{12}-\frac{Wl^2x}{16}+C' \quad \ldots \ldots \quad (156)$$

Where $x=0$, $y=0$; hence $C'=0$, and

$$y=\frac{W}{48EI}(4x^3-3l^2x). \quad \ldots \quad (157)$$

Since M is positive for any value of x less than $\frac{l}{2}$, the curve is concave upwards and is of the form shown in Fig. 18. The maximum value of y is where $x=\frac{l}{2}$, or

$$y_m=-\frac{Wl^3}{48EI}. \quad \ldots \ldots \quad (158)$$

Since the beam is centrally loaded, the curve of mean fiber will be symmetrical with respect to a vertical line through its middle point.

IV. Beam Resting on End Supports and Uniformly Loaded.— Let w = weight in pounds of load per unit of length. From previous deductions we have at C

$$M=\frac{EI\delta^2 y}{\delta x^2}=\frac{wlx}{2}-\frac{wx^2}{2}. \qquad (159)$$

Integrating,

$$\frac{EI\delta y}{\delta x}=\frac{wlx^2}{4}-\frac{wx^3}{6}+C. \qquad (160)$$

Because of symmetrical loading, where $x=\frac{l}{2}$, $\frac{\delta y}{\delta x}=0$; hence $C=-\frac{wl^3}{24}$. Substituting we have

$$\frac{EI\delta y}{\delta x}=\frac{wlx^2}{4}-\frac{wx^3}{6}-\frac{wl^3}{24}. \qquad (161)$$

Integrating,

$$EIy=\frac{wlx^3}{12}-\frac{wx^4}{24}-\frac{wl^3x}{24}+C', \qquad (162)$$

where $x=0$, $y=0$; hence $C'=0$, and

$$y=\frac{w}{24EI}(2lx^3-x^4-l^3x). \qquad (163)$$

For any value of x less than l, M is positive; the curve of mean fiber is therefore concave upwards, and of the form shown in Fig. 18. The value of y is a maximum where $x=\frac{l}{2}$, or

$$y_m=-\frac{5wl^4}{384EI}. \qquad (164)$$

V. A Beam without Weight Fixed Horizontally at Both Ends and Supporting a Weight W at its Middle Point.—In Fig. 19

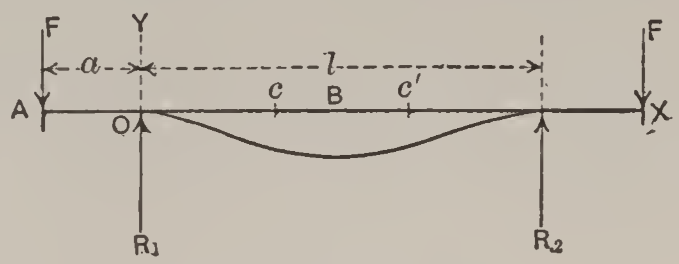

FIG. 19.

let AOX = axis of mean fiber of beam before deflection;
O = origin of coordinates;
l = length of beam, in inches, between the supports;
W = a weight in pounds acting downwards at the middle point B of the beam;
$R_1 = R_2$ = reactions;
$V_s' = V_s''$ = shears at R_1 and R_2;
c = any section between O and the middle point of the beam B;
$Oc = x$.

The effect of the weight W alone is to deflect the mean fiber of the beam as shown in Fig. 18. To make its tangent at O, $\frac{\partial y}{\partial x}$, equal to zero, requires the introduction of an additional force F to the left of O, acting in the same direction as W.

Let a = lever arm of F with respect to O.

$$R_1 = \frac{W}{2} + F = V_s' + F, \quad . \quad . \quad . \quad . \quad . \quad (165)$$

and

$$M = \frac{EI\partial^2 y}{\partial x^2} = \left(\frac{W}{2} + F\right)x - F(a+x) = \frac{Wx}{2} - Fa. \quad . \quad (166)$$

Integrating,

$$\frac{EI\partial y}{\partial x} = \frac{Wx^2}{4} - Fax + C. \quad . \quad . \quad . \quad . \quad . \quad (167)$$

By hypothesis, where $x = 0$, $\frac{\partial y}{\partial x} = 0$; hence $C = 0$, and

$$\frac{EI\partial y}{\partial x} = \frac{Wx^2}{4} - Fax. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (168)$$

Because of symmetrical loading, where $x=\frac{l}{2}$, $\frac{\delta y}{\delta x}=0$; hence

$$\frac{Wl^2}{16}-\frac{Fal}{2}=0, \qquad (169)$$

and

$$Fa=\frac{Wl}{8}. \qquad (170)$$

Substituting this value of Fa in equations (166) and (168) we have

$$\frac{EI\delta^2 y}{\delta x^2}=\frac{Wx}{2}-\frac{Wl}{8}, \qquad (171)$$

$$\frac{EI\delta y}{\delta x}=\frac{Wx^2}{4}-\frac{Wlx}{8}. \qquad (172)$$

Integrating again,

$$EIy=\frac{Wx^3}{12}-\frac{Wlx^2}{16}+C', \qquad (173)$$

where $x=0$, $y=0$; hence $C'=0$, and

$$y=\frac{W}{48EI}(4x^3-3x^2l). \qquad (174)$$

The curve of mean fiber has a point of inflection where $M=0$,

for which $$\frac{Wx}{2}-\frac{Wl}{8}=0, \quad \text{or} \quad x=\frac{l}{4}. \qquad (175)$$

It is of the form shown in Fig. 19.

The maximum deflection is at B, where the value of y is

$$y_m=-\frac{Wl^3}{192EI}. \qquad (176)$$

If the line of bending moments is constructed, it will be seen that the greatest values of M are $+\frac{Wl}{8}$ at the middle point, and $-\frac{Wl}{8}$ at each end. Hence $M_m = \pm\frac{Wl}{8}$.

The equations of the line of shear are

$$V_s' = +\frac{W}{2} \text{ to the left of } B, \text{ and } V_s' = -\frac{W}{2} \text{ to the right of } B. \quad (177)$$

VI. A Beam Fixed Horizontally at Both Ends and Uniformly Loaded.—Assume the same notation as above and replace W by a uniform load of w per lineal inch. Then at c

$$M = \frac{EI\delta^2 y}{\delta x^2} = \left(\frac{wl}{2} + F\right)x - \frac{wx^2}{2} - F(a+x) = \frac{wlx}{2} - \frac{wx^2}{2} - Fa$$

$$= V_s'x - \frac{wx^2}{2} - Fa. \quad \ldots \ldots \ldots \quad (178)$$

Integrating,

$$\frac{EI\delta y}{\delta x} = \frac{wlx^2}{4} - \frac{wx^3}{6} - Fax + C. \quad \ldots \ldots \quad (179)$$

By hypothesis, where $x=0$, $\frac{\delta y}{\delta x}=0$, hence $C=0$.

Because of symmetrical loading, where $x=\frac{l}{2}$, $\frac{\delta y}{\delta x}=0$; hence

$$Fa = \frac{wl^2}{12}. \quad \ldots \ldots \ldots \quad (180)$$

Substituting this value in equations (178) and (179) we have

$$M = \frac{EI\delta^2 y}{\delta x^2} = \frac{wlx}{2} - \frac{wx^2}{2} - \frac{wl^2}{12}, \quad \ldots \quad (181)$$

$$\frac{EI\delta y}{\delta x} = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^2 x}{12}. \quad \ldots \ldots \quad (182)$$

Integrating,

$$EIy = \frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl^2x^2}{24} + C', \quad . \quad . \quad . \quad . \quad (183)$$

where $x=0$, $y=0$; hence $C'=0$, and

$$y = \frac{w}{24EI}(2lx^3 - x^4 - l^2x^2). \quad . \quad . \quad . \quad . \quad (184)$$

The curve is of the form shown in Fig. 19, and has a point of inflection where $M=0$, or where

$$x = \frac{l}{2} \pm l\sqrt{\frac{1}{12}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (185)$$

The deflection is a maximum at the middle point where $x=\frac{l}{2}$; hence

$$y_m = -\frac{wl^4}{384EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (186)$$

The equation of bending moments

$$M = \frac{wlx}{2} - \frac{wx^2}{2} - \frac{wl^2}{12} \quad . \quad . \quad . \quad . \quad . \quad (187)$$

is the equation of a parabola whose axis passes through B, and whose curve intersects the axis of X at points $l\sqrt{\frac{1}{12}}$ on either side of the middle point, B. The greatest value of M is $-\frac{wl^2}{12}$ at the ends; at the middle point $M=\frac{wl^2}{24}$.

If equations (166) and (178) are examined, it will be seen that the bending moment at any section, as c, is equal to the algebraic sum of three moments: the bending moment at O, the moment of the shear at O with respect to c, and the moment of the load between O and c with respect to c.

VII. Beam without Weight Fixed Horizontally at One End, Resting on a Support at the Other, and Supporting a Weight at the Middle Point.—In Fig. 20 let

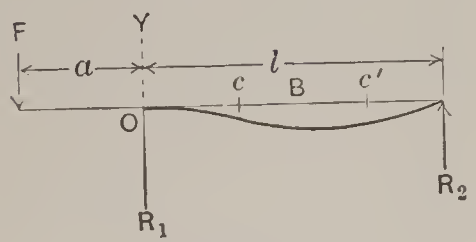

FIG. 20.

O = origin of coordinates;
W = weight applied at middle point;
R_1 = reaction at $O = F + V_s'$
$V_s = R_1 - F$ = shear at O;
R_2 = reaction at right end $= V_s''$;
Fa = moment required to keep neutral fiber at O horizontal.

For c, any section between O and B, we have

$$M = \frac{EI\delta^2 y}{\delta x^2} = -F(a+x) + (F+V_s')\,x = -Fa + V_s'x. \quad (188)$$

Integrating and remembering that where $x=0$, $\frac{\delta y}{\delta x}=0$; hence $C=0$, we have

$$\frac{EI\delta y}{\delta x} = -Fax + \frac{V_s'x^2}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (189)$$

Integrating again and remembering that where $x=0$, $y=0$; hence $C'=0$, we have

$$EIy = -\frac{Fax^2}{2} + \frac{V_s'x^3}{6}. \quad . \quad . \quad . \quad . \quad . \quad (190)$$

For c', any section between B and R_2 we have

$$M = \frac{EI\delta^2 y}{\delta x^2} = -F(a+x) + (F+V_s')x - W\left(x - \frac{l}{2}\right)$$

$$= -Fa + V_s'x - Wx + \frac{Wl}{2}. \quad . \quad . \quad . \quad . \quad (191)$$

Integrating,

$$\frac{EI\delta y}{\delta x} = -Fax + \frac{V_s'x^2}{2} - \frac{Wx^2}{2} + \frac{Wlx}{2} + C. \quad . \quad (192)$$

Integrating again,

$$EIy = -\frac{Fax^2}{2} + \frac{V_s'x^3}{6} - \frac{Wx^3}{6} + \frac{Wlx^2}{4} + Cx + C'. \quad . \quad (193)$$

where $x=l$, $y=0$; hence

$$C' = \frac{Fal^2}{2} - \frac{V_s'l^3}{6} + \frac{Wl^3}{6} - \frac{Wl^3}{4} - Cl.$$

Substituting,

$$EIy = -\frac{Fax^2}{2} + \frac{V_s'x^3}{6} - \frac{Wx^3}{6} + \frac{Wlx^2}{4} + Cx + \frac{Fal^2}{2} - \frac{V_s'l^3}{6} + \frac{Wl^3}{6} - \frac{Wl^3}{4} - Cl. \quad (194)$$

If the tangent to the mean fiber at B is represented by ϕ, we shall have, from (189) and (192),

$$EI \tan\phi = -\frac{Fal}{2} + \frac{V_s'l^2}{8}, \quad . \quad . \quad . \quad . \quad . \quad (195)$$

$$EI \tan\phi = -\frac{Fal}{2} + \frac{V_s'l^2}{8} - \frac{Wl^2}{8} + \frac{Wl^2}{4} + C. \quad . \quad . \quad (196)$$

Subtracting (195) from (196) we have

$$C = -\frac{Wl^2}{8}.$$

Representing the value of y, at B, by y', we have from (190) and (194), after substituting for C its value,

$$EIy' = -\frac{Fal^2}{8} + \frac{V_s'l^3}{48}, \quad . \quad . \quad . \quad . \quad . \quad (197)$$

$$EIy' = -\frac{Fal^2}{8} + \frac{V_s'l^3}{48} - \frac{Wl^3}{48} + \frac{Wl^3}{16} - \frac{Wl^3}{16} + \frac{Fal^2}{2} - \frac{V_s'l^3}{6} + \frac{Wl^3}{6} - \frac{Wl^3}{4} + \frac{Wl^3}{8}. \quad (198)$$

Subtracting (197) from (198) we have

$$\frac{Fal^2}{2} - \frac{V_s' l^3}{6} = -\frac{Wl^3}{48},$$

or

$$-Fa + \frac{V_s' l}{3} = \frac{Wl}{24}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (199)$$

The equation of equilibrium of moments about R_2 is

$$-Fa + V_s' l - \frac{Wl}{2} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (200)$$

Subtracting (199) from (200) and dividing by l,

$$\frac{2V_s'}{3} = \frac{11}{24}W, \quad \text{or} \quad V_s' = \frac{11}{16}W; \quad . \quad . \quad . \quad . \quad (201)$$

hence

$$R_2 = \tfrac{5}{16}W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (202)$$

and

$$Fa = \tfrac{3}{16}Wl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (203)$$

Substituting these values in equation (189) we have

$$\frac{EI\,\delta y}{\delta x} = -\frac{3}{16}Wlx + \frac{11}{32}Wx^2. \quad . \quad . \quad . \quad . \quad (204)$$

If the point of maximum deflection is between O and B, at that point $\frac{\delta y}{\delta x}$ must reduce to 0. Making this hypothesis and deducing the value of x we have

$$x = \tfrac{6}{11}l; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (205)$$

but this is beyond the middle point and equation (204) is not applicable.

Substituting values of V_s', etc., in equations (188), (189), (190), (191), (192), (193), and (194), we have for the segment between B and O

$$\frac{EI\delta^3 y}{\delta x^3}=V_s=\frac{11}{16}W, \quad (206)$$

$$\frac{EI\delta^2 y}{\delta x^2}=M=-\frac{3}{16}Wl+\frac{11}{16}Wx, \quad (207)$$

$$\frac{EI\delta y}{\delta x}=-\frac{3}{16}Wlx+\frac{11}{32}Wx^2, \quad (208)$$

$$EIy=-\tfrac{3}{32}Wlx^2+\tfrac{11}{96}Wx^3; \quad (209)$$

and for the segment between B and R_2,

$$\frac{EI\delta^3 y}{\delta x^3}=V_s=\frac{11}{16}W-W=-\frac{5}{16}W, \quad (210)$$

$$\frac{EI\delta^2 y}{\delta x^2}=M=-\frac{5}{16}Wx+\frac{5}{16}Wl, \quad (211)$$

$$\frac{EI\delta y}{\delta x}=-\frac{5}{32}Wx^2+\frac{5}{16}Wlx-\frac{Wl^2}{8}, \quad (212)$$

$$EIy=-\frac{3}{32}Wlx^2+\frac{11}{96}Wx^3-\frac{Wx^3}{6}+\frac{Wlx^2}{4}-$$

$$\frac{1}{8}Wl^2x+\frac{3}{16}\frac{Wl^3}{2}-\frac{11}{96}Wl^3+\frac{Wl^3}{6}-$$

$$\tfrac{1}{4}Wl^3+\tfrac{1}{8}Wl^3$$

$$=-\tfrac{5}{96}Wx^3+\tfrac{15}{96}Wlx^2-\tfrac{12}{96}Wl^2x+\tfrac{2}{96}Wl^3. \quad (213)$$

From equations (207) and (211) it may be shown that the bending moment has its greatest value at O where

$$M_m=-Fa=-\tfrac{3}{16}Wl. \quad (214)$$

The deflection is greatest in the segment between B and O, at the point when $\frac{\delta y}{\delta x}$ is equal to zero.

Under this hypothesis the maximum deflection will be found to be where $x=\frac{11}{20}l$ approximately.

If this value is substituted in equation (213), the maximum deflection will be found to be

$$y_m=\frac{Wl^3}{108EI}. \quad \ldots \ldots \ldots \quad (215)$$

VIII. Uniformly Loaded Beam Fixed Horizontally at One End and Resting on a Support at the Other.—Let the nomenclature be the same as above, and let w be the weight per unit of length of the uniform load. Then

$$M=\frac{EI\delta^2 y}{\delta x^2}=-Fa+V_s'x-\frac{wx^2}{2}. \quad \ldots \quad (216)$$

Integrating and remembering that where $x=0$, $\frac{\delta y}{\delta x}=0$, we have

$$\frac{EI\delta y}{\delta x}=-Fax+\frac{V_s'x^2}{2}-\frac{wx^3}{6}. \quad \ldots \ldots \quad (217)$$

Integrating and remembering that where $x=0$, $y=0$; we have

$$EIy=-\frac{Fax^2}{2}+\frac{V_s'x^3}{6}-\frac{wx^4}{24}. \quad \ldots \ldots \quad (218)$$

Where $x=l$, M in equation (216), and y in equation (218), are each equal to zero. Hence

$$-Fa+V_s'l-\frac{wl^2}{2}=0,$$

$$-\frac{Fal^2}{2}+\frac{V_s'l^3}{6}-\frac{wl^4}{24}=0.$$

Solving these equations for V_s' and Fa we have

$$V_s' = \tfrac{5}{8}wl,$$

$$R_2 = V_s'' = \tfrac{3}{8}wl,$$

$$Fa = \frac{wl^2}{8}.$$

Substituting these values we have

$$\frac{EI\partial^3 y}{\partial x^3} = V_s = -wx + \frac{5}{8}wl, \quad \ldots \ldots \quad (220)$$

$$\frac{EI\partial^2 y}{\partial x^2} = -\frac{wx^2}{2} + \frac{5}{8}wlx - \frac{wl^2}{8}, \quad \ldots \ldots \quad (221)$$

$$\frac{EI\partial y}{\partial x} = -\frac{wx^3}{6} + \frac{5}{16}wlx^2 - \frac{wl^2x}{8}, \quad \ldots \ldots \quad (222)$$

$$EIy = -\frac{wx^4}{24} + \frac{5}{48}wlx^3 - \frac{wl^2x^2}{16}. \quad \ldots \ldots \quad (223)$$

The bending moment will be greatest either at O or where $V_s = 0$.

At O, $M_m = -\frac{wl^2}{8}$; where $V_s = 0$ or $x = \frac{5}{8}l$, $M = \frac{9}{128}wl^2$.

The greatest deflection will be where $\frac{\partial y}{\partial x} = 0$, or where $x = \frac{9}{16}l$ approximately.

Substituting this value of x in equation (223) we have

$$y_m = \frac{wl^4}{180EI} \text{ approximately.} \quad \ldots \ldots \quad (224)$$

From data deduced in the preceding tables we may construct the following table:

TABLE OF MAXIMUM VALUES OF BENDING MOMENTS, FIBER STRESS, AND DEFLECTIONS IN BEAMS OF UNIFORM CROSS-SECTION, WHOSE LENGTH IS l, AND WHOSE LOAD IS EITHER W OR $wl=W$.

Method of Loading and Supporting.	Maximum Moments, M_m.	Maximum Fiber Stress, $s_m=\frac{M_m}{\frac{I}{y'}}$.	Maximum Deflections, y_m.
Cantilever loaded at end	Wl	$\frac{Wly'}{I}$	$\frac{1}{3}\frac{Wl^3}{EI}$
Cantilever uniformly loaded	$\frac{Wl}{2}$	$\frac{Wly'}{2I}$	$\frac{1}{8}\frac{Wl^3}{EI}$
Beam resting on end supports, loaded at middle point	$\frac{Wl}{4}$	$\frac{Wly'}{4I}$	$\frac{1}{48}\frac{Wl^3}{EI}$
Beam resting on end supports, uniformly loaded	$\frac{Wl}{8}$	$\frac{Wly'}{8I}$	$\frac{5}{384}\frac{Wl^3}{EI}$
Beam fixed horizontally at both ends, loaded at middle	$\frac{Wl}{8}$	$\frac{Wly'}{8I}$	$\frac{1}{192}\frac{Wl^3}{EI}$
Beam fixed horizontally at both ends, uniformly loaded	$\frac{Wl}{12}$	$\frac{Wly'}{12I}$	$\frac{1}{384}\frac{Wl^3}{EI}$
Beam fixed horizontally at one end and resting on end support at the other, loaded at middle	$\frac{3Wl}{16}$	$\frac{3Wly'}{16I}$	$\frac{1}{108}\frac{Wl^3}{EI}$
Beam fixed horizontally at one end and resting on end support at the other, uniformly loaded	$\frac{Wl}{8}$	$\frac{Wly'}{8I}$	$\frac{1}{180}\frac{Wl^3}{EI}$

Stiffness of Beams.—The stiffness of beams varies inversely with their maximum deflections under the same load. From the above table it follows, therefore, that the stiffness varies *inversely* with l^3, the cube of the length; and *directly* with E, the coefficient of longitudinal elasticity of the material. In a rectangular beam I is equal to $\frac{bd^3}{12}$, and the stiffness of beams varies *directly* with the *first power of the width* and the *third power of the depth* of the cross-section.

Strength of Beams.—The strength of beams of the same material or their power to support additional loads varies inversely with the stress on the surface fiber at the dangerous section. From the above table it follows, therefore, that the strength varies *inversely* with l, the length, and *directly* with the *section modulus*. If the beam is rectangular, the section modulus is $\frac{bd^2}{6}$; hence it varies directly with the *first power of the breadth* and the *second power of the depth of the cross-section.*

Maximum Deflections.—In the construction of the floors of buildings, to prevent injury to plastered ceilings, it is customary to limit the deflection in each beam to about $\frac{1}{360}$ of its span. Under this limitation the safe load of the beam may be determined from the formula for maximum deflection, page 86, and not from that giving the maximum fiber stress, since the first may give a smaller safe load than the second.

Resilience.—If a gradually applied load W is placed on the end of a cantilever without weight, the work performed by the weight, within the field of perfect elasticity, will be equal to the resilience of the beam, or

$$\text{Resilience} = \text{work} = \frac{Wy_m}{2}, \quad . \quad . \quad . \quad . \quad . \quad (225)$$

in which y_m is the deflection of the end of the cantilever.

Substituting for W its value in terms of s, the stress at the dangerous section, from the equation

$$s = \frac{Wl}{\frac{I}{y'}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (226)$$

or

$$W = \frac{s\frac{I}{y'}}{l} = \frac{sI}{y'l}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (227)$$

and for y_m, its value obtained by substituting the above value of W in the expression

$$y_m = \frac{Wl^3}{3EI}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (228)$$

$$y_m = \frac{3EIy'l}{sIl^3} = \frac{sl^2}{3Ey'}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (229)$$

we have

$$\text{Resilience} = \text{work} = \frac{\dfrac{sI}{y'l}\,\dfrac{sl^2}{3Ey'}}{2} = \frac{1}{3}\,\frac{s^2}{2E}\,\frac{I}{y'^2}l. \quad . \quad . \quad . \quad (230)$$

Substituting for I its value Ar^2, in which A is the area of cross-section and r is its radius of gyration with respect to the neutral axis, we have

$$\text{Resilience} = \frac{1}{3}\,\frac{s^2}{2E}\,\frac{r^2}{y'^2}Al \quad . \quad . \quad . \quad . \quad . \quad . \quad (231)$$

and

$$\frac{\text{Resilience}}{Al} = \frac{1}{3}\,\frac{s^2}{2E}\,\frac{r^2}{y'^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (232)$$

for the resilience in a unit volume.

If the stress is increased until s = limit of elasticity, $\frac{s^2}{2E}$ becomes the modulus of resilience of the material.

Similar expressions may be determined for resilience of beams otherwise supported and loaded.

Impact.—If a weight W falls through a distance h before it strikes the end of a cantilever without weight, the total amount of work performed before it ceases its motion downwards will be

$$\text{work} = W(h + y_m'), \quad . \quad . \quad . \quad . \quad . \quad . \quad (233)$$

in which W = weight of W in pounds;
h = distance of fall in inches;
y_m' = amount of deflection at end of cantilever in inches.

If the stress is within the field of perfect elasticity, the work of the weight will be equal to the resilience, or

$$W(h + y_m') = \frac{W'y_m'}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (234)$$

in which W' is the intensity of a static weight which applied at the end of the cantilever will produce the same stress and strain as the falling weight.

If we substitute for W' its value in terms of y_m', from the equation $y_m' = \frac{1}{3}\,\frac{W'l^3}{EI}$, or $W' = \frac{3EI}{l^3}y_m'$, we shall have

$$W(h + y_m') = \frac{3}{2}\,\frac{EI}{l^3}y_m'^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (235)$$

from which we can determine y_m', when all the other quantities are given.

If we substitute for $\frac{W'y_m'}{2}$ and y_m' their values from equations (229) and (230), we have

$$W\left(h+\frac{sl^2}{3Ey'}\right)=\frac{1}{3}\,\frac{s^2I}{2Ey'^2}l \quad . \quad . \quad . \quad . \quad (236)$$

or

$$W\left(h+\frac{l^2}{3Ey'}s\right)=\frac{1}{3}\,\frac{I}{2Ey'^2}s^2l, \quad . \quad . \quad . \quad . \quad (237)$$

from which we can determine the maximum stress at the dangerous section when all the other quantities are given.

Similar expressions can be determined for beams otherwise loaded and supported.

PROBLEMS.

30. What is the deflection produced by dropping a weight of 100 pounds from a height of 10 inches on the middle point of a wooden beam 20 feet long, 4 inches wide, 12 inches deep, which rests on end supports? Weight of beam not considered.
Ans. 0.85 inches.

31. What is the greatest unit stress produced in the above problem? *Ans.* 1593.8 pounds.

32. What is the safe uniformly distributed load of a 6-inch I beam resting on end supports 20 feet apart, if the deflection is limited to $\frac{1}{360}$ of the span. $E=30,000,000$; $I=24$; weight of beam 14.75 pounds per linear foot. *Ans.* 118.6 pounds per linear foot.

CHAPTER V.

CONTINUOUS BEAMS, HORIZONTAL SHEAR, COMBINED STRESSES, AND ECCENTRIC LOADING.

Continuous Beams.—A continuous beam is one resting on more than two supports.

From the definition of the bending moment it follows that an expression for the bending moment as a function of x can always be determined at every section of a beam, if we know its loads and the reactions at the supports. Having an expression for the bending moments in terms of x, we can readily deduce the equations and construct the curves of mean fiber, bending moment, and vertical shear.

If the loading is given, we can always determine the reactions at the supports, if the number of such reactions does not exceed the number of equations of equilibrium, which, for parallel forces, is only two.

If the number of supports exceeds two, the reactions cannot therefore be determined by the use of the equations of equilibrium alone; other equations are necessary. These are furnished by the *theorem of three moments*, if the bending moments at the end supports are known or can be readily determined. The theorem applies therefore to beams *not fixed* at the ends.

Theorem of Three Moments.—The deduction of the equation of the theorem of three moments depends upon the principles:

1. That the bending moment at any support of a beam, as B, Fig. 21, is equal to the bending moment at the preceding support A, plus the moment of the shear at the preceding support A about the point B, plus the moment of the load between the supports A and B about the point B. This was shown to be the case in the beam fixed horizontally at both ends, and in the beam fixed horizontally at one end and resting on an end support.

2. Since the mean fiber of an unbroken beam must be continuous, the tangent of the mean fiber at B must be the same, whether we discuss the segment AB or the segment BC.

To simplify the discussion, it will be assumed that each segment of the beam is uniformly loaded.

In Fig. 21 let A, B, C be three consecutive supports of the beam.

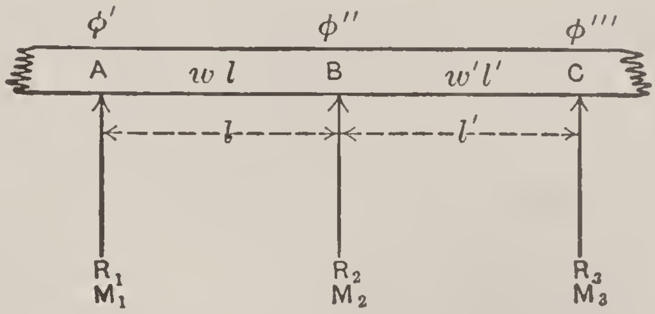

FIG. 21.

Let $AB = l$;
$BC = l'$;
w = load per lineal inch in AB;
w' = " " " " " BC;
M_1, M_2, and M_3 = bending moments at A, B, and C;
R_1, R_2, and R_3 = reactions at A, B, and C;
V_s', V_s'', and V_s''' = vertical shears at sections adjacent to supports A, B, and C on their right;
ϕ'' = angle made by mean fiber of beam with axis of x at B.

Taking the origin at A, and the forces from the left, we have for the bending moment at any section between A and B

$$M = \frac{EI\delta^2 y}{\delta x^2} = M_1 + V_s'x - \frac{wx^2}{2}. \quad . \quad . \quad . \quad . \quad (238)$$

Integrating,

$$\frac{EI\delta y}{\delta x} = M_1 x + \frac{V_s'x^2}{2} - \frac{wx^3}{6} + C. \quad . \quad . \quad . \quad . \quad (239)$$

For $x = l$, $\frac{\delta y}{\delta x} = \tan\phi''$; hence

$$C = EI\tan\phi'' - M_1 l - \frac{V_s'l^2}{2} + \frac{wl^3}{6}. \quad . \quad . \quad . \quad (240)$$

Substituting this value of C in equation (239),

$$EI\left(\frac{\partial y}{\partial x}-\tan\phi''\right)=M_1x+\frac{V_s'x^2}{2}-\frac{wx^3}{6}-M_1l-\frac{V_s'l^2}{2}+\frac{wl^3}{6}. \quad . \quad (241)$$

Integrating,

$$EI(y-\tan\phi''x)$$

$$=\frac{M_1x^2}{2}+\frac{V_s'x^3}{6}-\frac{wx^4}{24}-M_1lx-\frac{V_s'l^2x}{2}+\frac{wl^3x}{6}+C', \quad (242)$$

where $x=0$, $y=0$, hence $C'=0$. Making $x=l$, $y=0$, and we have

$$-EI\tan\phi''=\frac{M_1l}{2}+\frac{V_s'l^2}{6}-\frac{wl^3}{24}-M_1l-\frac{V_s'l^2}{2}+\frac{wl^3}{6} \quad (243)$$

$$=-\frac{M_1l}{2}-\frac{V_s'l^2}{3}+\frac{wl^3}{8}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (244)$$

Making $x=l$ and $M=M_2$ in equation (238) we have

$$M_2=M_1+V_s'l-\frac{wl^2}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (245)$$

hence

$$V_s'=\frac{M_2}{l}-\frac{M_1}{l}+\frac{wl}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (246)$$

Substituting this value in equation (244) we have

$$-EI\tan\phi''=-\frac{M_2l}{3}-\frac{M_1l}{6}-\frac{wl^3}{24}. \quad . \quad . \quad . \quad (247)$$

Taking the origin at B, we have for the bending moment at any section between B and C

$$M=EI\frac{\partial^2y}{\partial x^2}=M_2+V_s''x-\frac{w'x^2}{2}. \quad . \quad . \quad . \quad . \quad (248)$$

Integrating and remembering that where $x=0$, $\frac{\partial y}{\partial x}=\tan\phi''$, we have

$$EI\left(\frac{\partial y}{\partial x}-\tan\phi''\right)=M_2x+\frac{V_s''x^2}{2}-\frac{w'x^3}{6} \quad . \quad . \quad (249)$$

Integrating and remembering that where $x=0$, $y=0$, hence $C'=0$,

$$EI(y-\tan\phi''x)=\frac{M_2x^2}{2}+\frac{V_s''x^3}{6}-\frac{w'x^4}{24}. \quad . \quad . \quad (250)$$

Making $x=l'$, where $y=0$, we have

$$-EI\tan\phi''=\frac{M_2l'}{2}+\frac{V_s''l'^2}{6}-\frac{w'l'^3}{24}. \quad . \quad . \quad (251)$$

Making $x=l'$ and $M=M_3$ in equation (248) we have

$$M_3=M_2+V_s''l'-\frac{w'l'^2}{2},\ \text{hence}\ V_s''=\frac{M_3}{l'}-\frac{M_2}{l'}+\frac{w'l'}{2}. \quad (252)$$

Substituting this value of V_s'' in equation (251) we have

$$-EI\tan\phi''=\frac{M_2l'}{2}+\frac{M_3l'}{6}-\frac{M_2l'}{6}+\frac{w'l'^3}{12}-\frac{w'l'^3}{24}$$

$$=\frac{M_2l'}{3}+\frac{M_3l'}{6}+\frac{w'l'^3}{24}. \quad . \quad . \quad . \quad . \quad . \quad (253)$$

Equating the two values of $-EI\tan\phi$ in equations (247) and (253) and transposing we have

$$M_1l+2M_2(l+l')+M_3l'=-\frac{wl^3}{4}-\frac{w'l'^3}{4}. \quad . \quad . \quad (254)$$

This is the equation of the *theorem of three moments* deduced for continuous beams whose segments are unequal but each uniformly loaded.

It is evident that we can obtain as many such equations as there are supports to the beam less two. Thus we can determine $(n-2)$ independent equations for a beam resting on n

supports. If, therefore, we know the bending moments at two of the supports, we can determine all the others.

If the ends of a continuous beam are not fixed, the bending moments at these supports can always be determined when we know the forces acting on the beam outside of these supports. By substituting these values for M_1 and M_n in the equations above given, the bending moments at all the other supports may be ascertained.

Since the bending moment at each support is simply the resultant moment of the reactions and weights on one side of the support, we can form enough equations of condition containing reactions, loads, and bending moments at the supports to determine all the reactions.

Applications.

I. Beam Uniformly Loaded Resting on Three Supports Equally Spaced.

$$M_1 l + 2M_2(l+l') + M_3 l = -\frac{wl^3}{4} - \frac{w'l'^3}{4}. \quad . \quad . \quad (255)$$

If $l=l'$ and $w=w'$, this becomes

$$M_1 l + 4M_2 l + M_3 l = -\frac{wl^3}{2}. \quad . \quad . \quad . \quad . \quad (256)$$

If the beam terminates at the end supports, by definition, $M_1=0$ and $M_3=0$, and

$$4M_2 l = -\frac{wl^3}{2},$$

$$M_2 = -\frac{wl^2}{8}.$$

If R_1, R_2, and R_3 are the total reactions at the supports, we have

$$(a)\ M_2 = -\frac{wl^2}{8} = R_1 l - \frac{wl^2}{2}, \quad \therefore\ R_1 = \tfrac{3}{8}wl; \quad . \quad . \quad . \quad . \quad . \quad (257)$$

$$(b)\ M_3 = 2R_1 l + R_2 l - 2wl^2 = \tfrac{6}{8}wl^2 + R_2 l - 2wl^2 = 0,$$
$$\therefore\ R_2 = \tfrac{10}{8}wl; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (258)$$

$$(c)\ R_1 + R_2 + R_3 = 2wl = \tfrac{3}{8}wl + \tfrac{10}{8}wl + R_3, \quad \therefore\ R_3 = \tfrac{3}{8}wl. \quad (259)$$

The expression for the bending moment in the segment AB, Fig. 21, is therefore

$$M = R_1 x - \frac{wx^2}{2} = \frac{3}{8} wlx - \frac{wx^2}{2}. \quad . \quad . \quad . \quad (260)$$

In the segment BC, taking the origin at B,

$$M = M_2 + V_s'' x - \frac{wx^2}{2} = -\frac{1}{8} wl^2 + \frac{5}{8} wlx - \frac{wx^2}{2}. \quad . \quad (261)$$

In equation (261) V_s'' is the shear at B.

From these equations we may determine the equations and curves of bending moments, vertical shear, and mean fiber.

II. Beam Uniformly Loaded Resting on Four Supports Equally Spaced.

$$M_1 l + 2M_2(l + l') + M_3 l' = -\tfrac{1}{4} wl^3 - \tfrac{1}{4} w' l'^3, \quad . \quad . \quad (262)$$

$$M_2 l' + 2M_3(l' + l'') + M_4 l'' = -\tfrac{1}{4} w' l'^3 - \tfrac{1}{4} w'' l''^3. \quad . \quad (263)$$

If $l = l' = l''$, $w = w' = w''$, and $M_1 = M_4 = 0$, we have

$$4M_2 l + M_3 l = -\tfrac{1}{2} wl^3, \quad . \quad . \quad . \quad . \quad . \quad . \quad (264)$$

$$M_2 l + 4M_3 l = -\tfrac{1}{2} wl^3. \quad . \quad . \quad . \quad . \quad . \quad . \quad (265)$$

Combining and solving we have

$$M_2 = M_3 = -\tfrac{1}{10} wl^2.$$

If R_1, R_2, R_3, and R_4 are the total reactions at the supports, we have

$$M_2 = R_1 l - \frac{wl^2}{2} = -\tfrac{1}{10} wl^2,$$

$$\therefore \; R_1 = \tfrac{4}{10} wl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (266)$$

$$M_3 = 2R_1l + R_2l - 2wl^2 = \tfrac{8}{10}wl^2 + R_2l - 2wl^2 = -\tfrac{1}{10}wl^2,$$

$$\therefore\ R_2 = \tfrac{11}{10}wl. \qquad (267)$$

$$M_4 = 3R_1l + 2R_2l + R_3l - \tfrac{9}{2}wl^2 = \tfrac{12}{10}wl^2 + \tfrac{22}{10}wl^2 + R_3l - \tfrac{9}{2}wl^2 = 0,$$

$$\therefore\ R_3 = \tfrac{11}{10}wl. \qquad (268)$$

$$R_1 + R_2 + R_3 + R_4 = 3wl = \tfrac{4}{10}wl + \tfrac{11}{10}wl + \tfrac{11}{10}wl + R_4,$$

$$\therefore\ R_4 = \tfrac{4}{40}wl. \qquad (269)$$

The bending moments in these sections are

$$M = \frac{4}{10}wlx - \frac{wx^2}{2}, \text{ origin at first support} = R_1x - \frac{wx^2}{2}; \qquad (270)$$

$$M = -\frac{1}{10}wl^2 + \frac{5}{10}wlx - \frac{wx^2}{2},$$

$$\text{origin at second support} = M_2 + V_s''x - \frac{wx^2}{2}; \qquad (271)$$

$$M = -\frac{1}{10}wl^2 + \frac{6}{10}wlx - \frac{wx^2}{2},$$

$$\text{origin at third support} = M_3 + V_s'''x - \frac{wx^2}{2}. \qquad (272)$$

In which V_s'' and V_s''' are the shears at the second and third supports.

III. Beam Uniformly Loaded Resting on Five Supports Equally Spaced.

$$M_1l + 2M_2(l+l') + M_3l' = -\frac{wl^3}{4} - \frac{w'l'^3}{4}, \qquad (273)$$

$$M_2l' + 2M_3(l'+l'') + M_4l'' = -\frac{w'l'^3}{4} - \frac{w''l''^3}{4}, \qquad (274)$$

$$M_3l'' + 2M_4(l''+l''') + M_5l''' = -\frac{w''l''^3}{4} - \frac{w'''l'''^3}{4}. \quad . \quad (275)$$

If $M_1 = M_5 = 0$, $w = w' = w'' = w'''$ and $l = l' = l'' = l'''$, we have

$$+4M_2l + M_3l = -\frac{wl^3}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (276)$$

$$+M_2l + 4M_3l + M_4l = -\frac{wl^3}{2}, \quad . \quad . \quad . \quad . \quad (277)$$

$$+M_3l + 4M_4l = -\frac{wl^3}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (278)$$

Solving these equations we have

$$M_2 = M_4 = -\tfrac{3}{28}wl^2, \quad M_3 = -\tfrac{2}{28}wl^2. \quad . \quad . \quad . \quad (279)$$

If the total reactions at the supports are R_1, R_2, R_3, R_4, and R_5, we have

$$M_2 = -\frac{3}{28}wl^2 = R_1l - \frac{wl^2}{2}, \quad \therefore R_1 = \frac{11}{28}wl. \quad . \quad . \quad . \quad . \quad (280)$$

$$M_3 = -\tfrac{2}{28}wl^2 = 2R_1l + R_2l - 2wl^2 = \tfrac{22}{28}wl^2 + R_2l - 2wl^2,$$
$$\therefore R_2 = \tfrac{32}{28}wl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (281)$$

We may write out the other equations or assume from symmetry that

$$R_5 = R_1 = \tfrac{11}{28}wl, \quad R_4 = R_2 = \tfrac{32}{28}wl, \quad . \quad . \quad . \quad . \quad (282)$$

and $$R_3 = 4wl - (R_1 + R_2 + R_4 + R_5) = \tfrac{26}{28}wl. \quad . \quad . \quad . \quad (283)$$

The expressions for the moments will be

$$M = \frac{11}{28}wlx - \frac{wx^2}{2} = R_1x - \frac{wx^2}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (284)$$

$$M = -\frac{3}{28}wl^2 + \frac{15}{28}wlx - \frac{wx^2}{2} = M_2 + V_s''x - \frac{wx^2}{2}, \quad . \quad (285)$$

$$M = -\frac{2}{28}wl^2 + \frac{13}{28}wlx - \frac{wx^2}{2} = M_3 + V_s'''x - \frac{wx^2}{2}, \quad . \quad (286)$$

$$M = -\frac{3}{28}wl^2 + \frac{17}{28}wlx - \frac{wx^2}{2} = M_4 + V_s^{IV}x - \frac{wx^2}{2}. \quad . \quad (287)$$

From these expressions we may determine the equations and curves of bending moments, vertical shear, and mean fiber.

Horizontal Shear in Beams of Uniform Cross-section.—The horizontal shear in beams is the shear in surfaces parallel to the neutral surface; the movement of these surfaces on each other may be readily illustrated by making a beam of thin superposed strips.

In the cantilever fixed at its left end, shown in Fig. 22, let

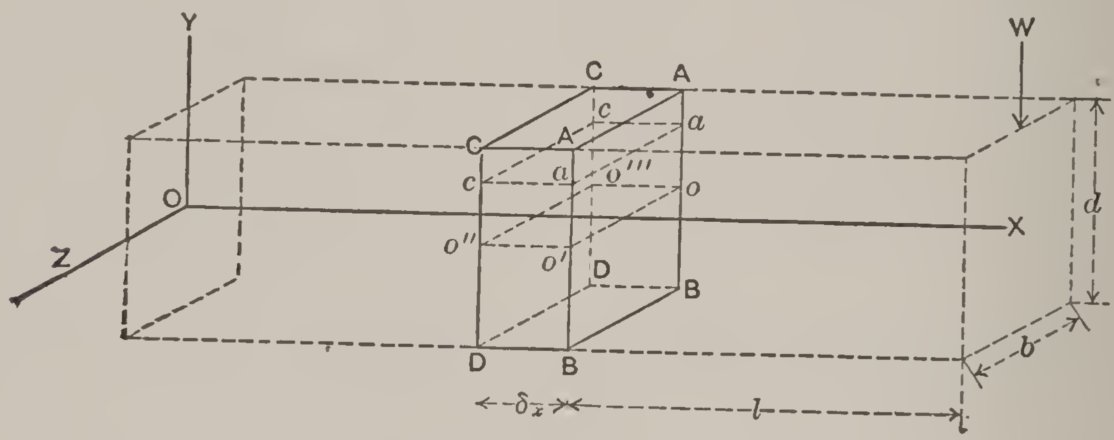

FIG. 22.

OX = its axis = the axis of X;
OY = axis of Y;
OZ = axis of Z;
$AABB$ = plane of cross-section;
l = distance in inches of plane $AABB$ from free end of beam;
$CCDD$ = plane of cross-section;
$l + \delta x$ = distance in inches of plane $CCDD$ from end of beam;
W = a concentrated weight applied at free end;
$M = Wl$ = bending moment at section $AABB$;
$M + \delta M = W(l + \delta x)$ = bending moment at section $CCDD$;
$oo'o''o'''$ = neutral surface of beam between the two planes of cross-section;
oo' = neutral axis of plane $AABB$;
$o''o'''$ = neutral axis of plane $CCDD$;

y = distance of any fiber of beam parallel to the neutral surface from that surface;

y'' = distance of plane $aacc$ from neutral surface;

b = width of beam;

d = depth of beam;

s_1''' = unit bending stress at a unit's distance from oo' in the plane AB;

s_2''' = unit bending stress at a unit's distance from $o''o'''$ in the plane CD;

s_s = unit shearing stress in the horizontal plane $aacc$.

Then $s_1'''y$ = unit tensile stress in a fiber of the plane AB at a distance y from oo';

$s_2'''y$ = unit tensile stress in a fiber of the plane CD at a distance y from $o''o'''$;

$s_1'''y\partial y\partial z$ = stress in the elementary area $\partial y\partial z$ of the plane AB at a distance y from the neutral axis;

$s_2'''y\partial y\partial z$ = stress in the elementary area $\partial y\partial z$ of the plane CD at a distance y from the neutral axis.

$$s_1'''\int_{z=-\frac{b}{2}}^{z=+\frac{b}{2}}\int_{y=y''}^{y=\frac{d}{2}} y\partial y\partial z = \text{tensile stress on area } AAaa \text{ of the plane } AB, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (288)$$

$$s_2'''\int_{z=-\frac{b}{2}}^{z=+\frac{b}{2}}\int_{y=y''}^{y=\frac{d}{2}} y\partial y\partial z = \text{tensile stress on area } CCcc \text{ of the plane } CD. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (289)$$

In the above equations, each double integral is the sum of the products of all the elementary areas of the total area considered, by their distances from the neutral axis; hence each is the static moment of the area considered about the neutral axis.

If A' = area $AAaa$ of the plane AB or of $CCcc$ of the plane CD,

y' = distance of center of gravity of $AAaa$ from oo' or of $CCcc$ from $o''o'''$,

then

$A'y'$ = static moment of area $AAaa$ with respect to oo', or of area $CCcc$ with respect to $o''o'''$.

Substituting this value for the double integral in equations (288) and (289) we have

$s_1'''A'y'$ = stress in area $AAaa$,
$s_2'''A'y'$ = stress in area $CCcc$, and
$(s_2''' - s_1''')A'y'$ = difference of tensile stress on areas $AAaa$ and $CCcc$.

Since the small prism $AAaaCCcc$ is free on all sides but $aacc$, and is in a state of rest, the shearing stress on the base $aacc$ must be equal to the difference between the tensile stresses on $AAaa$ and $CCcc$, or

$$s_s b\,\delta x = (s_2''' - s_1''')A'y'. \quad . \quad . \quad . \quad . \quad . \quad . \quad (290)$$

From equation 82 we have

$$s_1''' = \frac{M}{I} \quad \text{and} \quad s_2''' = \frac{M+\delta M}{I}, \quad . \quad . \quad . \quad (291)$$

in which I = moment of inertia of cross-section $AABB$ or $CCDD$ about the neutral axis.

Substituting in equation (290) we have

$$s_s b\delta x = \left\{ \left[\frac{M+\delta M}{I}\right] - \frac{M}{I} \right\} A'y' = \frac{(\delta M)A'y'}{I}, \quad . \quad (292)$$

or

$$s_s = \left(\frac{\delta M}{\delta x}\right)\frac{A'y'}{bI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (293)$$

But $\dfrac{\delta M}{\delta x} = V_s$ = vertical shear in the cross-section $AABB$. Substituting this value in equation (293) we have

$$s_s = \frac{V_s A'y'}{bI}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (294)$$

or, *the unit horizontal stress, at any point of a surface parallel to the neutral surface, is equal to the quotient obtained by dividing the product of the vertical shear at the point, and the static moment of* *the area of cross-section between this surface and the top of the beam, by the product of the breadth of the beam and the moment of inertia of the cross-section of the beam at the point considered. The units employed are inches and pounds.*

If H_s = total shear in any horizontal surface of a beam,
$(H_s)_m$ = maximum value of H_s,
s_s'' = allowable unit stress in shear,
A = area of horizontal surface in shear,

for safety $s_s''A =$ or $> (H_s)_m$.

If the beam is of uniform cross-section, b and I are constant. Hence in the same horizontal surface, for which $A'y'$ is constant, the value of s_s will vary with V_s; or, *the unit shearing stress in any surface of a beam of uniform cross-section, which is parallel to the neutral surface, will be uniform along the line cut out by any plane of cross-section, and will vary with the vertical shear along any line parallel to the axis of the beam.*

The unit horizontal shear will therefore be uniform at every point of a horizontal surface of a beam whose line of shear is parallel to the axis of X, and the total shear will be equal to s_sbl. The unit horizontal shear will be variable in the horizontal surfaces of a beam whose line of shear is not parallel to the axis of X. The total shear in the plane will be equal to the mean value of s_s multiplied by bl.

In the same *plane of cross-section* V_s will be constant, and the value of s_s will vary with $A'y'$; or, *the unit horizontal shear in any plane of cross-section will be uniform along any line parallel to the neutral axis, and will vary with the static moment of the area included between the surface of the beam and the surface of horizontal shear, along any line perpendicular to the neutral axis.*

In a rectangular beam of uniform cross-section whose depth is d and whose breadth is b,

$$A' = b\left(\frac{d}{2} - y\right), \quad y' = \frac{\frac{d}{2} + y}{2}, \quad \text{and} \quad A'y' = \frac{b}{2}\left[\left(\frac{d}{2}\right)^2 - y^2\right]. \qquad (295)$$

The unit stress will, in any plane of cross-section, vary in a direction perpendicular to the neutral axis with the expression $\left(\frac{d}{2}\right)^2 - y^2$; it will be zero at the surface where $y = \frac{d}{2}$, and will be a maximum at the neutral surface where y = zero. The expression

$$s_s = \frac{V_s A' y'}{bI} = \frac{V_s}{bI} \cdot \frac{b}{2} \cdot \left[\left(\frac{d}{2}\right)^2 - y^2\right] \quad . \quad . \quad . \quad (296)$$

is the equation of a parabola when s_s and y are the only variables; hence the unit horizontal shearing stress in a cross-section of a rectangular beam will, in the direction perpendicular to the neutral axis, vary with the ordinates of a parabola.

In Chapter II we have shown that the unit vertical and horizontal shears at any point in a beam must be equal to each other, hence the vertical shear cannot be uniform in the plane of cross-section as heretofore assumed, but must also be a maximum at the neutral axis.

The error introduced in assuming that the unit vertical shear is uniform in the cross-section of a beam is unimportant, since beams are designed ordinarily to resist bending forces and have a large excess strength to resist vertical shear.

If, in equation (296), we substitute for I its value $\frac{1}{12}bd^3$, and for y its value zero at the neutral axis, we have

$$s_s = \frac{3}{2}\left(\frac{V_s}{bd}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (297)$$

or the unit horizontal shear at the neutral fiber is $\frac{3}{2}$ the unit vertical shear assumed to be uniformly distributed over the area of cross-section.

In a rectangular beam, therefore, the true unit vertical shear at the axis is $\frac{3}{2}$ its assumed value.

PROBLEMS.

33. A cantilever 6 inches wide, 8 inches deep, 10 feet long, supports a weight of 1000 pounds at its outer extremity. Find the maximum unit horizontal shear in the beam. *Ans.* $31\frac{1}{4}$ pounds.

34. A wooden built-up beam 6 inches wide and 12 inches deep is made of three 4×6-inch beams bolted together. When the beam supports a load of 2000 pounds at its middle point, what is

the unit shear at the planes of contact, and what is the total shear on the bolts? *Ans.* Unit shear 18 pounds.
Total " 12,960 "

35. The 8-inch 18-pound I beam is designed to support a load of 5000 pounds placed at its middle point when resting on supports 15 feet apart. Determine the unit horizontal shear at the neutral axis, assuming that the center of gravity of each half-section is 3.6 inches from the neutral axis.* *Ans.* 1564 pounds.

Combined Stresses.

Tension and Flexure.—If a beam is subjected simultaneously to tensile and bending forces, the maximum unit stress will be in the fiber in which the sum of the tensile stresses due to the two forces will be a maximum. This fiber will ordinarily be the surface fiber on the tension side of the dangerous section.

Let F = intensity of the tensile force;
M = bending moment at dangerous section due to the bending force;
A = area of cross-section of beam;
I = moment of inertia of A about the neutral axis;
y' = distance from the extreme fiber to the neutral axis;
s_m = maximum longitudinal unit fiber stress in beam. Then

$$s_m = \frac{F}{A} + \frac{My'}{I} \qquad (298)$$

For safety s_m should not exceed the allowable stress of the material either in tension or in flexure.

Fig. 23 represents an inclined rectangular beam of uniform cross-section, one end of which is fastened in a wall. At the free end it bears a weight W, whose action line makes an angle ϕ with the axis of the beam. Its component $W \cos \phi$ produces tension in the beam, and its component $W \sin \phi$ produces flexure.

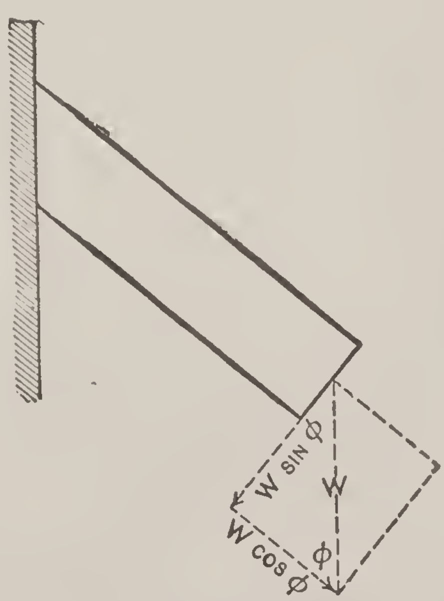

FIG. 23.

* Cambria Handbook, p. 156.

Let l = length of the beam from the free end to the dangerous section;
b = breadth of cross-section;
d = depth of cross-section.

Substituting the above value in equation (298) we have

$$s_m = \frac{W \cos \phi}{bd} + \frac{6Wl \sin \phi}{bd^2}, \quad . \quad . \quad . \quad . \quad . \quad (299)$$

in which s_m is the stress in the fibers of the upper surface in the section at the wall.

Compression and Flexure.—If a beam is subjected simultaneously to compressive and bending forces, the maximum unit stress will be in the fiber in which the sum of the compressive stresses due to the two forces is a maximum. This fiber will ordinarily be the surface fiber on the compression side of the dangerous section.

For compression and bending equation (298) will give the maximum longitudinal unit fibre stress if F = the intensity of the compressive force.

For safety s_m should not exceed the allowable stress of the material either in compression or flexure.

If in Fig. 23 the beam is inclined upwards instead of downwards, making the same angle with the vertical, equation (299) will give the stresses in the fibers of the lower surface in the section at the wall.

PROBLEM.

36. A square wooden cantilever inclined at an angle of 60 degrees projects 12 feet from a wall. What should be its dimensions to sustain a weight of 300 pounds if the allowable unit stress is 1000 pounds? *Ans.* 6.09 inches square.

Flexure and Shear.—A force of flexure produces longitudinal stresses in the fibers of the cross-section of a beam, and shearing stresses in each horizontal plane and in each vertical plane of cross-section.

Thus if $ABCD$, Fig. 24, represents an elementary particle in the tensile side of the beam, on the face AC there will be a bending and shearing stress, and on the face AB a shearing stress.

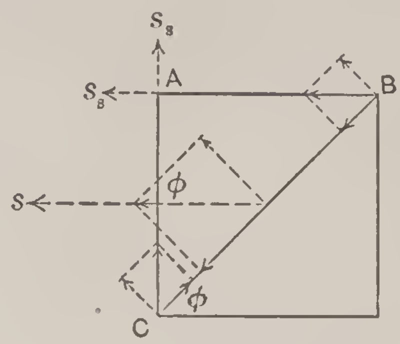

FIG. 24.

The joint effect of the stresses will be to bring a combined stress on some oblique plane BC.

Let it be required to find the maximum unit bending stress and the maximum unit shear on the plane BC.

Let s = unit bending stress on AC;
s_s = " shear on AC and AB. Then

$$s \times AC = \text{total bending stress on } AC;$$
$$s_s \times AC = \text{" shear on } AC;$$
$$s_s \times AB = \text{" " " } AB.$$

Let ϕ = angle ABC;
s' = unit bending stress on BC;
s_s' = " shear on BC.

Resolving the three elementary stresses into their components, perpendicular and parallel to BC, we have for the *total bending stress on BC*

$$s' \times BC = s \times AC \sin\phi + s_s \times AC \cos\phi + s_s \times AB \sin\phi, \quad (300)$$

or

$$s' = \frac{s \times AC \sin\phi}{BC} + \frac{s_s AC \cos\phi}{BC} + \frac{s_s AB \sin\phi}{BC}. \quad (301)$$

But

$$\frac{AC}{BC} = \sin\phi \quad \text{and} \quad \frac{AB}{BC} = \cos\phi;$$

hence

$$s' = s \sin^2\phi + 2s_s \sin\phi \cos\phi.$$

Substituting for $\sin\phi$ its value $\sqrt{\frac{1-\cos 2\phi}{2}}$, and for $2\sin\phi\cos\phi$ its value $\sin 2\phi$,

$$s'=\tfrac{1}{2}s-\tfrac{1}{2}s\cos 2\phi+s_s\sin 2\phi. \quad . \quad . \quad . \quad . \quad (302)$$

To determine the value of 2ϕ which will make s' a maximum, we have

$$\frac{\partial s'}{\partial(2\phi)}=\frac{1}{2}s\sin 2\phi+s_s\cos 2\phi. \quad . \quad . \quad . \quad . \quad (303)$$

Placing the first differential coefficient equal to zero,

$$\tan 2\phi=-\frac{2s_s}{s}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (304)$$

There may then be two angles 2ϕ, differing by 180 degrees, one with a positive sine and negative cosine, the other with a negative sine and positive cosine, each giving a mathematical maximum or minimum, the numerically greater of which will be the value sought. That is,

$$\left.\begin{aligned}\sin 2\phi&=\pm\frac{2s_s}{\sqrt{s^2+4s_s^2}},\\ \cos 2\phi&=\mp\frac{s}{\sqrt{s^2+4s_s^2}}.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (305)$$

By substitution in (302) we have

$$(s')_m=\frac{s}{2}\pm\sqrt{s_s^2+\frac{s^2}{4}}. \quad . \quad . \quad . \quad . \quad . \quad (306)$$

Obviously the upper sign will give the greater, and hence the greatest, numerical value of unit bending stress on any oblique *plane due to combined bending and shearing stresses.*

For the *total shear on BC* we have

$$s_s'\times BC=s\times AC\cos\phi+s_s\times AB\cos\phi-s_s\times AC\sin\phi; \quad (307)$$

or $$s_s' = s\frac{AC}{BC}\cos\phi + s_s \times \frac{AB}{BC}\cos\phi - s_s \times \frac{AC}{BC}\sin\phi \quad . \quad . \quad . \quad (308)$$

$$= s\sin\phi\cos\phi + s_s\cos^2\phi - s_s\sin^2\phi$$

$$= s\sin\phi\cos\phi + s_s(\cos^2\phi - \sin^2\phi).$$

Substituting for $\sin\phi\cos\phi$ its value $\frac{\sin 2\phi}{2}$, and for $\cos^2\phi - \sin^2\phi$ its value $\cos 2\phi$, we have

$$s_s' = \tfrac{1}{2}s\sin 2\phi + s_s\cos 2\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad (309)$$

Differentiating with respect to 2ϕ, we have

$$\frac{\partial s_s'}{\partial(2\phi)} = \frac{1}{2}s\cos 2\phi - s_s\sin 2\phi. \quad . \quad . \quad . \quad . \quad . \quad (310)$$

Placing the first differential coefficient equal to zero, and solving, we have

$$\tan 2\phi = \frac{s}{2s_s}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (311)$$

Hence

$$\cos 2\phi = \pm\frac{2s_s}{\sqrt{s^2+4s_s^2}}, \quad \sin 2\phi = \pm\frac{s}{\sqrt{s^2+4s_s^2}}. \quad . \quad (312)$$

Substituting these values in equation (309), *the maximum value of the unit shearing stress on any oblique plane due to a combined bending and shearing stress* becomes

$$(s_s')_m = \pm\sqrt{s_s^2 + \frac{s^2}{4}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (313)$$

In equations (304), (305) and (306), if $s_s = 0$, as it is in the *surface fiber*, then $\tan 2\phi = 0$, $\cos 2\phi = -1$, $\sin 2\phi = 0$, $2\phi = 180°$, $\phi = 90°$, or BC and AC coincide and $(s')_m = (s)_m$, as it should under this hypothesis. If $s=0$, as it is at the *neutral fiber*, then $\tan 2\phi=\infty$, $\cos 2\phi=0$, $\sin 2\phi=1$, $2\phi=90^\circ$, $\phi=45^\circ$, and $(s')_m=s_s$.

In equations (311), (312) and (313) if $s_s=0$, $\tan 2\phi=\infty$, $\cos 2\phi=0$, $\sin 2\phi=1$, $2\phi=90^\circ$, $\phi=45^\circ$, $(s_s')_m=\frac{s}{2}$, which is the conclusion deduced on page 35; and if $s=0$, $\tan 2\phi=0$, $\cos 2\phi=1$, $\sin 2\phi=0$, $2\phi=360^\circ$, $\phi=180^\circ$. Then $(s_s')_m=s_s$, which is the case in simple shear.

For safety the maximum value of s' for the surface fiber of the beam at the dangerous section must not exceed s'', the safe unit stress in bending for the material, nor must s_s' at the neutral fiber in the section of greatest bending moment exceed s_s'', the safe unit stress in shear. The value of s at any distance from the neutral fiber may be found from equation (87),

$$s=s'''y=\frac{My}{I}.$$

The value of s_s may be found from equation (294),

$$s_s=\frac{V_sA'y'}{bI}.$$

All quantities in these equations must be expressed in inches and pounds.

PROBLEM.

37. In a cantilever 6 inches wide, 8 inches deep, 10 feet long, carrying a weight of 1000 pounds at its outer extremity, find the maximum tensile and shearing stress in surface fiber, in neutral fiber, and in fiber midway between surface and axis at the dangerous section.

Ans. $(s')_m=1875$ pounds at surface, 0 pounds at neutral fiber, 937.9 pounds midway between surface and neutral fiber.

$(s_s)_m=0$ pounds at surface, 31.25 pounds at neutral fiber, 469.2 pounds midway between surface and neutral fiber.

Longitudinal Stress and Torsion.—As torsion is a shearing stress, the formulas above given are also applied to stresses produced by a combined longitudinal and torsional force.

Let s = unit longitudinal stress in the fiber considered;
s_t = " torsional " " " " "

s' = maximum unit longitudinal stress in fiber considered due to combined stress;

s_t' = maximum unit shearing stress in fiber considered due to combined stress. Then

$$s' = \frac{s}{2} + \sqrt{s_t^2 + \frac{s^2}{4}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (314)$$

$$s_t' = \sqrt{s_t^2 + \frac{s^2}{4}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (315)$$

These formulas may also be applied to the determination of the combined stresses of bending and torsion in fibers farthest from the neutral axis of a circular shaft subjected to bending and torsion. They may therefore be applied to the determination of the combined stresses in the surface fiber of the dangerous section due to flexure. This follows from the fact that in these fibers there is no shear due to the bending force.

The value of s may be obtained from equation (88),

$$s = \frac{My'}{I},$$

and s_t from equation (66),

$$s_t = \frac{MR}{I_p}.$$

All quantities in these equations must be expressed in inches and pounds.

PROBLEMS.

38. A circular steel shaft 6 inches in diameter, resting on supports 10 feet apart, transmits 50 horse-power when making 225 revolutions per minute. Find the maximum unit tensile and unit torsional stress in the surface fibers in a vertical plane through the axis due to the combined bending and torsional moments.

Ans. 815.5 pounds per square inch.

39. A circular wrought-iron shaft 12 feet long between bearings while making 130 revolutions per minute transmits 50 horse-power. What should be its diameter if the pulleys are so placed

as to produce a bending moment of 600 (foot-pounds) at its middle point and the maximum combined stress in the upper fiber is limited to 10,000 pounds? *Ans.* 2.54 inches.

Distribution of Stress under Eccentric Loading.—The cross-section of a rod or column is said to be *centrally loaded* when the action line of the resultant force of tension or compression parallel to its axis coincides with that axis. It is said to be *eccentrically loaded* when this action line does not coincide with the axis.

The theoretical distribution of stress over the area of cross-section depends on the hypotheses:

1. The plane of cross-section does not warp or change form under the action of the force.

2. All fibers offer equal resistances to tensile and compressive forces.

3. The amount of stress developed in a fiber is proportional to its strain.

Let *ABCD*, Fig. 25, be any cross-section of a column whose

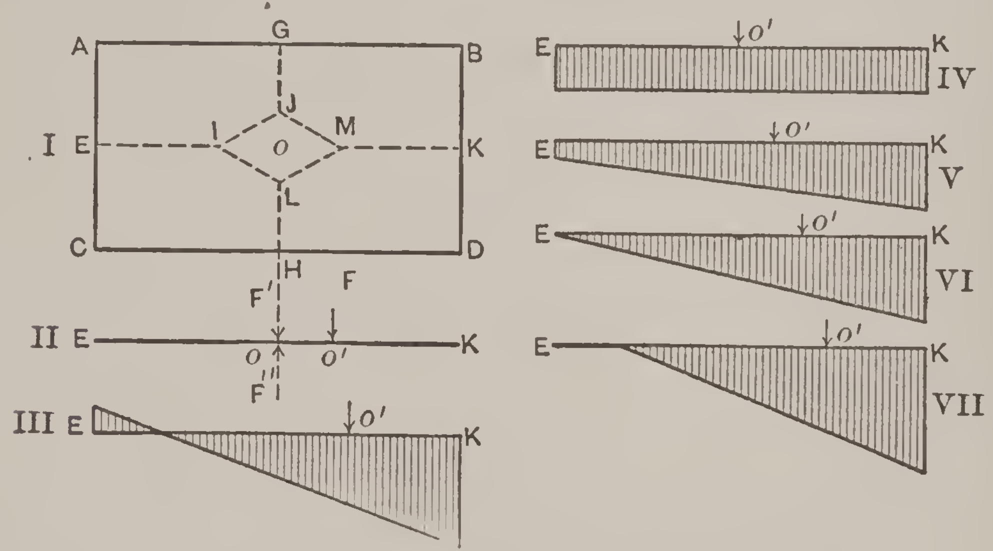

FIG. 25.

lower end is firmly supported, and let *EK*, Fig. 25, be the line cut out of this cross-section by a vertical plane.

Let o = center of gravity of *ABCD*;

A = area of cross-section;

EK and *GH* = axes of symmetry;

$$EK=d;\ \ IM=\frac{d}{3};\ \ JL=\frac{b}{3};$$

$GH=b$;

F = resultant force of compression, parallel to the axis of the column and perpendicular to *ABCD*;

o' = center of pressure or point of application of F in the plane $ABCD$;
s = unit longitudinal stress, due to F, at any point in the area $ABCD$;
o = origin of coordinates;
oK = axis of X;
oG = axis of Y;
x_f and y_f = coordinates of point o', the center of pressure;
x and y = coordinates of any point of area $ABCD$;
r' = radius of gyration of area $ABCD$ about GH;
r'' = " " " " " " " EK.

When the Center of Pressure is on an Axis of Symmetry.— If o' is on the axis EK, as shown in II, Fig. 25, we may replace F by F', an equal and parallel force acting through the center of gravity o in the same direction as F, and the couple $F''F$, in which F'' is a force equal and parallel to F, acting through the center of gravity o in a contrary direction to F'.

The stress on the fibers of $ABCD$ produced by the force F' and the couple FF'' must be the same as those produced by the force acting at o'.

From Chapter I we know that the force F' acting at the center of gravity produces uniform stress over the area of cross-section, and hence if s_1 is the unit stress at any point due to F',

$$s_1 = \frac{F'}{A} = \frac{F}{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (316)$$

The couple FF'' rotates the area $ABCD$ about some line in its plane. Under the hypothesis above given this axis must, as in bending, pass through the center of gravity of the area. Since the point o' is on EK, one of the axes of symmetry, the axis of rotation must be GH, the other axis of symmetry.

Since the cross-section rotates about a line in its plane, the unit stress at any point varies directly with its distance from that axis. The law of distribution must therefore be the same as that for bending, which is

$$s'''y = \frac{My}{I} = \frac{My}{A\,r^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (317)$$

in which $s'''y$ = unit stress at any point;

M = bending moment or moment producing rotation;

y = distance of any point from the axis of rotation or the neutral axis;

I = moment of inertia of cross-section;

A = area of cross-section;

r = radius of gyration of area A about the neutral axis.

If in this expression we substitute s_2 for $s'''y$, Fx_f for M, x for y, and r' for r, the resulting expression

$$s_2 = \frac{Fx_f x}{Ar'^2} \quad \ldots \ldots \quad (318)$$

gives the unit stress at any point of the area $ABCD$ when the moment of rotation is Fx_f and the axis of rotation is GH.

The total unit stress s due to the force F' and the couple FF'' will therefore be

$$s = s_1 + s_2 = \frac{F}{A} + \frac{Fx_f x}{Ar'^2} = \frac{F}{A}\left(1 + \frac{x_f x}{r'^2}\right). \quad \ldots \quad (319)$$

The sign of the second term must be positive, since the couple FF'' will produce in the area $GBDH$, where x is positive, the same kind of stress, compression, as that produced by the force F'; in the area $ACGH$, where x is negative, the stress produced by the couple FF'' will be tension or contrary to that produced by F'. Positive values of s will therefore represent compression, and negative values tension. The area $GBDH$ is the positive and the area $ACGH$ is the negative part of the area $ABCD$ due to the couple acting alone.

In equation (319), if $x_f = 0$, that is the center of pressure is at the center of gravity of the cross-section, $s = \frac{F}{A}$ for all values of x, positive and negative, or the stress over the area $ABCD$ is uniform.

This is in accordance with our hypothesis that a central load *in a column always produces uniform stress in every cross-section.*

If $x=0$, s will be equal to $\frac{F}{A}$ for all values of x_f. *Hence the unit stress at the center of gravity of a cross-section will be the same whether the column be centrally or eccentrically loaded.* The stress at the center of gravity is the *mean stress.*

On the positive side of the cross-section the value of s increases algebraically with x and x_f. If, therefore, we assume any point in the cross-section by making x constant, the corresponding value of s will be greatest when x_f is greatest or equal to $\frac{d}{2}$. If we assume the center of pressure as fixed by making x_f constant, s will be greatest when x is equal to $\frac{d}{2}$. The maximum value of s must be when x and x_f have each their greatest values or are each equal to $\frac{d}{2}$.

Hence *the unit compressive stress at any point of the cross-section of a column eccentrically loaded is greatest when the center of pressure is on the same side of the center of gravity as the point considered, and as far from the axis of rotation as possible.*

For any position of an eccentric load the unit compressive stress is greatest at the point farthest from the axis of rotation on the same side of that axis as the center of pressure.

The maximum unit compressive stress is at the center of pressure when the center of pressure is as far from the axis of rotation as possible.

On the negative side of the cross-section, for which x is negative and x_f positive, the value of s decreases algebraically with x and x_f. If, therefore, we assume any point in the cross-section by making x constant and negative, the corresponding value of s will be algebraically least when x_f is positive and greatest, or equal to $+\frac{d}{2}$. If we assume the center of pressure as fixed and positive by making x_f constant, s will be algebraically least where x is numerically greatest and negative, or equal to $-\frac{d}{2}$.

The minimum value of s must be when x and x_f have each their greatest numerical values but x is negative and x_f is positive, or $x = -\frac{d}{2}$ and $x_f = +\frac{d}{2}$.

The least algebraic stress may be the least positive or compressive stress, or it may be the greatest negative or tensile stress, depending on the sign of s.

Hence *the unit compressive stress at any point of the cross-section of a column is least or the unit tensile stress is greatest when the center of pressure is on the opposite side of the axis of rotation and as far from that axis as possible.*

For any position of the eccentric load the unit compressive stress is least or the tensile stress is greatest at the point farthest from the axis of rotation on the side not containing the center of pressure.

The minimum unit compressive stress or the maximum unit tensile stress is at the point farthest from the axis of rotation on one side of that axis when the center of pressure is as far as possible from the axis on the opposite side.

For any value of x_f all points having the same value of x have the same value of s. Hence *for any position of the center of pressure the unit pressure is uniform along all lines parallel to the axis of rotation of the plane of cross-section.*

To find the position of the center of pressure when the value of s at E is zero, or the distribution of the stress along EK is as shown in VI, Fig. 25, we must substitute for x in equation (319) $-\frac{d}{2}$, make $s = 0$, and solve for x_f. The resulting value of x_f is $\frac{2r'^2}{d}$. For a rectangular cross-section $r' = \sqrt{\frac{d^2}{12}}$, hence

$$x_f = \frac{d}{6}.$$

For a smaller numerical value of x_f, s is positive or the stress at E is compressive; for a larger numerical value of x_f, s is negative or the stress at E is tensile.

Hence the stress over the entire cross-section is compressive if the center of pressure is on the axis of symmetry EK and not more *than* $\frac{2r'^2}{d}$ *from the center of gravity; or in the case of a rectangular cross-section more than* $\frac{d}{6}$ *from the center of gravity.*

If, in equation (319), we make $x_f=\frac{2r'^2}{d}$ and $x=\frac{d}{2}$, we shall have

$$s=s_1+s_2=\frac{2F}{A}=2s'. \quad . \quad . \quad . \quad . \quad . \quad (320)$$

That is, *when the unit stress is zero at one extremity of the axis of symmetry containing the center of pressure, the unit stress at the other extremity is twice the mean stress.*

If the center of pressure is nearer the center of gravity, the distribution of the pressure is as shown in V, Fig. 25; if farther from the center, it is as shown in III, Fig. 25.

If, in equation (319), we make $x_f=\frac{d}{2}$ and $x=\frac{d}{2}$, we shall have

$$s=s_1+s_2=\frac{F}{A}\left(1+\frac{d^2}{4r'^2}\right). \quad . \quad . \quad . \quad . \quad . \quad (321)$$

In the case of a rectangle, in which $r'^2=\frac{d^2}{12}$, this becomes $\frac{4F}{A}$; or, *the maximum unit stress at any point of a rectangular cross-section is four times the mean stress and of the same character as the mean stress.*

If, in equation (319), we make $x_f=\frac{d}{2}$ and $x=-\frac{d}{2}$, we have

$$s=s_1+s_2=\frac{F}{A}\left(1-\frac{d^2}{4r'^2}\right).$$

In the case of a rectangle this becomes $-\frac{2F}{A}$; or, *the minimum unit stress at any point of a rectangular cross-section is twice the mean stress, but of opposite character.*

When the Center of Pressure is Not on One of the Axes of Symmetry.—If the point o' is not on one of the axes of symmetry, the resultant F may be resolved into two components whose points of application are on these axes.

These points of application must also lie on the same straight line through o'.

Let F = intensity of force applied at o';

x_f and y_f = coordinates of point o';

F_1 and F_2 = components of F acting at o'' and o''' on the axes of X and Y;

x_f' = abscissa of F_1;

y_f' = ordinate of F_2;

s = unit stress at any point of surface $ABCD$ due to F;

s_3 = " " " " " " " " " " F_1;

s_4 = " " " " " " " " " " F_2.

Then from equation (316) we have

$$s_3 = \frac{F_1}{A}\left(1 + \frac{x_f'x}{r'^2}\right), \quad \ldots \ldots \quad (322)$$

$$s_4 = \frac{F_2}{A}\left(1 + \frac{y_f'y}{r''^2}\right); \quad \ldots \ldots \quad (323)$$

hence

$$s = s_3 + s_4 = \frac{F_1}{A}\left(1 + \frac{x_f'x}{r'^2}\right) + \frac{F_2}{A}\left(1 + \frac{y_f'y}{r''^2}\right). \quad \ldots \quad (324)$$

If o'' and o''' are equally distant from o', then

$$F_1 = F_2 = \frac{F}{2}, \quad x_f' = 2x_f, \quad \text{and} \quad y_f' = 2y_f.$$

Substituting we have

$$s = \frac{F}{A}\left(1 + \frac{x_f x}{r'^2} + \frac{y_f y}{r''^2}\right). \quad \ldots \ldots \quad (325)$$

For a rectangular cross-section, since $r'^2 = \frac{d^2}{12}$ and $r''^2 = \frac{b^2}{12}$, this becomes

$$s = \frac{F}{A}\left(1 + \frac{12x_f x}{d^2} + \frac{12y_f y}{b^2}\right). \quad \ldots \quad (326)$$

If we determine the stress at any point on EK for any position of o', and then find the point on GH which has the same stress, by substituting this value for s in equation (326), making $x=0$, and solving for y, the line joining these points must be a line of uniform stress, and the line parallel to it through the center of gravity o must be the axis of rotation. The line through o perpendicular to this axis is the line of greatest variation in stress.

It follows that if the center of pressure o' is in the area $oGBK$, the greatest stress at any point of the area $ABCD$ will be at B, and the least stress will be at C.

It also follows that the stress will be compressive at C if the center of pressure lies between o and JM; it will be zero at C if the center of pressure lies on JM; and it will be tension at C if the center lies between JM and B.

Distribution of Stress in Masonry Columns Eccentrically Loaded.—The joints of such a column offer no reliable tensile resistance, hence the minimum stress should never be negative. The stress developed by the force F must always be compressive, and the resultant of the total compressive stress must always pass through o', VII, Fig. 25, which is the center of gravity of the triangle of pressures, if the hypothesis is adopted that the pressure is always uniformly varying.

If $ABCD$ represents a joint in such a column, and the center of pressure is on the axis of symmetry EK, the distribution of stress along EK will be as shown in IV, Fig. 25, if the center of pressure is at the center of gravity; as shown in V, Fig. 25, if the center of pressure is between the center of gravity and $\frac{2r'^2}{d}$; and as shown in VI, Fig. 25, if at a distance $\frac{2r'^2}{d}$ from the center of gravity.

If beyond $\frac{2r'^2}{d}$ from the center of gravity, the distribution will be as shown in VII, Fig. 25, the point of zero pressure on EK being twice the distance from o' that K is from o'.

Since the total compressive stress remains constant as o' moves towards K, the pressure midway between K and the point of zero pressure will be $\frac{F}{A'}$, and the pressure at K will be $\frac{2F}{A'}$, in which A' = area between BD and the limiting line of zero pressure.

As o' approaches K, the area A' will decrease; and finally when o' reaches its limit K, $\frac{F}{A'}$ will be infinite.

As it is desirable to have the pressure over a masonry joint as uniform as possible, the condition is usually imposed that the center of pressure shall never be more than $\frac{2r'^2}{d}$ from the center of gravity of the joint, or in a rectangular joint the center of pressure must never be more than one-sixth the depth of the joint from the center of gravity.

In masonry joints the resultant pressure is often oblique to the joint. The center of pressure of any horizontal joint is, however, the point where this resultant pierces it. If the resultant pressure is oblique, F becomes the component of the resultant pressure normal to the surface of the joint.

PROBLEMS.

40. The base of a block is 7×10 feet, and the center of pressure is one foot from the center of the base, measured parallel to the longer side. The total pressure being 40,000 pounds, find the unit pressure at the corners.

Ans. 228.6 and 914.3 pounds per square foot.

41. At the base of a wall 6 feet wide the pressure at one edge is 20 pounds per square inch; at the opposite edge it is 5 pounds per square inch. What is the mean pressure and where is the center of pressure? *Ans.* 12.5 pounds per square inch.
$x_f = 7.2$ inches.

CHAPTER VI.

MOVING OR LIVE LOADS ON A BEAM.

Bending Moments.—A load which moves over a structure and is not continuously supported by it in the same position is called a *live or moving load.* It is *concentrated* if it acts at a single point; it is *uniformly distributed* if it is either a continuous load of uniform weight or made up of many equal concentrated loads separated by short intervals. A railway train is a live or moving load on a railway bridge. The wheel loads are concentrated loads; the entire train is often treated as a uniformly distributed load. The live load on each elementary part of the structure is so much of the entire live load as is transmitted through it.

A load which is continuously supported by a structure is its *dead or permanent load.* The weight of the superstructure and track is the dead load of a railway bridge. The dead load is usually treated as a uniformly distributed load, but may also be treated as a number of concentrated loads. The dead load in each elementary part of a structure is so much of the entire dead load as is transmitted through it.

Concentrated Live Loads.—I. If a single concentrated load is moved over a beam without weight resting on end supports, *the bending moment at any section of the beam will be numerically greatest when the load is at the section.* Thus in the beam shown in Fig. 26 the bending moment at the cross-section C will be greatest when the weight is at C.

Let clockwise moments be positive.

O = the origin of coordinates;
l = length of the beam;
x = distance of center of gravity of load from O;
d = " " section from O;
W = weight of concentrated load;

R_1 = reaction at $O = \dfrac{W(l-x)}{l}$;

M' = bending moment at C;

M'' = " " " the load;

M_m = maximum bending moment in beam due to live load.

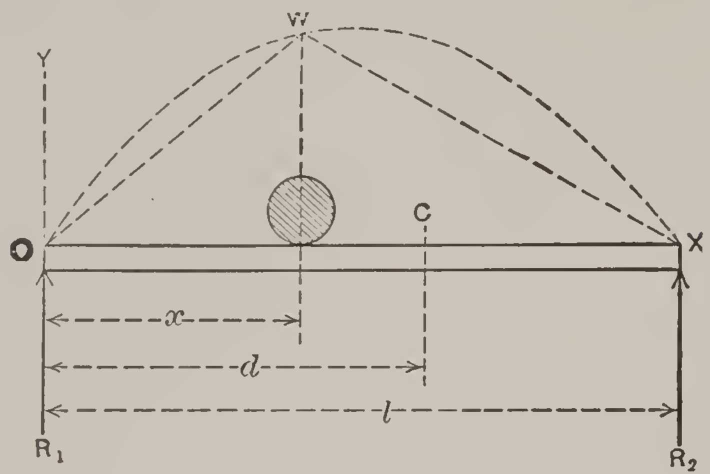

FIG. 26.

1. *While the load moves from O to C* we have

$$M' = R_1 d - W(d-x) = \frac{Wd(l-x)}{l} - W(d-x) = Wx\left(1 - \frac{d}{l}\right). \quad (327)$$

Since $d < l$, M' must be positive and will increase with x, until x reaches its maximum value, d, when we shall have

$$M' = Wd - \frac{Wd^2}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (328)$$

2. *While the load moves from C to X* we have

$$M' = R_1 d = Wd - \frac{Wxd}{l} = Wd\left(1 - \frac{x}{l}\right). \quad . \quad . \quad . \quad (329)$$

Since $x < l$, M' will be positive and will decrease while x increases, until x reaches its maximum value, l, when we shall have

$$M' = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (330)$$

Therefore the live-load bending moment at the section C is greatest when the live load is at C.

II. *The dangerous section is the section at the middle point of the beam.*

If, in equation (328), we substitute x for d, M' becomes M'', the bending moment at the load, and

$$M''=Wx-\frac{Wx^2}{l}. \quad \ldots \ldots \quad (331)$$

If we differentiate this equation with respect to x, and place the first differential coefficient equal to zero, the resulting value of x will be the distance of the concentrated load from O, when the bending moment at the load is a numerical maximum.

$$\frac{\delta M''}{\delta x}=W-\frac{2Wx}{l}=0, \text{ whence } x=\frac{l}{2}. \quad \ldots \quad (332)$$

The bending moment at the load is therefore a maximum when the load is at the middle point of the beam, and

$$M_m=\frac{Wl}{4}. \quad \ldots \ldots \quad (333)$$

These same conclusions are shown graphically in Fig. 26, where the parabola OWX is the curve constructed from equation (331), and the broken line OWX is the line of bending moments from equations (327) and (329), if d is variable and x constant.

Since the broken line lies below the parabola at every point, except W, for every position of the load, it follows that the bending moment at C is greatest when the load is at C, and its bending moment is represented by the ordinate of the parabola.

Since the ordinate of the parabola at any point represents the greatest bending moment at that point, it follows that the middle point, where this ordinate is greatest, will be the dangerous section if the beam is of uniform cross-section.

III. *Propositions I and II are equally true if the beam is of uniform cross-section and weight, and its weight is considered; or if we combine the live with a uniform dead load.*

The resulting bending moment at any cross-section is the sum of the bending moments due to the dead and live loads acting separately. Since the bending moment at any section due to the dead load is independent of the position of the live load, their sum will have its greatest value for any section when the bending moment at that section due to the live load alone has its greatest value, or when the live load is at the section considered.

Since the bending moment at the middle point of the beam is a maximum for both live and dead loads separately, it must be a maximum for their sum. This will therefore be the dangerous section.

IV. If two unequal concentrated loads, separated by a fixed distance, move over a beam without weight resting on end sup-

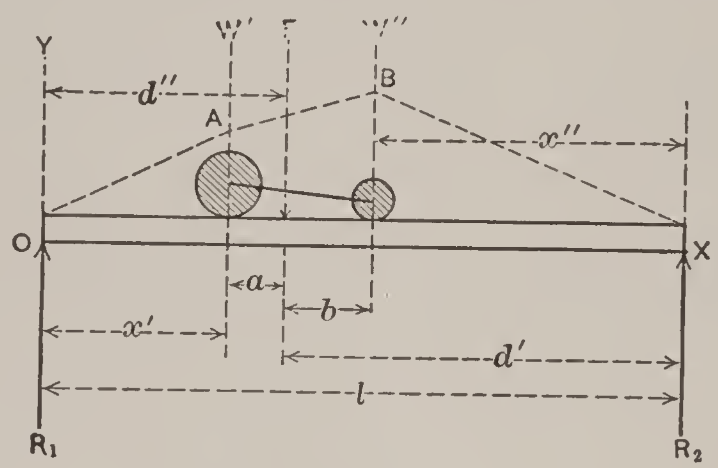

FIG. 27.

ports, *the dangerous section will be at the greater load when that load is as far from one support as the resultant of the loads is from the other.*

In Fig. 27 let W' = greater load,

W'' = lesser load,

F = resultant of W' and W'',

O = origin of coordinates,

d'' = distance of F from O,

d' = " " F " X,

a = " " F " W',

b = " " F " W'',

x' = " " W' " O,

x'' = " " W'' " X,

R_1 = reaction at $O = \dfrac{Fd'}{l}$,

R_2 = " " $X = \dfrac{F(l-d')}{l} = \dfrac{Fd''}{l}$,

$M' =$ bending moment at W',
$M'' =$ " " " W''.

To prove the above proposition, it is necessary to show—

1st. That the bending moment at one of the loads is equal to or greater than the bending moment at any other section of the beam.

2d. That the bending moment under each load is a maximum when the load is as far from one support as the resultant of the two is from the other, or when the middle point of the beam bisects the distance between the load and the resultant.

3d. That the maximum value of M', or M_m', is greater than the maximum value of M'', or M_m''.

1. If we construct the line of bending moments for any position of the two loads, as that shown in Fig. 27, it will be the broken line $OABX$. For different positions of the load, the line AB may be horizontal or slope towards O or X. That is, the bending moment at one or the other of the loads will be equal to or greater than the bending moment at any other section of the beam. This might also be shown analytically.

2. To prove the second statement, for the section at W' we have

$$M' = R_1 x' = \frac{Fx'd'}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (334)$$

But $d' = l - x' - a$, hence

$$M' = \frac{F}{l}(lx' - x'^2 - ax'). \quad . \quad . \quad . \quad . \quad . \quad . \quad (335)$$

If this expression is differentiated with respect to x' and its first differential coefficient is placed equal to zero, the value of x' deduced will be the distance of W' from O when M' is a maximum.

$$\frac{\delta M'}{\delta x'} = \frac{F}{l}(l - 2x' - a). \quad . \quad . \quad . \quad . \quad . \quad . \quad (336)$$

Making the first differential coefficient equal to zero we have

$$\frac{F}{l}(l - 2x' - a) = 0, \quad \text{or} \quad x' = \frac{l-a}{2}. \quad . \quad . \quad . \quad . \quad (337)$$

Therefore when W' is at a distance of $\frac{l-a}{2}$ from O, M' is a maximum. Substituting this value of x' in the value of d' given below equation (334), we have

$$d'=l-x'-a=(l-a)-\frac{l-a}{2}=\frac{l-a}{2}, \quad . \quad . \quad . \quad (338)$$

which is the same as the value of x'. Hence when M' is a maximum, $x'=d'$, or W' is as far from O as F is from X.

If the origin is taken at X, we have for the bending moment at W''

$$M''=R_2x''=\frac{Fx''d''}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (339)$$

But $d''=l-x''-b$, hence

$$M''=\frac{F}{l}(lx''-x''^2-bx''), \quad . \quad . \quad . \quad . \quad (340)$$

$$\frac{\delta M''}{\delta x''}=\frac{F}{l}(l-2x''-b). \quad . \quad . \quad . \quad . \quad . \quad . \quad (341)$$

Making $\frac{\delta M''}{\delta x''}=0$ we have

$$x''=\frac{l-b}{2}.$$

Therefore when W'' is at a distance $\frac{l-b}{2}$ from X, M'' is a maximum.

Substituting this value of x'' in the value of d'' above, we have $d''=\frac{l-b}{2}$. Hence when M'' is a maximum, $x''=d''$, or W'' is as far from X as F is from O.

3. To compare the maximum values of M' and M'', we must substitute for x' and x'', in equations (334) and (339), the values

which correspond to the maximum values of M' and M''. We shall then have

$$M_m' = \frac{Fd'^2}{l} \quad \text{and} \quad M_m'' = \frac{Fd''^2}{l}. \quad . \quad . \quad . \quad (342)$$

Since $a < b$, the value of d', when the middle point of the beam bisects a, will be greater than the value of d'' when the middle point bisects b; hence the maximum value of M' is greater than the maximum value of M''.

Therefore when the greater load is as far from one support as the resultant is from the other, the section at the greater load is the dangerous section.

Uniformly Distributed Live Load.—V. If a live load of uniform weight moves over a beam without weight, resting on end supports, separated by a distance less than the length of the live load, *the bending moment at every section of the beam will be greatest when the live load entirely covers the beam.*

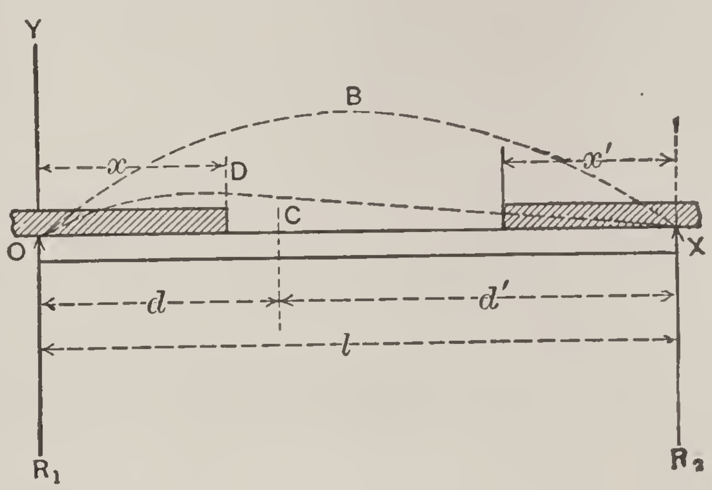

FIG. 28.

In Fig. 28 let

l = length of beam;

a = or $> l$ = length of moving load;

d = distance of section C from O;

d' = " " " " " X;

x = loaded part of beam when load moves on;

x' = " " " " " " " off.

R_1 = reaction at O due to moving load $= \dfrac{wx\left(l - \dfrac{x}{2}\right)}{l}$;

R_2 = " " X;

w = weight of moving load per unit of length;
M' = bending moment at C when D is between O and C;
M_m' = greatest numerical value of M';
M'' = bending moment at C when D is between C and X;
M_m'' = greatest numerical value of M'';
M_m = maximum bending moment in beam due to live load.

1. *While the head of the live load moves from O to C.* The bending moment at C is

$$M'=R_1d-wx\left(d-\frac{x}{2}\right)=\frac{wxd\left(l-\frac{x}{2}\right)}{l}-wx\left(d-\frac{x}{2}\right)$$

$$=wxd-\frac{wx^2d}{2l}-wxd+\frac{wx^2}{2}=\frac{wx^2}{2}\left(1-\frac{d}{l}\right). \quad . \quad . \quad (343)$$

Since $d<l$, M' is positive and increases as x^2 increases, and will be zero when x is zero, and greatest when $x=d$. Substituting d for x in equation (343) we have for the bending moment at C when the head of the load is at C,

$$M_m'=\frac{wd^2}{2}\left(1-\frac{d}{l}\right)=\frac{wd}{2}\times\frac{d}{l}(l-d). \quad . \quad . \quad . \quad (344)$$

2. *While the head of the moving load moves from C to X.* The bending moment at C is

$$M''=R_1d-\frac{wd^2}{2}=\frac{wxd\left(l-\frac{x}{2}\right)}{l}-\frac{wd^2}{2}=wxd-\frac{wx^2d}{2l}-\frac{wd^2}{2}. \quad (345)$$

If this equation is differentiated with respect to x, and the first differential coefficient is placed equal to zero, the resulting value of x will give the distance of D from O when M'' is a maximum:

$$\frac{\delta M''}{\delta x}=wd-\frac{wdx}{l}=0. \quad . \quad . \quad . \quad . \quad . \quad (346)$$

Solving for x we have $x=l$. Substituting this value in equation (345) we have

$$M_m''=\frac{wd}{2}(l-d). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (347)$$

M_m'' must be greater than M_m', since M_m' is equal to M_m'' multiplied by the factor $\frac{d}{l}$, which is less than unity.

To find the value of M_m, the maximum bending moment for any position of the load, we must make d in equation (347) variable and find the maximum value of M_m'':

$$\frac{\partial M_m''}{\partial d}=\frac{wl}{2}-wd=0. \quad . \quad . \quad . \quad . \quad . \qquad (348)$$

Hence $d=\frac{l}{2}$ corresponds to a maximum value of M_m''.

Substituting this value in equation (347) we have

$$M_m=\frac{wl^2}{8}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (349)$$

The middle section is therefore the dangerous section.

If the origin is taken at X, and the distance from X to the tail of the load is represented by x', similar expressions to those given above may be obtained for the bending moment at C, as the tail of the load moves from O to C, and from C to X.

From these expressions it may also be shown as above that the bending moment at C is a maximum when the live load covers the entire beam.

These results are shown graphically in Fig. 28.

The parabola OBX is the curve of bending moments when the load covers the entire beam.

The parabola OD and the straight line DX together form the curve of bending moments for the live load extending from O to D.

It is evident that if $x=0$, the line ODX will coincide with OX, and if $x=l$, the line ODX will coincide with the parabola OBX.

Hence the maximum bending moment M_m'' at C must be the ordinate of OBX at C; and the maximum bending moment of the beam or M_m will be the ordinate of the parabola OBX at the middle point.

VI. *Proposition V is equally true if the beam upon which the live load is moved is of uniform cross-section and weight, and its weight is considered, or if we combine the live with a uniform dead load.*

The resulting bending moment at any cross-section is the sum of the bending moments due to the dead and live loads acting separately. Since the bending moment at any section due to the dead load is independent of the position of the live load, their sum will have its greatest value for that section when the bending moment due to the live load alone has its greatest value or when the live load covers the entire beam.

Since the bending moment at the middle point of the beam is a maximum for both the live and the dead load separately, it must be a maximum for their sum. This will therefore be the dangerous section.

VII. If the uniform live load is shorter than the beam upon which it moves, *the maximum bending moment at any section will be greatest when the section divides the load into segments whose lengths are proportional to the lengths of the segments of the beam on either side of the section.* If the segments of the load and beam to the left of C are L' and l', and on the right of the section L'' and l'', then will the bending moment at C be a maximum when $L' : l' :: L'' : l''$. This proposition may be proved in a manner similar to the other proofs.

PROBLEMS.

42. A wheel weighing 500 pounds is rolled over a beam 20 feet long whose weight is 50 pounds per linear foot. Find the maximum bending moment in the beam in inch-pounds.

Ans. 60,000 (inch-pounds).

43. Two carriage-wheels, separated by a fixed distance of 8 feet, move over a beam without weight, 25 feet long. The load on one wheel is 3000 and on the other 2000 pounds; find the maximum bending moment in the beam.

Ans. 23,762 (foot-pounds).

44. A beam 25 feet long, weighing 50 pounds per linear foot, is subjected to a moving load 30 feet long whose weight per linear foot is 100 pounds. Find the greatest bending moment at a section of the beam 10 feet from either support.

Ans. 11,250 (foot-pounds).

SHEARING STRESS.

Concentrated Load.—VIII. If a concentrated load is moved over a beam without weight resting on end supports, *the shear at any section will be numerically greatest when the load is at the section.*

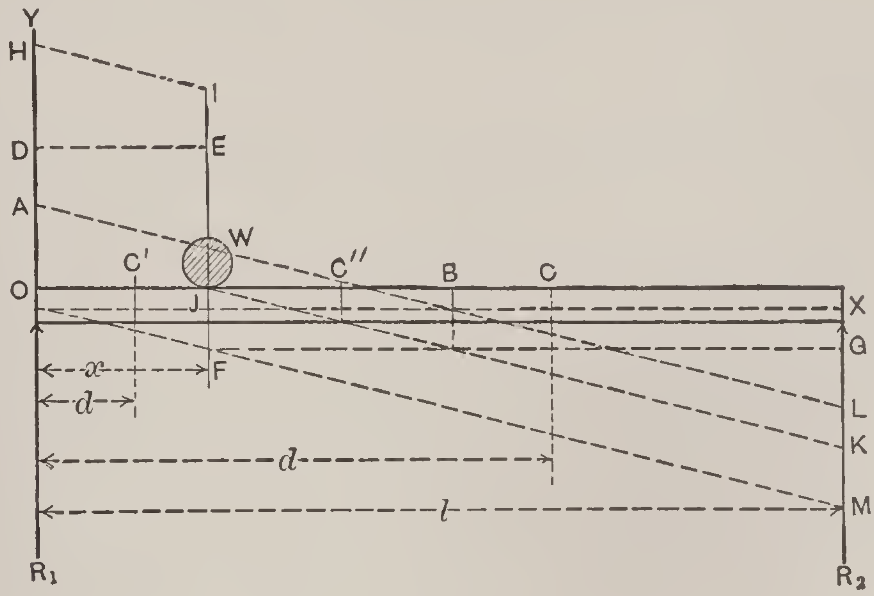

FIG. 29.

In Fig. 29 let O = origin of coordinates;

l = length of beam;

w = weight per unit of length of beam;

W = weight of concentrated load;

d = distance of any cross-section from O;

x = distance of concentrated load from O;

R_1 = reaction at $O = \dfrac{W(l-x)}{l}$ for live load alone,

$= \dfrac{W(l-x)}{l} + \dfrac{wl}{2}$ for both live and dead loads;

V_s'' = shear due to the concentrated load, or the live-load shear;

V_s = resultant shear of live and dead loads.

Considering the live load alone, we have for any section between O and J, as the one marked C',

$$V_s'' = \frac{W(l-x)}{l}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (350)$$

which is positive since $x<l$. In this expression V_s'' will be great-

est when x is least, that is when $x=d$; or the weight is at the section considered or is moved back to C'.

For any section between J and X, as the one marked C, we have

$$V_s''=\frac{W(l-x)}{l}-W=-\frac{Wx}{l}, \quad . \quad . \quad . \quad . \quad (351)$$

which is negative.

This value of V_s'' increases numerically as x increases, and is greatest when $x=d$ or the load moves to C. Hence the shear at any section due to the live load alone is a maximum when the concentrated weight is at that section.

This is shown graphically in the figure. If the line of live-load shear $DEFG$ is plotted from O to X, its ordinates will be positive between O and J, and negative between J and X. The positive ordinate OD increases as the weight W is moved towards O, and the negative ordinate XG increases as the load is moved towards X.

The maximum shear at the supports is greater numerically than the maximum shear at any other section, since the expression $V_s''=\dfrac{W(l-x)}{l}$ is greatest when $x=0$, and $V_s''=-\dfrac{Wx}{l}$ is greatest when $x=l$.

IX. If a concentrated load moves over a uniformly loaded beam, the dead- and live-load shears will have *like signs between the load and the nearer support*, as well as *between the middle point of the beam and the farther support;* but *unlike signs between the load and the middle point.*

For any section between O and W, as the one marked C', Fig. 29, we have

$$V_s=R_1-wd=w\left(\frac{l}{2}-d\right)+W\left(1-\frac{x}{l}\right). \quad . \quad . \quad . \quad (352)$$

In this expression $w\left(\dfrac{l}{2}-d\right)$ is the shear due to the dead load, and $W\left(1-\dfrac{x}{l}\right)$ is the shear due to the live load. If we consider only the part of the beam between O and J, $\dfrac{l}{2}>d$ and $1>\dfrac{x}{l}$, their signs are both positive and their sum is numerically greater than either. This is shown graphically in the figure. At any section between O and J the ordinates of HI, the resultant line of shear, are greater than the ordinates of DE or AB, the component lines of shear.

For any section between B and X, as the one marked C, we have

$$V_s = R_1 - wd - W = w\left(\frac{l}{2} - d\right) - \frac{Wx}{l}, \quad . \quad . \quad . \quad (353)$$

in which $w\left(\frac{l}{2} - d\right)$ is the shear due to the dead load, and $-\frac{Wx}{l}$ is the shear due to the live load. Since $\frac{l}{2} < d$, both terms of the second member are negative, hence the resultant shear at the section is negative and is numerically equal to the sum of the shears of the dead and live loads.

This is shown graphically in the figure. At any section between B and X the ordinate of the resultant line of shear, JK, is equal to the sum of the ordinates of BL and FG, the component lines of shear.

For any section between J and B, as the one marked C'', we have

$$V_s = R_1 - wd - W = w\left(\frac{l}{2} - d\right) - \frac{Wx}{l}. \quad . \quad . \quad . \quad (354)$$

Since $\frac{l}{2} > d$, the first term of the second member, which is the shear due to the dead load, is positive. The resultant shear will be positive if $w\left(\frac{l}{2} - d\right) > W\frac{x}{l}$; it will be zero when $w\left(\frac{l}{2} - d\right) = W\frac{x}{l}$, or when at any section the shear due to the live and concentrated loads is equal; it will be less than zero or negative when $w\left(\frac{l}{2} - d\right) < \frac{Wx}{l}$, or the shear due to the dead load is numerically less than the shear due to the live load.

This is shown graphically in Fig. 29, in which ABL is the line of shear due to the dead load, $DEFG$ is the line of shear due to the live load in the position shown, and $HIJK$ is their resultant line of shear. The ordinates of $HIJK$ are numerically greater than those of ABL between O and J and between B and X; and algebraically less than those of ABL between B and J. The resultant line passes through zero, the point of zero shear being either at W or between W and B.

If we draw the line OM, whose ordinate $XM=EF$, it is the locus of the point F as the live load moves from O to X; its ordinate at any point, as C'', will give the negative live-load shear between C'' and X when the load is at C''. Since there can be no section of zero shear to the left of the load W when the load is to the left of B, the point where the ordinate of AB is equal to that of OM is the farthest limit to which the point of zero shear can move from the middle point to the left. This distance depends upon the relative values of w and W as shown by the value of d deduced from the equation $w\left(\frac{l}{2}-d\right)=\frac{Wx}{l}$; by making $x=d$,

$$d=\frac{wl^2}{2W+2wl}. \qquad (355)$$

There will be a corresponding point on the opposite side of B equally distant from it, since, when the load is at C, the live-load shear between B and C will be positive and the dead-load shear negative. These two points will fix the limits between which the shear at every section must change its sign as the load moves from O to X.

The resultant shear V_s at any section, due to the combined moving and dead loads, will be a numerical maximum when the load is at that section. This arises from the fact that the dead-load shear at any section is constant, and the live-load shear is a maximum of the same sign as the dead-load shear when the load is at the section.

When the live load is at the section adjacent to either support, the shear at that section will be the maximum shear in the beam, since the ordinate OH of the resultant shear will then have its maximum value.

Uniformly Distributed Live Load.—X. If a uniform live load is moved over a uniformly loaded beam, resting on end supports separated by a distance less than the length of the live load, *the shear at any cross-section has one limiting value algebraically when the live load covers the greater segment into which the section divides the beam, and another when the load covers the smaller segment.*

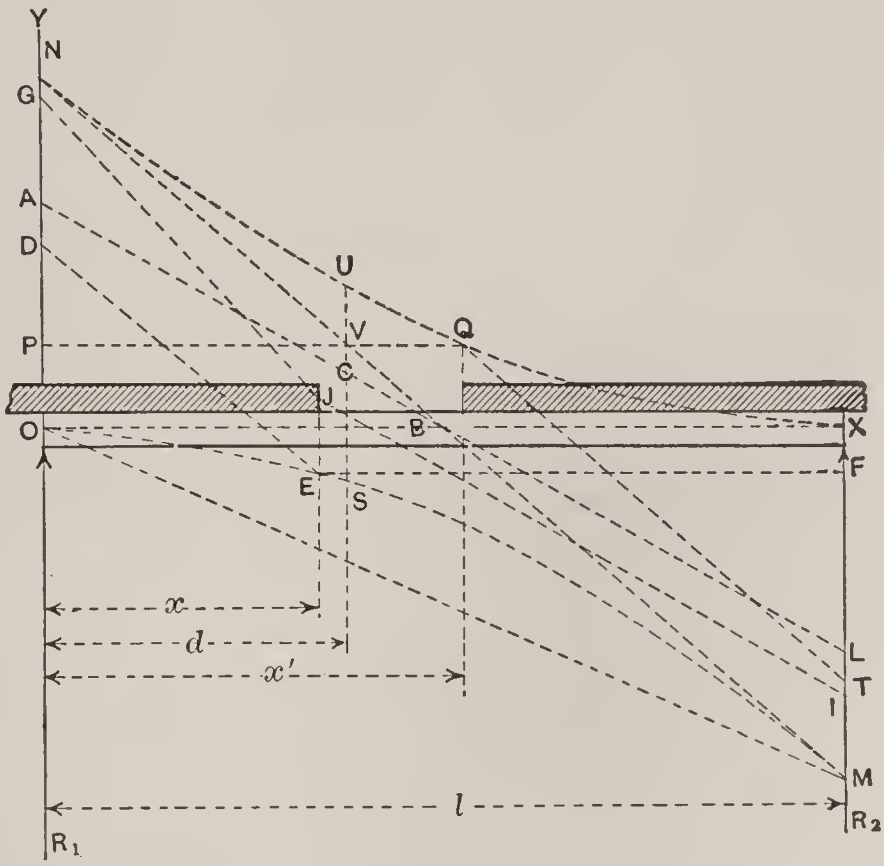

FIG. 30.

In Fig. 30 let l=length of beam;

a=total length of moving load, =or>l;

w=weight per unit of length of beam;

w'=weight per unit of length of live load;

d=distance to any section, as C, at which $d<\frac{l}{2}$, from O;

x=distance of head of live load from O;

x'=distance of tail of live load from O;

R_1=reaction at O;

R_2=reaction at X;

V_s'=dead-load shear at C;

V_s''=live-load shear at C;

$V_s=V_s'+V_s''$=resultant shear at C.

Then will V_s be a minimum when $x=d$, and a maximum when $x'=d$.

1. While the head of the live load moves from O to C we have

$$V_s = R_1 - wd - w'x$$

$$= \frac{wl}{2} + \frac{w'x\left(l - \frac{x}{2}\right)}{l} - wd - w'x$$

$$= w\left(\frac{l}{2} - d\right) - \frac{w'x^2}{2l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (356)$$

The first term of the second member is the dead-load shear at C which is constant for all values of x, and is positive since $\frac{l}{2} > d$. The second term is the live-load shear at C which increases numerically as x^2 increases. The resultant shear will therefore decrease algebraically as x increases, and will have its greatest value when $x = 0$, and its least value when $x = d$.

When $x = 0$,

$$V_s = w\left(\frac{l}{2} - d\right) = V_s'; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (357)$$

when $x = d$,

$$V_s = w\left(\frac{l}{2} - d\right) - \frac{w'd^2}{2l} = V_s' - V_s''. \quad . \quad . \quad . \quad . \quad (358)$$

In equation (358) V_s may be positive, negative, or zero, depending on the relative values of $w\left(\frac{l}{2} - d\right)$ and $\frac{w'd^2}{2l}$; the relative values of these quantities depend upon the ratio $\frac{w}{w'}$. The minimum value is not a numerical minimum, unless the value of V_s in (358) is positive or zero.

2. While the head of the load moves from C to X,

$$V_s = R_1 - (w + w')d$$

$$= \frac{wl}{2} + \frac{w'x\left(l - \frac{x}{2}\right)}{l} - (w + w')d$$

$$= w\left(\frac{l}{2} - d\right) + w'\left(x - d - \frac{x^2}{2l}\right). \quad . \quad . \quad . \quad (359)$$

The first term of the second member is the dead-load shear at C and is identical with the first term of the equation (356). The second term is the live-load shear at C and increases algebraically as x increases, since its first differential coefficient is positive. The resultant shear will therefore increase algebraically as x increases, and will have its least value when $x=d$, and its greatest value when $x=l$.

When $x=d$,

$$V_s=w\left(\frac{l}{2}-d\right)-\frac{w'd^2}{2l}; \quad \ldots \quad (360)$$

When $x=l$,

$$V_s=w\left(\frac{l}{2}-d\right)+w'\left(\frac{l}{2}-d\right)=(w+w')\left(\frac{l}{2}-d\right). \quad \ldots \quad (361)$$

The value of V_s in equation (360) may be positive, zero, or negative, depending upon the relative values of $w\left(\frac{l}{2}-d\right)$ and $w'\frac{d^2}{2}$; the value of V_s in equation (361) will always be positive, since $\frac{l}{2}>d$.

3. While the tail of the load moves from O to C,

$$\begin{aligned} V_s &= R_1-wd-w'(d-x') \\ &= \frac{wl}{2}+\frac{w'(l-x')^2}{2l}-wd-w'(d-x') \\ &= w\left(\frac{l}{2}-d\right)+w'\left(\frac{l}{2}-d+\frac{x'^2}{2l}\right). \quad \ldots \quad (362) \end{aligned}$$

The first term of the second member is the dead-load shear at C and is identical with its values in the preceding equations. The second term is the live-load shear; and since $\frac{l}{2}>d$, it is positive and increases with x'. The resultant shear therefore increases with x', and is least when $x'=0$, and greatest when $x'=d$.

When $x'=0$,

$$V_s=(w+w')\left(\frac{l}{2}-d\right); \quad \ldots \ldots \quad (363)$$

when $x'=d$,

$$V_s=w\left(\frac{l}{2}-d\right)+w'\left(\frac{l}{2}-d+\frac{d^2}{2l}\right). \quad \ldots \quad (364)$$

Since $\frac{l}{2}>d$, the values of V_s given by equations (363) and (364) are both positive.

4. While the tail of the load moves from C to X,

$$V_s=R_1-wd$$

$$=\frac{wl}{2}+\frac{w'(l-x')^2}{2l}-wd$$

$$=w\left(\frac{l}{2}-d\right)+w'\left(\frac{l}{2}-x'+\frac{x'^2}{2l}\right). \quad \ldots \quad (365)$$

The first term of the second member is the dead-load shear and is identical with its values previously given. The second term of the second member is the live-load shear and decreases as x' increases, since its first differential coefficient is negative. The resultant shear will therefore decrease as x' increases, and will be greatest when $x'=d$, and least when $x'=l$.

When $x=d$,

$$V_s=w\left(\frac{l}{2}-d\right)+w'\left(\frac{l}{2}-d+\frac{d^2}{2l}\right); \quad \ldots \quad (367)$$

when $x'=l$,

$$V_s=w\left(\frac{l}{2}-d\right). \quad \ldots \ldots \quad (368)$$

Since $\frac{l}{2}>d$, the values of V_s in equations (367) and (368) are both positive.

Hence we see that the resultant shear at C decreases algebraically while the head of the live load moves from O to C; it increases algebraically while the head of the load moves from C to X and while the tail of the live load moves from O to C, and finally decreases while the tail of the live load moves from C to X. Had the section been taken beyond B, as at Q, so that $\frac{l}{2} < d$, the maximum resultant shear would have been when the head of the load was at Q, and the minimum when the tail was at Q.

Section of Zero Shear.—If, in equation (356), we make $V_s = 0$, d will be the distance from O to the section of zero shear. Solving with respect to d we have

$$d = \frac{l}{2} - \frac{w'}{w} \frac{x^2}{2l}.$$

If we assume $x = 0$, the value of d will give the position of the section of zero shear before the live load comes on the beam. If $x = 0$, $d = \frac{l}{2}$, or the section of zero shear is at the middle point of the beam.

As x increases, d decreases, and the section of zero shear approaches O. When $x = d$, its maximum value in equation (356), the section of zero shear is at the head of the live load and

$$d = \frac{l}{2} - \frac{w'}{w} \frac{d^2}{2l}. \quad \quad (369)$$

Hence

$$d = l \frac{w}{w'} \left(-1 \pm \sqrt{1 + \frac{w'}{w}} \right), \quad \quad (370)$$

or

$$\frac{d}{l} = \frac{w}{w'} \left(-1 \pm \sqrt{1 + \frac{w'}{w}} \right).$$

The ratio of $\frac{d}{l}$ depends therefore on the ratio $\frac{w}{w'}$.

If, in equation (359), we make $V_s = 0$, d will also be the distance from O to the section of zero shear. Solving with respect to d we have

$$d = \frac{\frac{wl}{2} + w'x - \frac{w'x^2}{2l}}{w + w'}. \quad \quad (371)$$

This value of d increases as x increases, hence it will be least when $x=d$ and greatest when $x=l$. Making $x=d$, we have as before

$$d=\frac{lw}{w'}\left(-1\pm\sqrt{1+\frac{w'}{w}}\right). \quad . \quad . \quad . \quad . \quad (372)$$

Making $x=l$, we have $d=\frac{l}{2}$.

These last equations show that as soon as the section of zero shear meets the head of the live load, the section of zero shear moves towards the middle point of the beam and reaches the middle point when the live load covers the entire beam.

As the head of the live load therefore moves from O to X, the section of zero shear moves from B and meets it at some point between B and O, whose distance from O depends upon the ratio $\frac{w}{w'}$; it then moves back to the middle point of the beam.

In a similar manner it may be shown that while the tail of the load moves from O to X, the section of zero shear moves to a corresponding point between B and X, which it reaches simultaneously with the tail of the live load; it then moves back to the middle point. If we substitute for w and w' their values in equations (370) and (372), we may find the distance from O and X to the extreme limits of the position of the section of zero shear; between these limits the shear at every section must change its sign as the live load moves over the beam.

The effect of a live load of uniform weight may also be shown graphically.

In Fig. 30 let

O = origin of coordinates, and OX and OY = coordinate axes;

$OA=\frac{wl}{2}$;

$ON=\frac{w'l}{2}$;

ABL = dead-load line of shear from equation $V_s'=w\left(\frac{l}{2}-x\right)$;

NBM = live-load line of shear when live load extends from O to X from equation $V_s''=w'\left(\frac{l}{2}-x\right)$;

OEM = parabola whose ordinates give live-load shear at the head of the live load, from equation $V_s'' = \frac{-w'x^2}{2l}$; (see equation (356);)

NQX = parabola whose ordinates give the live-load shear at tail of live load from equation $V_s'' = w'\left(\frac{l}{2} - x' + \frac{x'^2}{2l}\right)$; (see equation (362);)

DEF = live-load line of shear when head of live load is as shown to the left of B; OD = reaction at O, XF = reaction at X;

PQT = live-load line of shear when the tail of live load is as shown to the right of B; OP = reaction at O, XT = reaction at X.

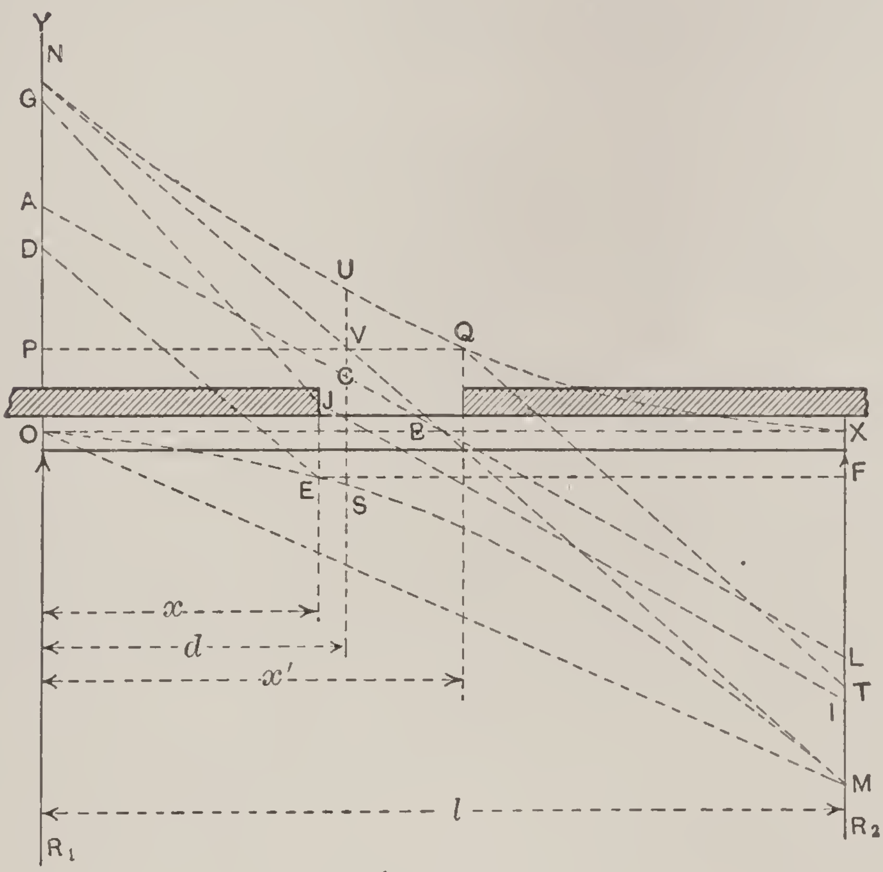

FIG. 30.

The locus of the point E is the parabola OEM, and the locus of the point Q is the parabola NQX.

When the head of the live load is at O, the point E is also at O, and the line EF coincides with OX. The live-load shear is therefore zero at every point, and the resultant shear at any point is given by the ordinates of the line ABL; at the point C the resultant shear is positive.

When the head of the live load is at J, as shown in the figure, the ordinates of the lines ABL and DEF at some point between J and B are equal and the resultant shear at this point is zero. The resultant shear at any other point is found by taking the algebraic sum of the ordinates of lines ABL and DEF as in GJI; at the point B the resultant shear is negative.

When the head of the live load is at X or the tail at O, the resultant shear at any point is found by taking the algebraic sum of the ordinates of ABL and NBM; at the point C the resultant shear is positive, and at B zero.

When the tail of the live load is at Q, as shown in the figure, the resultant shear at any point is the algebraic sum of the ordinates of the lines ABL and PQT.

When the tail of the live load is at X, the live-load shear at every point is zero and the resultant shear at any point is the ordinate of the line ABL.

PROBLEMS.

45. A beam weighing 50 pounds per lineal foot rests on end supports 25 feet apart; a wheel weighing 500 pounds is rolled across it. Find the maximum and minimum shear at the section 10 feet from left support.

Ans. Maximum +425 pounds.
Minimum − 75 "

46. A beam weighing 50 pounds per lineal foot rests on end supports 25 feet apart and also supports a uniformly distributed load which is 30 feet long and weighs 40 pounds per lineal foot which moves over the beam. Find the maximum and minimum shear at the section 10 feet from left support.

Ans. Maximum +305 pounds.
Minimum + 45 "

CHAPTER VII.

COLUMNS.

A COLUMN *is a strut whose length is such that it will perceptibly bend or buckle under a compressive force before it actually breaks.* This takes place when the ratio of *length* to the *least dimension of cross-section* has a certain value depending upon the character of the material. This ratio varies from 5 to 20 for different materials.

It is assumed that in a column, when the unit compression reaches a certain limit, the axial fiber will be deflected in a manner similar to the mean fiber of a beam supporting a load concentrated at its middle point.

If the ends of a column are *square*, that is the end faces are perpendicular to the axis, the curve of the axial fiber will be similar to the curve of mean fiber of a beam fixed at both ends. It will be tangent to the original axis at the ends and will have two points of inflection, one half-way between each end and the middle point (A, Fig. 31). If one end of the column is *square*, and the other *round* or held by a *pin*, the curve of the axial fiber will be similar to the curve of the mean fiber of a beam, having one end fixed and the other resting on a support. It will be tangent to the original axis at the square end and there will be but one point of inflection, which will be about one-third of the height of the column from the square end (B, Fig. 31). If the column has two *round* or *pin-connected* ends, the curve of the axial fiber is similar to the curve of mean fiber of a beam resting on two supports. It has no points of inflection (C, Fig. 31).

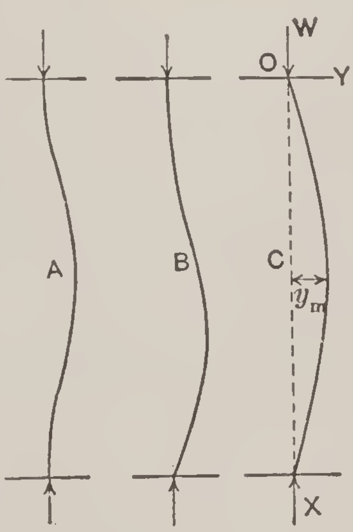

FIG. 31.

Euler's Formula.—The first attempt to deduce a rational formula for the breaking load of a column of uniform cross-section was made by Euler. He assumed a bent column with round ends and ascertained the least intensity of the force which, acting in the line of the original axis, would keep the axis of the column in a bent condition. He assumed this as its breaking weight.

In C, Fig. 31, let

W''' = weight in pounds which placed on the column C will hold it in its bent condition;

l = length of the column in inches;

y_m = maximum deflection of the axial curve in inches;

x and y = coordinates of any point on the axial curve.

The axis of X coincides with the axis of the column before deflection, and the origin is at one end.

Then we have for equilibrium between the moments of the external force and the internal stresses

$$\frac{EI\partial^2 y}{\partial x^2} = -W'''y, \quad \ldots \ldots \quad (373)$$

in which the first member is the moment of resistance and the second is the moment of flexure. Multiplying by $2\partial y$ we have

$$\frac{EI(2\partial y\partial^2 y)}{\partial x^2} = -2W'''y\partial y. \quad \ldots \ldots \quad (374)$$

Integrating,

$$\frac{EI\partial y^2}{\partial x^2} = -W'''y^2 + C. \quad \ldots \ldots \quad (375)$$

Where $y = y_m$, $\frac{\partial y}{\partial x} = 0$, hence $C = W'''y_m^2$. Substituting,

$$\frac{EI\partial y^2}{\partial x^2} = W'''y_m^2 - W'''y^2. \quad \ldots \quad (376)$$

Solving with respect to ∂x^2 we have

$$\partial x^2 = \frac{EI}{W'''}\frac{\partial y^2}{y_m^2 - y^2}. \quad \ldots \ldots \quad (377)$$

Extracting the square root,

$$\partial x = \sqrt{\frac{EI}{W'''}}\frac{\partial y}{\sqrt{y_m^2 - y^2}}. \quad \ldots \ldots \quad (378)$$

Integrating,

$$x=\sqrt{\frac{EI}{W'''}}\sin^{-1}\frac{y}{y_m}+C'. \quad . \quad . \quad . \quad . \quad . \quad (379)$$

Where $x=0$, $y=0$, hence $C'=0$. Transposing and taking the sine of both members, we have

$$\text{sine}\left(x\sqrt{\frac{W'''}{EI}}\right)=\frac{y}{y_m}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (380)$$

which is the equation of a sinusoid. For $x=l$, $y=0$, hence

$$\text{sine}\left(l\sqrt{\frac{W'''}{EI}}\right)=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (381)$$

The arc whose sine is zero is π or some multiple of π. Hence

$$l\sqrt{\frac{W'''}{EI}}=\pi \text{ or some multiple of } \pi. \quad . \quad . \quad . \quad (382)$$

The least value of W''' is derived from solving equation (382), or

$$W'''=\frac{EI\pi^2}{l^2}=\frac{AEr^2\pi^2}{l^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (383)$$

This is Euler's formula for the breaking load of a column with round ends; in it A=area of cross-section, and r=least radius of gyration of cross-section about an axis through its center of gravity.

It will be observed that the value of W''' is independent of y_m, hence a bent column, with any deflection, must be considered unsafe.

To determine expressions for the breaking weight of columns with square ends or with square and round ends, it was assumed that their breaking loads were inversely proportional to their maximum deflections, or to the maximum deflections of their corresponding beams. The maximum deflections of the corresponding beams are given in the table, page 86, under beams loaded at the middle point. The maximum deflection of the beam fixed at both ends is $\frac{1}{192}\frac{Wl^3}{EI}$, that of a beam fixed at one end only is

$\frac{1}{108}\frac{Wl^3}{EI}$, and that of a beam simply resting on its supports is $\frac{1}{48}\frac{Wl^3}{EI}$. These deflections are to each other as $\frac{1}{4}:\frac{4}{9}:1$.

Recent experiments have caused $\frac{1}{2}$, $\frac{3}{4}$, and 1 to be substituted for the former values, $\frac{1}{4}$, $\frac{4}{9}$, and 1.

If W' = breaking load of column A,
W'' = " " " " B,
W''' = " " " " C, then

$$W':W'''::1:\tfrac{1}{2}, \quad \text{or} \quad W'=2W'''=\frac{2EI\pi^2}{l^2}=\frac{2AEr^2\pi^2}{l^2},$$

$$W'':W'''::1:\tfrac{3}{4}, \quad \text{or} \quad W''=\tfrac{4}{3}W'''=\frac{\frac{4}{3}EI\pi^2}{l^2}=\frac{\frac{4}{3}AEr^2\pi^2}{l^2}.$$

As Euler's formulas are based on the hypothesis that the column yields by bending alone, they are more applicable to long columns than to short ones.

Gordon's Formula.—This empirical formula credited to both Tredgold and Gordon is of the following form:

$$W'=\frac{s_c'A}{1+\frac{q'l^2}{b^2}}, \qquad (386)$$

in which W' = breaking load of A, the square-end column, in pounds;
A = area of the cross-section in square inches;
l = length of the column in inches;
b = least dimension of the cross-section in inches;
s_c' = modulus of crushing of the material;
q' = a coefficient, determined by experiments on square-end columns.

The values deduced by Gordon for s_c' and q' were from experiments made by Hodgkinson about 1840, and are no longer employed.

Rankine's Formula.—Gordon's formula for columns with square ends was modified by Rankine, who substituted for b its value in terms of r, the least radius of gyration of the cross-section. The formula then became

$$W' = \frac{s_c'A}{1 + \frac{ql^2}{r^2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (387)$$

For a column of rectangular cross-section $r^2 = \frac{I}{A} = \frac{1}{12}b^2$, hence for a solid column of this cross-section $q = \frac{1}{12}q'$.

Rankine's formula may be deduced from the formula giving the greatest unit stress in a beam due to a force of compression and a force of flexure in the following manner. In A, Fig. 31, let

W' = weight in pounds which will break the column;
s_c' = modulus of crushing of the material;
A = area of cross-section in square inches;
M = bending moment in inch-pounds at the middle point;
I = moment of inertia of the cross-section about an axis through its center of gravity;
x and y = coordinates of the axial fiber of the column;
y' = distance of extreme fiber in the cross-section from the neutral axis.

From equation (298) we have

$$s_c' = \frac{W'}{A} + \frac{My'}{I}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (388)$$

Substituting for M its value at the dangerous section, $W'y_m$, in which y_m = maximum deflection of the column, we have

$$s_c' = \frac{W'}{A} + \frac{W'y_m y'}{I}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (389)$$

Substituting as before for I its value Ar^2, in which r is the least radius of gyration of the cross-section about the neutral axis, we have

$$s_c' = \frac{W'}{A} + \frac{W'y_m y'}{Ar^2}, \quad \text{or} \quad W' = \frac{s_c'A}{1 + \frac{y_m y'}{r^2}}. \quad . \quad . \quad (390)$$

In this formula we have the value of W' in terms of the maximum deflection, and quantities depending on the character of the material and upon the form and dimensions of the cross-section. If we can obtain the value of the maximum deflection in terms of the length, and substitute it in this formula for y_m, we shall have a working formula applicable to columns of all materials, of all lengths, and of all dimensions and forms of cross-section. To obtain such a value of y_m we assume that the ratio of the maximum stress to the maximum deflection is the same in this column as in a beam fixed horizontally at both ends and loaded at the middle point. From the table, page 86, we have

$$s_m = \frac{Wly'}{8I} \quad \text{and} \quad y_m = \frac{Wl^3}{192EI}.$$

Solving each equation with respect to $\frac{Wl}{I}$, and equating the results, we have

$$y_m = \frac{s_m l^2}{24Ey'}.$$

If we replace $\frac{s_m}{24E}$ by q, we have

$$y_m = \frac{ql^2}{y'},$$

which substituted in equation (390) gives

$$W' = \frac{s_c'A}{1 + \frac{ql^2}{r^2}}.$$

This is Rankine's formula for the breaking load of a column with square ends.

The suppositions made in this deduction are only approximately true for columns, as may be seen by comparing the values of q obtained by experiment with those obtained by substituting the modulus of rupture of any material for s_m and its coefficient of longitudinal elasticity for E in the equation

$$q = \frac{s_m}{24E}.$$

Round-end Column, etc.—In the Rankine formula for the breaking load of square-end columns, which transformed is

$$s_c' = \frac{W'}{A} + \frac{W'ql^2}{Ar^2},$$

the unit stress, s_c', is the sum of the unit stress $\frac{W'}{A}$ due to simple compression, and the unit stress $\frac{W'ql^2}{Ar^2}$ due to flexure. The unit stress due to compression is assumed to be independent of the form of the ends; if, therefore, we represent the breaking loads of square, square and round, and round-end columns by W', W'', and W''', the unit compressive stresses will be $\frac{W'}{A}, \frac{W''}{A}, \frac{W'''}{A}$. The unit stress due to flexure, however, is assumed to vary for the same load with the form of the ends. Assuming that this variation is as in Euler's formulas, the unit stresses in flexure for square-end, round and square-end, and round-end columns will be as $1 : \frac{3}{2} : 2$. The total unit stresses in the round and square-end, and round-end columns will therefore be

$$s_c' = \frac{W''}{A} + \frac{3W''ql^2}{2Ar^2}, \quad \text{or} \quad W'' = \frac{s_c'A}{1 + \frac{3ql^2}{2r^2}};$$

$$s_c' = \frac{W'''}{A} + \frac{2W'''ql^2}{Ar^2}, \quad \text{or} \quad W''' = \frac{s_c'A}{1 + \frac{2ql^2}{r^2}}.$$

Working Formulas.

If W' = breaking load of a square column,

A = its area of cross-section,

b = least dimension of cross-section,

$\frac{W'}{A}$ = its breaking load per square inch of cross-section,

f = its factor of safety,

w' = its safe load,

$\frac{w'}{A}$ = its safe load per square inch of cross-section, we shall have for *Gordon's formulas:*

$$W' = \frac{W'A}{A} = fw' = \frac{w'fA}{A} = \frac{s_c'A}{1 + \frac{q'l^2}{b^2}} \text{ (square-end column);}$$

$$W'' = \frac{W''A}{A} = fw'' = \frac{w''fA}{A} = \frac{s_c'A}{1+\dfrac{3q'l^2}{2b^2}} \left.\right\} \text{square and round-end column;}$$

$$W''' = \frac{W'''A}{A} = fw''' = \frac{w'''fA}{A} = \frac{s_c'A}{1+\dfrac{2q'l^2}{b^2}} \left.\right\} \text{round-end column.}$$

Rankine's formulas may be derived from Gordon's formulas by substituting $\frac{q}{r^2}$ for $\frac{q'}{b^2}$.

The factors now used with these formulas are the following:

WOODEN COLUMNS (GORDON'S FORMULA).

	s_c'	q'
Yellow pine	5,000	$\frac{1}{250}$
Oak	4,500	"
White pine	3,500	"

CAST-IRON COLUMNS (GORDON'S FORMULA).

	s_c'	q' (circular).	q' (rectangular).
Hollow cast iron	80,000	$\frac{1}{800}$	$\frac{1}{1067}$

STEEL AND WROUGHT-IRON COLUMNS (RANKINE'S FORMULA).

	s_c'	q
Medium steel	50,000	$\frac{1}{36000}$
Soft steel	45,000	"
Wrought iron	40,000	"

Factors of Safety.—The factors of safety employed with these formulas are: *five* for *wooden columns; eight* for *cast-iron columns; four* for *steel and wrought-iron columns* in *buildings; five* for *steel and wrought-iron columns* in *bridges.*

By the use of either the Gordon or the Rankine formula it is easy to determine either W', the breaking load, or w', the safe load, $\frac{W'}{A}$, the breaking load per square inch of cross-section, or $\frac{w'}{A}$, the safe load per square inch of cross-section, if all the dimensions of the column are given. The problem of determining the dimensions of cross-section corresponding to a given breaking or safe load is a much more difficult problem. This involves the solution of equations of a higher degree than the second.

For the above reason tables of breaking or safe loads for columns of different lengths and different dimensions of cross-section are found in all standard engineering manuals, and the engineer makes use of these whenever he is required to find the dimensions of cross-section corresponding to any given load and length.

Other Formulas.—The formulas now generally applied to wooden columns are right-line formulas, or those containing only the first powers of l and b. These formulas are easy of application. They are usually of the form

$$\frac{w'}{A}=s_c-\frac{cl}{b}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (396)$$

in which w' = total safe load of a square column;
A = area of cross-section;
s_c = safe unit stress of material in compression;
l = length of column in inches;
b = least dimension of cross-section in inches;
c = a constant determined by experiment.

Stanwood's right-line formula for white-pine columns with square ends is of this form:

$$w_1=800-\frac{10l}{b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (397)$$

In this formula w_1 is the safe load per square inch, or $\frac{w'}{A}$.

Other formulas of this class are found in engineering handbooks.*

Eccentric Loading.—The column formulas given are based on the hypothesis that the load acts along the axis of the column. If the load is placed eccentrically, the safe load must be decreased so that the maximum fiber stress in the cross-section shall not exceed its safe value.

Built-up Columns.—In order that there should be no waste of material, it is desirable that a column should be equally strong to resist bending in all directions. This is only true when the column is of circular cross-section either solid or hollow. If the cross-section is one of the regular polygonal forms which can be inscribed in a circle, as a hexagon or square, it approximately fulfills the required condition.

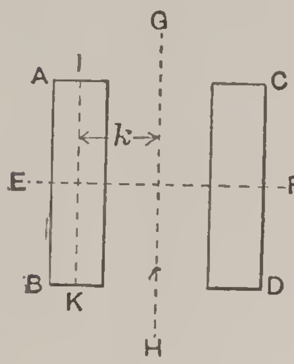

FIG. 32.

A *built-up column* is one made of two or more longitudinal pieces, so fastened together as to act as a single strut. The aim in designing built-up columns is to design a cross-section which shall act likè a regular inscribed polygon, and be as nearly as practicable equally strong to resist bending in all directions.

If a column is made up of two rectangular pieces, as those shown in cross-section in Fig. 32, it will fulfill the required conditions as nearly as practicable if the unit stresses on the edges AB and AC are equal. They will be equal to each other if the moment of inertia or radius of gyration of the combined section about GH is equal to that about EF. This follows from the values of s_c', page 147.

Therefore let it be required to find the distance between the inner faces of the rectangles AB and CD when the moments of inertia of the combined cross-section about EF and GH are equal to each other.

Let A = area of each rectangle;
I' = moment of inertia of the rectangle AB about EF;
r = radius of gyration " " " " " ";
i' = moment of inertia " " " " " IK;
r' = radius of gyration " " " " " ";
I'' = moment of inertia " " " " " GH.

* Cambria Handbook, pp. 368, 369.

The moment of inertia of the combined section about EF is therefore

$$2I'=2\ Ar^2. \quad \ldots \ldots \ldots \quad (398)$$

The moment of inertia of the combined section about GH may be determined from the following principle of mechanics:

The moment of inertia of any mass with reference to any axis is equal to the moment of inertia about a parallel axis through its center of gravity, plus the product of the mass into the square of the distance between the two axes.

Hence we have for the moment of inertia of the rectangle AB about the axis GH through the center of gravity of the combined section

$$I''=i'+Ak^2=Ar'^2+Ak^2, \quad \ldots \ldots \quad (399)$$

in which k = distance between the axes IK and GH, or between the center of gravity of the area of the rectangle AB and the center of gravity of the combined section.

For the combined section we have

$$2I''=2Ar'^2+2Ak^2. \quad \ldots \ldots \quad (400)$$

By hypothesis the distance between IK and GH is such that $2I''=2I'$, hence

$$2Ar^2=2Ar'^2+2Ak^2, \quad \ldots \ldots \quad (401)$$

or

$$r^2=r'^2+k^2, \quad \text{or} \quad k=\sqrt{r^2-r'^2}, \quad \ldots \ldots \quad (402)$$

from which we can determine the value of k, when r and r' are known. The distance between the inner faces of the two rectangular pieces is $2k-b$, in which b is the breadth of the cross-section AB.

If r' is so small with respect to r that its value in the second member may be omitted, we have

$$k=r. \quad \ldots \ldots \ldots \quad (403)$$

This value of k is slightly greater than its true value.

In practice it is not unusual to make $k=r$ in built-up metal columns.

Application.—Let the dimensions of each of the rectangular beams be 16×4 inches. Then

$$r^2 = \frac{I'}{A} = \frac{d^2}{12} = \frac{16\times16}{12} = \frac{64}{3} = 21.3,$$

$$r'^2 = \frac{i'}{A} = \frac{b^2}{12} = \frac{4\times4}{12} = \frac{4}{3} = 1.3;$$

hence $r^2 = r'^2 + k^2$ becomes

$$21.3 = 1.3 + k^2,$$
$$k^2 = 20,$$
$$k = 4.47 + \text{ inches.}$$

In this problem if k had been made equal to r, its value would have been $\sqrt{21.3}$ or $4.62+$; the difference between the two values is only 0.15 of an inch.

Having determined the value of k we may easily find the distance between the inner faces of the rectangles, which may be represented by b', by substituting the value of k and b in the equation

$$b' = 2k - b = 4.94 \text{ inches.}$$

In structural-metal handbooks the values of r, r', and the distance of the center of gravity from the inside face are given for all structural forms used in designing columns of wrought iron and steel.* The spacings for standard channels are also given in handbooks.†

Column Design.—Wooden columns may be solid, and of square or circular cross-section, or built-up in the form of a box or other design; cast-iron columns are hollow and of circular or rectangular cross-section; wrought-iron and steel columns are built-up columns formed of plates, channels, angles, Z and I bars, so arranged as to give a cross-section whose moments of inertia are approximately equal about bisecting lines parallel to different sides of the cross-section. A very common form is made of two channels placed back to back or face to face (Fig. 33), so spaced

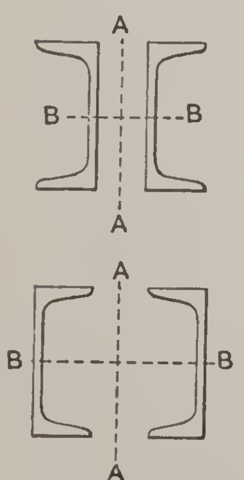

FIG. 33.

* Cambria Handbook, pp. 158–183. † Ibid., p. 219.

that the radius of gyration of the cross-section about AA shall be equal to the radius of gyration about BB.

To find the number of square inches in the cross-section of a column to support any given weight, or to determine the breaking weight of any given column, we must substitute in the formulas given the values of the factors c and q, the values of A and r in terms of d in inches, and the value of l in inches. The resulting equations must then be solved either for W or d. The values of r in terms of d for the cross-sections employed in the design of wooden and cast-iron columns are:

Rectangle, $d =$ shorter side, $r^2 = \dfrac{d^2}{12}$;

Circle, $d =$ diameter, $r^2 = \dfrac{d^2}{16}$;

Hollow square, $d =$ outer diameter, $r^2 = \dfrac{d^2 + d'^2}{12}$;

Hollow circle, $d' =$ inner diameter, $r^2 = \dfrac{d^2 + d'^2}{16}$.

The values of r in terms of d for steel and wrought-iron shapes employed in constructing built-up columns will be found in structural-metal handbooks. To save the work of calculating areas of cross-section, tables are given in engineering handbooks in which the ultimate unit stresses are tabulated for columns of different materials, to correspond to all ordinary ratios of $\dfrac{l}{d}$ or $\dfrac{l}{r}$.*

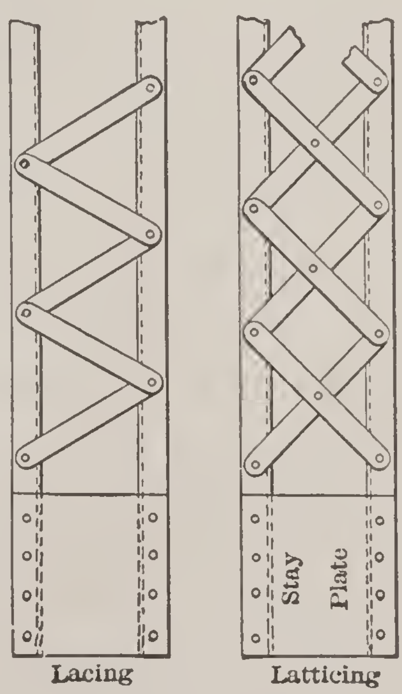

FIG. 34.

Stay-plates, etc.—At their ends, the channels of a steel or wrought-iron built-up column are fastened together by *stay-* or *batten-plates*, and between the stay-plates the channels are held together by *lacing-* or *lattice-bars* (Fig. 34). Lacing is formed of

* Cambria Handbook, pp. 192–195.

single strips; latticing or double lacing is formed of double strips, often riveted together at their intersections as shown in the figure.

The length of the stay-plates is usually at least equal to the greatest dimension of the cross-section of the column. The greatest distance between the rivets of the lacing or latticing on each channel may be determined by the proportion

$$l:r'::L:r, \quad \ldots \ldots \ldots \quad (404)$$

in which l = distance between the lacing-rivets of the same channel;
r' = *least radius of gyration* of a single channel;
L = length of the column;
r = least radius of gyration of the column section, which is also the *greatest radius of gyration* of a single channel.

Each channel is therefore divided into short columns between rivets, each of which is as stiff to resist buckling as the column itself. In practice the distance between rivets is less than this theoretical distance; the angle between the lacing-strips is usually 60 degrees, and between latticing-strips 90 degrees. The values of r' and r are given in structural-metal handbooks.*

The thickness of the stay-plates and the lacing-bars may be computed on the theory that the column, if used as a beam, should be as strong to resist shearing when the channels are placed with the axis BB, Fig. 33, vertical as it is when placed with the axis BB horizontal. In the first position we may assume the total shear to be resisted by the lacing-strips, and in the second by the webs of the channels themselves.

If we determine the greatest central load which can be borne by the channel webs without buckling or shearing, the stay-plates and lacing-bars may be proportioned to bear the same load. They may be computed as a truss by the methods hereafter described.

Application.—Let it be required to design, with the aid of a structural-metal handbook, a medium-steel bridge column of two channels with pin-connected ends. The column is to be 20 feet long and to safely resist a compressive force of 40,000 pounds.

Consulting a handbook we find a table giving the *strength*

* Cambria Handbook, pp. 162–165, columns 11 and 14.

of steel columns,* in which the unit breaking stress of columns with different ends is given for the different values of the ratio of the length in feet to the least radius of gyration in inches, or $\frac{L}{r}$. We shall call this Table A. Before we can use this table, therefore, we must know approximately the radius of gyration of the channel about the axis BB, Fig. 33. If we turn to the table giving the *properties of standard channels*,† which we shall call Table B, we find that the values of the radii of gyration about the axis BB, or the greatest radii of gyration, except for the very lightest and very heaviest channels, vary between 2 and 4 inches.

For light columns we shall therefore have $r=2$ inches, for intermediate columns $r=3$ inches, and for heavy columns $r=4$ inches. Unless the character of the column is known, it is usual to enter Table A with the value of r for intermediate columns, or 3 inches.

Since the length of our column is 20 feet, the most probable value of $\frac{L}{r}$ is 6.7; opposite the ratio 6.7 in Table A ‡ we find the ultimate unit strength of a pin-connected column, by interpolation, to be 36,800, and above the table we find the factor of safety for bridges to be 5. The allowable unit stress is therefore 7360 pounds. Since each channel must support a weight of 20,000 pounds, its area of cross-section should be approximately 2.7 square inches. Turning to Table B § we find the *lightest* channel having this area to be the 7-inch, 9.75-pound channel whose area is 2.85 square inches and whose radius of gyration is 2.72 inches. The ratio $\frac{L}{r}$ for this channel is 7.4 and the corresponding allowable unit stress 6954 pounds. Its total allowable stress is therefore 6954×2.85, or 19,819 pounds, a little less than 20,000. The next lightest channels are the 6-inch, 10.5-pound channel, whose area of cross-section is 3.09 square inches and whose radius of gyration is 2.21 inches; and the 8-inch,

* Cambria Handbook, pp. 194, 195.
† Ibid., pp. 162–164.
‡ Ibid., p. 194.
§ Ibid., p. 162.

11.25-pound channel, whose area of cross-section is 3.35 square inches and whose radius of gyration is 3.10 inches. The value of $\frac{L}{r}$ for the former is 9, and for the latter 6.4. The latter corresponds more nearly to the probable value in our table, hence we will adopt it in our next trial. The allowable unit stress for this channel is 7532 pounds; hence the allowable stress of the entire cross-section is $3.35 \times 7532 = 25,232$ pounds, which is greater than 20,000 pounds. The adopted channel is therefore the 8-inch, 11.25-pound channel.

For the distance between the backs of the channels we may employ the rule heretofore deduced that *the distance between the centers of gravity of the channels should be equal to twice the radius of gyration of each channel about the axis through its center of gravity perpendicular to the web.* From Table B we find $R' = 3.10$ inches;* the distance between the centers of gravity of the channels should therefore be 6.20 inches. From the same table we see that the distance between the centre of gravity and the back of the channel selected is .58 inch.† Hence the distance between the channels back to back will be $6.20 - 1.16$ inches $= 5.04$ inches. Since the width of the flange of the channel from the same table is 2.26 inches, the outside dimensions of the column will be 8×9.56 inches. The stay-plates of such a column would be made up of $\frac{5}{16}$-inch plates; the lacing-strips would be 2 inches $\times \frac{5}{16}$ inch in cross-section, and would make an angle of 60 degrees with each other. They would be fastened to the channels with $\frac{3}{4}$-inch rivets.‡ The proper column to be employed to bear a given load may also be taken from tables in handbooks.§

PROBLEMS.

47. Find the safe load on a hollow round cast-iron column with square ends when the external diameter is 12 inches, thickness $1\frac{1}{4}$ inches, length 14 feet, factor of safety 8.

Ans. 338,956 pounds.

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* Cambria Handbook, p. 162, column 9.
† Ibid., p. 163, column 13.
‡ Ibid., pp. 250, 251.
§ Ibid., pp. 220–282.

48. A hollow cast-iron column with square ends is to carry safely a load of 30,000 pounds; its thickness is $\frac{3}{8}$ inch, length 12 feet. Find the external diameter using a factor of safety of 8.

Ans. 5.5 inches.

49. A medium-steel column 14 feet long, with square ends, is made up of two 12-inch, 20.5-pound steel channels placed back to back. Factor of safety 4. Determine the proper spacing of the channels and the load that the column will carry safely.

Ans. 145,516 pounds, and 7.82 inches.

CHAPTER VIII.

RIVETS AND PINS.

Rivets.—A rivet is a short cylinder of malleable metal, usually headed at one end, which is employed to fasten together the wrought-iron or steel pieces in an engineering structure. (Fig. 35,

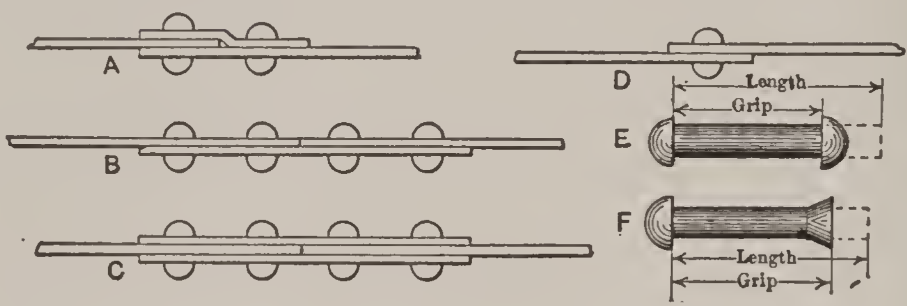

FIG. 35.

E and *F*.) The rivet *E* has a *full head*, and *F* a *countersunk head* on the right end, made by *upsetting* the headless end.

The ordinary sizes of rivets vary from $\frac{3}{8}$ to 1 inch in diameter by differences of $\frac{1}{8}$ of an inch; intermediate sizes are also made.* The $\frac{5}{8}$-, $\frac{3}{4}$-, and $\frac{7}{8}$-inch rivets are the ones commonly used.

To make a riveted joint, holes are *punched* or *drilled* in the pieces to be fastened together, and the rivets, after being passed through the holes, are secured in place by upsetting the headless ends by successive blows of a hammer or by pressure. The headless ends of rivets over one-half inch in diameter must be heated to a red heat before using; the upsetting produces a second head, hemispherical or conical, similar to the first. The holes for rivets are usually punched, as drilling is more expensive. In plates $\frac{5}{8}$ of an inch in thickness, or less, the holes are punched to full size; in thicker plates the holes are punched $\frac{1}{8}$ of an inch less than the full size and then *reamed* to the full size. The reaming removes the metal immediately about the hole which has been injured in

* Cambria Handbook, pp. 334, 335.

punching. In very important work the holes in the thin plates are punched and reamed, and those in plates over $\frac{3}{4}$ inch in thickness are drilled. The diameter of the rivet-hole is usually made $\frac{1}{16}$ inch greater than that of the rivet, but in computing the net area of the piece at a rivet-hole it is assumed as $\frac{1}{8}$ inch greater than the rivet.

If the holes for the rivets have been carefully laid out, the holes should line up accurately when the pieces are superposed. In work carelessly done all the holes do not line up accurately, and a conical steel pin, called a *drift-pin*, is often used to bring the holes into line. Drifting is prohibited in all specifications for high-grade work.

Riveted Joints.—When pieces are fastened together at or near their ends, the joint is called a *lap-joint* if the pieces overlap (Fig. 35, *A* and *D*), and a *butt-joint* if the ends abut against each other (Fig. 35, *B* and *C*). A *cover-strip*, or *fish-plate*, is a third piece which is riveted to both pieces to be joined (Fig. 35, *A*, *B*, and *C*). A joint is *single-riveted* when each piece is fastened by a single

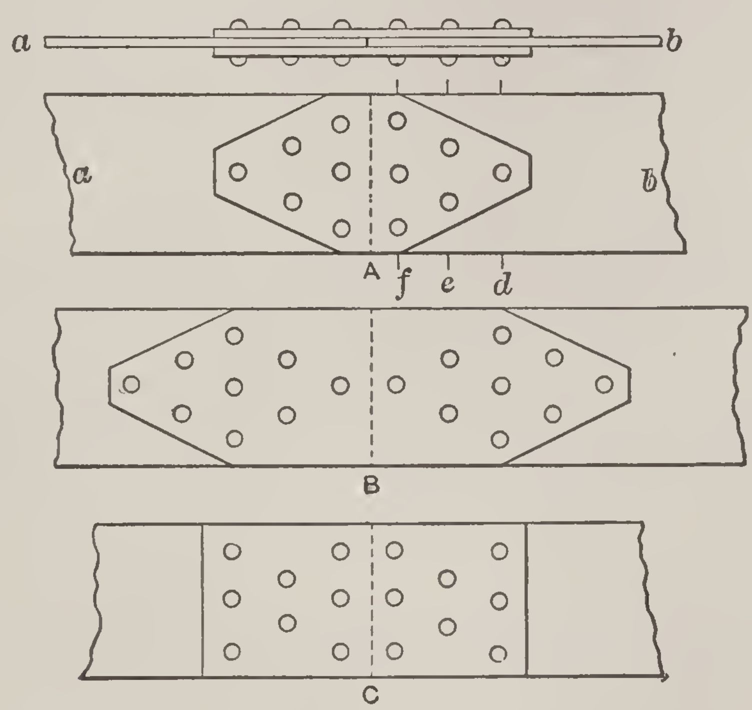

FIG. 36.

row of rivets perpendicular to its axis (Fig. 35, *A* and *D*). It is *double-riveted* when each piece is fastened by two rows of rivets (Fig. 35, *B* and *C*). It is *chain-riveted* when it is fastened by more than two rows of rivets (Fig. 36). The rivets are *staggered* when they are placed in quincunx order (Fig. 36). The *pitch* or *spacing* of the rivets is the distance between the centers of consecutive rivets.

Riveted Tension-joints.—A riveted tension-joint may rupture by the yielding of the rivets, by the yielding of the pieces joined, or by the yielding of the cover-strips.

I. *The rivets may yield by shearing off at one or more cross-sections of the rivets themselves.*

If each rivet which yields shears off along a single plane of cross-section, as in Fig. 35, *B* and *D*, the rivets are said to be in *single shear;* if each rivet can yield only by shearing off along two planes of cross-section simultaneously, as in Fig. 35, *C*, the rivets are said to be in *double shear.*

To resist rupture by *single shear* we must have the total resistance of all the rivets in single shear equal to the force to be transmitted, or

$$\frac{ns_s''\pi d^2}{4} = 0.7854 s_s'' n d^2 = F, \quad . \quad . \quad . \quad . \quad (405)$$

in which s_s'' = allowable unit shearing stress of the rivets in single shear;

n = least number of rivets in any of the pieces to be joined;

d = diameter of each rivet in inches;

F = total tensile force to be transmitted through the joint in pounds.

From this formula we may determine the diameter of a single rivet when the number of rivets in the joint is assumed or known.

Formula (405) may be written in the form

$$n = \frac{F}{0.7854 s_s'' d^2} = \frac{F}{\text{single-shear value of one rivet}}, \qquad (406)$$

in which $0.7854 d^2 s_s''$ is the *single-shear value of one rivet whose diameter is d.* The single-shear values of rivets of different diameters and for different values of s_s'' are given in handbooks on structural metal.*

To find the number of rivets of a given size required to trans-

* Cambria Handbook, p. 308, third column.

mit a given tensile stress, we need only take the single-shear value of a rivet from the table and substitute it with F in the formula (406) and deduce the value of n.

To resist rupture in *double shear* the formulas become

$$1.5708 s_s'' n d^2 = F, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (407)$$

or

$$n = \frac{F}{1.5708 s_s'' d^2} = \frac{F}{\text{double-shear value of one rivet}}, \quad (408)$$

in which $1.5708 s_s'' d^2$ is the *double-shear value of one rivet whose diameter is d.* The double-shear values of rivets are also tabulated in the handbooks.*

II. *The plate may yield by the rivets crushing into the sides of the holes.*

The *plate* is then said to be deficient in *bearing area.* The area of material crushed by each rivet is dt, in which d is the diameter of the rivet, and t the thickness of the plate. To resist crushing, therefore, we must have the total bearing resistance of the plate at the rivet-holes equal to the force to be transmitted, or

$$s_b'' n d t = F, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (409)$$

in which s_b'' = allowable unit stress of the material in bearing;
n = number of rivets;
d = diameter of the rivets in inches;
t = thickness of the plate in inches;
F = total tensile stress to be transmitted in pounds.

From this formula we may determine the thickness of the plate when the number of rivets and the diameter of each rivet are known, or we may determine either the number of rivets of an assumed diameter, or the diameter of an assumed number of rivets, corresponding to any given thickness of plate. Formula (409) may be written in the form

$$n = \frac{F}{s_b'' dt} = \frac{F}{\text{bearing value of the plate and a single rivet}}, \quad (410)$$

* Cambria Handbook, p. 308, fourth column.

in which $s_b''dt$ is the *bearing value* of the plate, also called the bearing value of the rivet. The bearing values of plates of different thicknesses when subjected to pressure of rivets of different diameters and for different values of s_b'' are given in structural-metal handbooks.*

To find the number of rivets of a given size required to transmit a given tensile stress we need only take from the table the bearing value corresponding to the rivet and plate and substitute it with F in the formula (410).

If the rivets are in single shear, the value of n must be determined from equation (406) if the single-shear value of the rivet is less than its bearing value; if the latter is less, the value of n is determined from equation (410). If the rivets are in double shear, the value of n is determined from equation (408) if the double-shear value of a rivet is less than its bearing value, and from equation (410) if the bearing value is less. In the tables of bearing values of rivets, the limits within which the bearing values are less than the single- and double-shear values are indicated by horizontal lines.

III. *One of the pieces or the cover-strips may yield by tearing apart along one of the transverse lines of rivets.*

In a *single-riveted joint*, to resist this mode of yielding the resistance of the net section of each piece or of the cover-strips must be equal to the force transmitted, or

$$s_e''(bt-vdt)=F, \quad \ldots \ldots \quad (411)$$

in which s_e'' = allowable unit stress in tension or elongation;
b = breadth of each piece in inches;
t = thickness of the weakest piece, or of the combined thickness of the cover-strips, in inches;
v = numerical coefficient, which, for a single-riveted joint, is equal to the number of rivets;
d = diameter of each rivet in inches plus $\frac{1}{8}$ inch;
F = total stress transmitted in pounds;
$(bt-vdt)$ = *net section* of the piece or of the combined cover-strips at their weakest section;

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* Cambria Handbook, pp. 308, 309.

bt = *gross section* of the piece or of the combined cover-strips at their weakest section.

When the joint is a double- or chain-riveted one, the location of the weakest section and the value of v in the first member of the above formula will depend upon the arrangement of the rivets.

In the butt-joint Fig. 36, *A*, the total area bt of the piece at the cross-section d is decreased by the area dt, or the meridian section of one rivet-hole; at the section e the total area is decreased by the area $2dt$, or the area of the meridian sections of two rivet-holes; but the piece cannot rupture along this section unless the rivet in the section at d first fails; at the section f the total area of the piece is reduced by $3dt$, but the piece cannot rupture at this section without the three rivets in the sections d and e first failing. *If the resistance to shearing or bearing of a single rivet is equal to or greater than the tensile resistance of the area* dt *of the piece*, it is evident that the rivets may always be arranged (as in Fig. 36, *A*) to make the minimum strength of a piece at a joint equal to the tensile strength of its total cross-section minus tensile strength of the meridian section of one rivet hole, or $(b-d)t$.

In the joint Fig. 36, *A*, if the cover-plates are made of the same material as the pieces themselves, the combined thickness of the cover-plates must be greater than the thickness of either piece, since the section of weakness of the cover-plates is the cross-section f, which has been reduced by three rivet-holes, while the weakest section of each piece has been practically weakened by only one.

In the joint Fig. 36, *B*, the plane of weakness in both pieces and cover-plate passes through a single rivet-hole, but the cover-plate is a very long one. The arrangement of rivets is usually that shown in Fig. 36, *A* or *C*.

Values of dt corresponding to different values of d and t are tabulated in structural-metal handbooks. By the use of these tables the value of vdt can be readily determined.*

Pitch and Size of Rivets.—In applying the formulas under I, II, and III, it is necessary to assume either the size or the number of the rivets. In the direction of the stress the pitch of the rivets

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* Cambria Handbook, pp. 312, 313.

should not exceed either 6 inches, or sixteen times the thickness of the thinnest plate; in a perpendicular direction it should not exceed thirty-two times the thickness of the thinnest plate. The *minimum pitch of rivets* in any direction is *three diameters.* The distance between the center of the outside rivet and the edge of a plate should be not less than $1\frac{1}{2}$ diameters nor more than eight times the thickness of the plate.

The maximum spacing of rivets in the flanges of girders is 6 inches; in pin-plates at the end of columns, girders, etc., it is usually four diameters. The diameter of a rivet-hole should never be less than the thickness of the thickest plate which is to be fastened by it; in thin plates it should usually be at least one-third to one-half greater. In structural-metal handbooks tables are given showing the size of rivets which should be used with different I beams, channels, angles, and plates,* and also rivet-spacing.†

Compression-joints.—The only difference between the design of riveted compression- and tension-joints is that in the former no allowance need be made for the rivet-holes in determining the strength of the compression-pieces or cover-plates at the joints; the rivets are assumed to fit the holes perfectly, and to replace the material of the hole. In ordinary riveted compression-joints the rivets themselves are designed to transmit the entire stress, and no reliance is placed on the abutting surfaces to transmit it. This, however, is not true of the joints in the compression-chords of a bridge-truss; the surfaces in these joints may be so accurately planed and fitted that the stress need not all be transmitted through the rivets.

APPLICATION.

Let it be required to design a riveted tension butt-joint of structural steel which will safely transmit a tensile force of 60,000 pounds.

Then $F = 60{,}000$ pounds.

Let $s_e'' = 15{,}000$ pounds = safe unit stress in tension for main and cover plates;

$s_b'' = s_s'' = 12{,}000$ pounds = safe unit stress in bearing and double shear for rivets;

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* Cambria Handbook, p. 312.

† Ibid., pp. 313, 314.

$d = \frac{3}{4}$ inch = diameter of each rivet;

$t = \frac{1}{2}$ inch = thickness of ties.

In a butt-joint there are usually two cover-plates, hence the rivets will be in double shear.

Number of Rivets.—In the handbook the double-shear value of a $\frac{3}{4}$-inch rivet is given as 5301 pounds, and the bearing value of a $\frac{3}{4}$-inch rivet in a $\frac{1}{2}$-inch plate as only 4500 pounds.* The latter must therefore determine the number of rivets.

Assuming equation (410) we have

$$n = \frac{60{,}000}{4500} = 13.3.$$

Fourteen rivets will therefore be required to fasten each main plate to the cover-plates.

Width of Main Plates.—If there were no rivet-holes in the plates which are united, the width of each would be given by the formula

$$bt = \frac{60{,}000}{15{,}000} = 4 \text{ square inches, whence } b = 8 \text{ inches.}$$

We will assume the rivets to be placed in rows across the plate, four in each row, except the outside rows, which contain but two.

The gross width of the plates through the outside rows must therefore be $8 + 2(\frac{7}{8})$ inches $= 9\frac{3}{4}$ inches.

The gross width through the next row is deduced from the requirement that the tensile strength of the plate, minus four rivet-holes, plus the strength of two rivets in bearing must be equal to the force transmitted, or

$$15{,}000 \times \tfrac{1}{2} \times [b - 4(\tfrac{7}{8})] + 2(4500) = 60{,}000,$$

or
$$b = 10.3 \text{ inches.}$$

This is therefore the weakest section of the plate, as each section through rows nearer the middle of the joint has the same reduction in area, but is strengthened by the resistance of a greater number of rivets.

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* Cambria Handbook, pp. 308, 309.

It is necessary to ascertain whether this width allows proper rivet-spacing. The rivets in each row may be placed at intervals of $2\frac{1}{2}$ inches and an interval of $1\frac{1}{4}$ inches must be left between the outside rivet and the edge of the plate; as this spacing requires a plate only 10 inches wide, the main plates are sufficiently wide.

Thickness of Cover-plates.—The weakest section of the cover-plates is through the rows of rivets on either side of the center of the joints, and hence at a point where the cover-plates are reduced by four rivet-holes.

The combined thickness of the cover-plates is therefore deduced from formula (411) by making $v=4$ and substituting the proper values for the other quantities in the formula, or

$$t=\frac{60,000}{15,000(10.3-3.5)}=0.59.$$

Each plate should therefore be 0.30 inch thick. The standard plate nearest in thickness to this is $\frac{5}{16}$ inch thick.

Since the combined thickness of the two cover-plates is more than the thickness of each tie, it is unnecessary, in this case, to test the cover-plates for bearing, since their bearing value for each rivet must be greater than the bearing value of either tie.

PROBLEM.

50. A double-cover butt-joint unites two $8\times\frac{5}{8}''$ plates. Each fish-plate is $8\times\frac{3}{8}''$. Eleven $\frac{3}{4}$-inch rivets unite each tension- and fish-plate; they are arranged in six rows, which from the middle of the joint contain respectively two, three, three, two, and one rivet. What is the maximum stress which can be transmitted through the joint? Stresses as on page 164. *Ans.* 58,311 pounds.

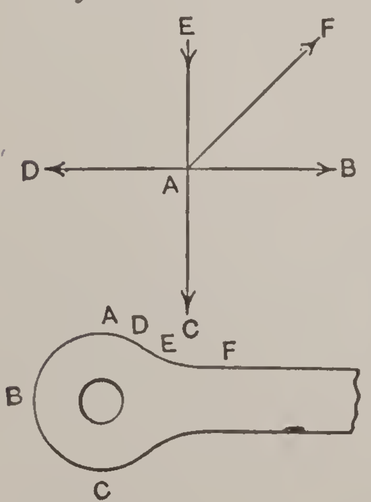

FIG. 37.

Pins and Pin-joints.—Whenever three or more pieces of an engineering structure meet at a common vertex, and the stresses in the pieces are longitudinal stresses in equilibrium, a *pin-joint* may be employed. (Fig. 37.)

The tensile stresses are usually carried by eye-bars, and the compressive stresses by columns; each piece is provided with a pin-hole at its extremity to receive the pin.

Eye-bars.—An eye-bar is a wrought-iron or steel bar of circular or rectangular section, having at one or both extremities an enlarged, flattened head, with a circular hole or eye to receive the pin. (Fig. 37.) The axis of the pin-hole intersects the axis of the bar, and the hole is about $\frac{1}{50}$ of an inch larger than the diameter of the pin. Eye-bars are usually made by upsetting the ends of straight bars and forging the ends into shape by hydraulic pressure in suitable dies. The form and thickness of the metal about the pinhole is so designed that the weakest cross-section in an eye-bar shall be in the straight portion of the bar. This is effected in a bar of uniform width by proportioning the depth of the eye-bar as follows: (Fig. 37.)

Depth of bar.	Radius of Hole.	Radii of *ABC* and *DEF*.
1	From $\frac{5}{6}$ to $2\frac{1}{3}$	$2\frac{1}{3}$ to $3\frac{1}{3}$

Standard eye-bars vary in width from 2 to 10 inches, and are made to fit pins varying in diameter from $1\frac{3}{4}$ to 10 inches. (See tables in structural-metal handbooks.*)

Columns.—Pin-connected steel or wrought-iron columns are usually made in the form of two channels laced together; the pin-hole is made through the webs of the channels, or through the stay-plate. The metal about the pin-hole is reinforced by additional plates riveted on to prevent the pin from crushing into the web. The thickness of the reinforced area of the web is determined by the condition that the safe bearing resistance of the reinforced web shall be at least equal to the force transmitted through the columns, or

$$ts_b''d=F, \quad \therefore\ t=\frac{F}{s_b''d}, \quad . \quad . \quad . \quad . \quad . \quad (412)$$

in which t = total thickness of a single web and its reinforcing-plate in inches;

F = total compressive stress in pounds transmitted through a single web;

s_b'' = allowable unit stress of material in bearing;

d = diameter of the pin in inches;

$ts_b''d$ = bearing value of the pin on the reinforced web.

* Cambria Handbook, pp. 330, 331, 337–339.

The bearing values of pins of various diameters in one-inch plates are tabulated in handbooks of structural metals for convenience in computation.*

Pins.—Pins are circular forged steel cylinders having either a head and a nut or two nuts at their extremities. The standard sizes vary in diameter from 1 to 10 inches (see tables in structural-metal handbooks).†

A pin is simply a beam acted upon by a system of non-coplanar bending forces in equilibrium. The pin must be designed to resist the maximum bending moment and the maximum shear to which it is subjected. As the forces acting on a pin do not, as in the beams heretofore considered, all act in the same plane containing the axis of the pin, to determine the maximum bending moment and maximum shear each force is resolved into its vertical and horizontal components, and hence the entire system into two coplanar systems, one in the vertical and the other in the horizontal plane through the axis of the pin.

Let M = resultant moment at any section in inch-pounds;
M_h = moment at the same section due to the horizontal components in (inch-pounds);
M_v = moment at the same section due to the vertical components in (inch-pounds);
S_s = resultant shear at any section in pounds;
S_h = shear at the section due to the horizontal components in pounds;
S_v = shear at the same section due to the vertical components in pounds.

Then at every section of the beam we shall have

$$M = \sqrt{M_h^2 + M_v^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (413a)$$

and

$$S_s = \sqrt{S_h^2 + S_v^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (413b)$$

The dangerous section of the pin will be when M is a maximum. The pin may be tested for shearing strength at the section where S_s is a maximum, but this is usually unnecessary.

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* Cambria Handbook, p. 315.

† Ibid., pp. 336, 337.

The bearing strength of the ties and struts which abut on the pin should always be tested, as they usually determine its size.

The location of the dangerous section along the axis of the pin may usually be ascertained by constructing and inspecting the curves of bending moments for the vertical and horizontal components separately.

The location of the section of greatest shear may usually be ascertained by constructing and inspecting the lines of shear for the vertical and horizontal components separately.

Application.—Assume five forces shown in Fig. 37 as intersecting at a common point A. Let $AB=90{,}000$ pounds, $AC=30{,}000$ pounds, $AD=150{,}000$ pounds, $AE=30{,}000$ pounds, and $AF=84{,}850$ pounds. Substitute for the force AB two equal components, and let each be transmitted through an eye-bar, of the form shown in Fig. 38, to a pin whose axis is perpendicular

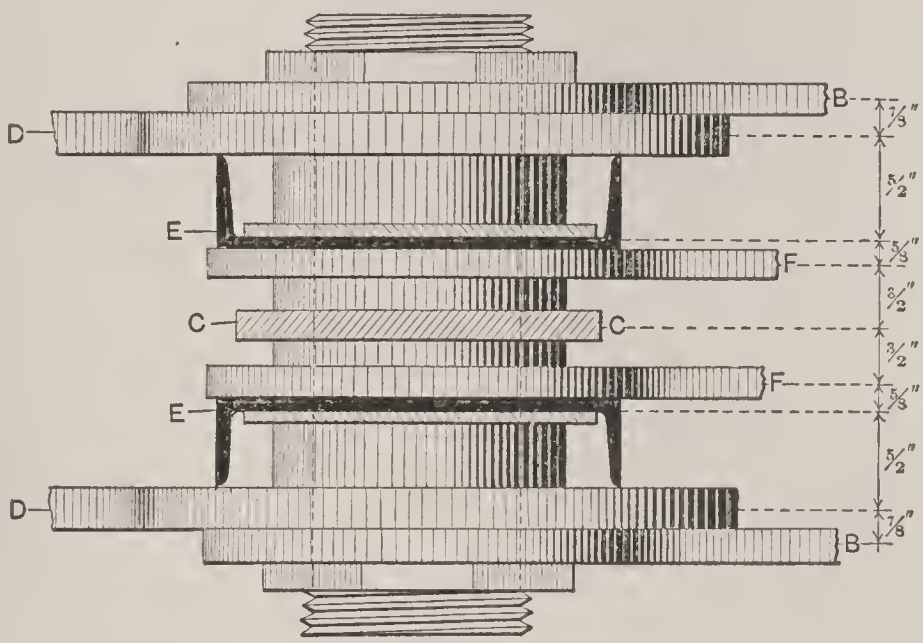

FIG. 38.

to the plane of the paper at A, Fig. 37. In a similar manner let AD and AF be transmitted through two eye-bars, so arranged that the force transmitted through each bar shall be the same, and the resultants of each pair of forces shall act through the middle point of the axis of the pin. The angle $EAF=45$ degrees.

Let the force AE be transmitted through a column made up of two laced channels, as in Fig. 38, also symmetrically disposed with respect to the middle point of the pin. Assume the distance between the channels to be $3\frac{3}{4}$ inches, and the flange of each channel to be $2\frac{1}{4}$ inches wide, the web to be $\frac{1}{4}$ inch thick, with a reinforcing-plate $\frac{1}{4}$ inch thick.

Let the force AC be transmitted through a rectangular plate $\frac{3}{4}$ inch thick, as in Fig. 38, whose axis passes through the middle point of the pin. We shall then have a pin-connected joint in which the pin has neither motion of translation nor rotation. Such a joint is shown in plan in Fig. 38.

Maximum Shearing and Bending Moments.—From Figs. 37 and 38 we may construct the table given below. In the first column we write the pieces in their order beginning with the outside of the pin. The second column is obtained by dividing the total stress in the piece by the allowable unit stress in tension, assumed as 15,000 pounds; in the third column are standard eye-bars taken from the table which have approximately the dimensions of cross-section required; the fourth column gives the total stress in each piece; the fifth column gives the horizontal components of these stresses; the sixth column gives the shears in the pin due to the horizontal components; the seventh and eighth columns give the vertical components and vertical stress; and the ninth column gives the maximum shearing stresses in the pin, obtained by solving the equation $s_s = \sqrt{s_h^2 + s_v^2}$.

1 Piece	2 Area Required	3 Eye-bars Selected.	4 Stresses.	5 Horizontal Components.	6 Horizontal Shear.	7 Vertical Components.	8 Vertical Shears.	9 Resultant Shear.
	Sq. In.	Ins.	Pounds.	Pounds.	Pounds.	Pounds.	Pounds.	Pounds.
AB	3	$4\times\frac{3}{4}$	45,000	+45,000				
					45,000			45,000
AD	5	5×1	75,000	−75,000				
								30,000
AE	.....	.....	15,000	....	30,000	−15,000		
							15,000	33,540
AF	2.83	$4\times\frac{3}{4}$	42,425	+30,000		+30,000		
							15,000	15,000
AC	2.0	.....	30,000	....	0	−30,000		
							15,000	15,000
AF	2.83	$4\times\frac{3}{4}$	42,425	+30,000		+30,000		
							15,000	33,540
AE	.....	.....	15,000	....	30,000	−15,000		
								30,000
AD	5	5×1	75,000	−75,000				
					45,000			45,000
AB	3	$4\times\frac{3}{4}$	45,000	+45,000				

Forces acting to the *right* and those acting *upwards* are considered *positive*, those acting in a *contrary direction* are considered *negative*.

We may also write out a second table to determine the maximum bending moment. In the table given below, the first column gives the pieces in their order; the second gives the total stresses; the third gives the horizontal component of the stresses; the fourth gives the distances between the axes of the pieces; the fifth gives the moments of these horizontal components about points on the axis of the pin; the sixth gives the vertical component of the stresses; the seventh gives the distance between the axes of the pieces; the eighth gives the moments of the vertical components; the ninth gives the maximum bending moment derived from the solution of the equation

$$M = \sqrt{M_h^2 + M_v^2}.$$

1 Piece	2 Stresses.	3 Horizontal Components.	4 Between Bars.	5 Moments.	6 Vertical Components.	7 Between Bars.	8 Moments.	9 Resultant Moment
	Pounds.	Pounds.	Inches.	(Inch-lbs.)	Pounds.	Inches.	(Inch-lbs.)	(Inch-lbs.)
AB	45,000	+45,000		0			0	
			$\frac{7}{8}$					
AD	75,000	−75,000		39,375			0	39,375
			$\frac{5}{2}$					
AE	15,000	....		35,625	−15,000		0	35,625
			$\frac{5}{8}$			$\frac{5}{8}$		
AF	42,425	+30,000		54,375	+30,000		9,375	55,177
			$\frac{3}{2}$			$\frac{3}{2}$		
AC	30,000	....		54,375	−30,000		13,125	55,936

In forming this table the stress of each piece is assumed to act along its axis. A simpler method of determining the bending stresses in pins will be given hereafter under Graphic Statics.

In *packing* the pin, or in arranging the order of the pieces on it, it will be observed that the smaller the maximum stress in any of the pieces, the smaller will be the maximum shear and the maximum bending moment; hence if the required pin diameter is too large, it may be decreased by substituting two or more bars for each of the two single bars carrying the greatest stress. It will be observed that the maximum shear and the maximum bending moment are diminished by alternating positive and negative bars, and by placing the bar with the greatest negative stress adjacent to the bar with the greatest positive stress. The single bar is usually the tension-bar having the smallest stress.

If the bars are separated by any considerable distance, washers are used between them; if no washers are used, an allowance of $\frac{1}{16}$ of an inch is made for each space between bars, in determining the length of the pin. The flanges of the channels are removed in the vicinity of the pin, when by so doing the maximum bending moment may be greatly decreased. In the example given the flanges have been left on for clearness, and the distance between the channels of the column has been reduced to make Fig. 38 small.

Design.—Having determined the maximum shear and the maximum bending moment in the pin, its cross-section is designed by employing the usual formulas.

For Shear.—

$$\frac{s_s''\pi d^2}{4}=F, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (414)$$

in which s_s'' = allowable unit stress in shear = 10,000 pounds;
d = diameter of pin in inches;
F = maximum shearing force = 45,000 pounds.

Substituting and solving for d we have $d=2.4+$ inches.

For Flexure.—

$$\frac{s''I}{y'}=M, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (415)$$

in which s'' = allowable stress in flexure = 20,000 pounds;
I = moment of inertia $=\frac{\pi d^4}{64}=0.049d^4$;
$y'=\frac{d}{2}$;
M = maximum bending moment = 55,936 (inch-pounds).

Substituting and solving for d, we have $d=3.06$ inches.

The required diameter of a pin to resist a given bending moment may also be taken directly from the tables in handbooks.*

* Cambria Handbook, pp. 310, 311.

From the table of eye-bars we see that the diameters of the largest pins corresponding to the bars selected vary from $4\frac{1}{8}$ to $6\frac{1}{8}$ inches.*

In this example the $3\frac{1}{8}$-inch pin is sufficiently large to resist the maximum bending moment, but is not large enough for bearing on any of the bars.

For Bearing.—The bar requiring the largest bearing area and also the largest pin is the bar AD, which transmits a tensile force of 75,000 pounds. The bearing area required by AD may be found from the formula

$$td=\frac{F}{s_b''}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (416)$$

in which t=thickness of bar in inches=1 inch;

d=diameter of pin in inches;

s_b''=allowable unit stress in bearing in pounds=15,000;

F=force transmitted through bar in pounds=75,000.

Solving the equation with respect to d, we find that the diameter of the pin must be 5 inches.

Bearing Value of Reinforced Web.—Having determined the diameter of the pin, the next step is to ascertain whether the webs of the struts AE have sufficient bearing area without a reinforcing-plate.

In the formula

$$td=\frac{F}{s_b''}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (417)$$

making $t=\frac{1}{4}$ inch,

d=5 inches,

s_b''=15,000 pounds,

the resulting value of F is 18,750 pounds. As each strut transmits a force of 15,000 pounds only, a reinforcing-plate will not actually be required at the pin-hole, but would ordinarily be used to strengthen the web, as shown in the figure.

The bearing value of a 5-inch pin in a 1-inch plate might also have been taken directly from the handbook,† and the bearing value of a $\frac{1}{4}$-inch plate found by dividing the value given by four.

* Cambria Handbook, p. 331.

† Ibid., p. 315.

CHAPTER IX.

SOLID BUILT BEAMS, I BEAMS, AND PLATE GIRDERS.

Solid Built Beams.—A *simple beam* is a beam cut out of a single piece of timber. A *solid built beam* is a beam made of two or more simple beams, so fastened together as to act as a single beam. If the two pieces are of equal length, they may be bolted together as shown in Fig. 39.

The strength of a rectangular beam, or its power to resist additional loads, is given by the equation

$$\frac{1}{s}=\frac{bd^2}{6mWl}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (418)$$

in which m = a coefficient depending on the manner of loading and supporting the beam;

b = breadth of cross-section in inches;

d = depth of cross-section in inches;

l = length of beam in inches;

W = total load in pounds;

s = maximum unit fiber stress in the beam in pounds.

From this equation it appears that, all other conditions being the same, the strength of a rectangular beam varies with the *first power of the breadth* and the *square of the depth.*

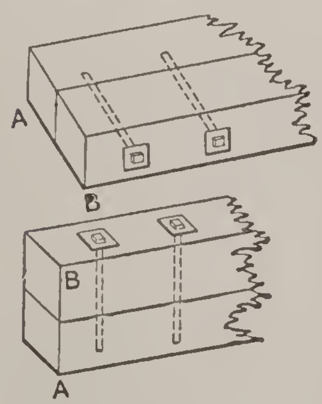

FIG. 39.

If, therefore, two beams of equal breadth and depth are placed side by side, as shown in the upper drawing, the strength of the two beams will be *twice* the strength of a single beam.

This will be equally true whether the beams are simply laid side by side, or fastened together as shown in Fig. 39. In the latter case there will be no stress in the bolts, since their axes lie in the neutral surface or in a surface parallel to it.

If the beams are superposed, as shown in the lower drawing, but *not fastened together*, neglecting friction, the strength of the two beams will also be *twice* the strength of a single beam, since the beams will act independently, each having its own neutral surface and the same stress in the fibers at its upper and lower surfaces. If the beam rests on two supports, under the action of the external forces the lower surface of the upper beam is lengthened, and the upper surface of the lower beam is compressed; hence these surfaces slide along each other. If, however, the beams are bolted together as shown in Fig. 39, and this sliding is prevented, the neutral surface of the solid built beam will be at the surface of contact of the two beams, and the strength of the solid built beam will be *four times* that of a single beam. The upper beam will now be wholly in compression, the lower one wholly in tension, and the bolts will be subjected to a horizontal shearing stress due to the tendency of the beams to slide upon each other.

Rolled I Beams.—A rolled I beam is a solid metal beam of steel or wrought iron, whose uniform cross-section is of the form of the letter I. The upper and lower horizontal branches of the I are called the *flanges;* the vertical connecting part is called the *web.*

Rolled I beams are made of standard sizes whose dimensions may be found in structural-metal handbooks.* To determine the load which may be placed on such a beam we utilize the formula

$$M_m = \frac{s''I}{y'} = \frac{2s''Ar^2}{d}, \quad \ldots \ldots \quad (419)$$

in which M_m = bending moment at the dangerous section in terms of the load in pounds and the length of the beam in inches;

s'' = safe unit stress of material in pounds;

$\frac{I}{y'}$ = section modulus of beam, in which dimensions are in inches;

A = area of cross-section in square inches;

* Cambria Handbook, pp. 158–161.

r = radius of gyration of cross-section about the neutral axis in inches;

d = depth of the beam in inches.

Application.—What weight may be safely distributed uniformly along an 8-inch 18-pound I beam, 12 feet long, if $s'' =$ 15,000 pounds?

From the table, page 86, we have $M_m = \frac{1}{8}Wl$; from the handbook $\frac{I}{y'} = 14.2$. Substituting these values in (419) we have

$$\tfrac{1}{8}W \times 12 \times 12 = 15{,}000 \times 14.2,$$

whence $W = 11{,}833$ pounds. This includes the weight of the beam, 12×18 pounds, or 216 pounds.

Plate Girder.—A plate girder, or built-up I beam, is a steel I beam in which the flanges and web are separate pieces fastened together by rivets, as shown in Fig. 40. The web is a metal plate whose thickness is usually from $\frac{3}{8}$ to $\frac{1}{2}$ inch, and whose depth varies from $\frac{1}{8}$ to $\frac{1}{12}$ the span, in heavily loaded girders, and from $\frac{1}{12}$ to $\frac{1}{20}$ the span, in light ones.

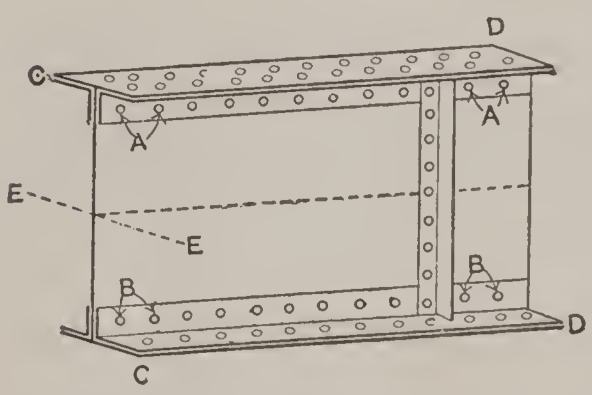

FIG. 40.

Each flange consists of two *angles* which are riveted to the upper and lower edges of the web as shown. The angles are made with equal or unequal legs, varying in width from 2 to 7 inches, and in thickness from $\frac{1}{8}$ to $\frac{7}{8}$ inch. The area of each flange may be increased by one or more *flange-plates* riveted to the horizontal legs of the angles. The area of cross-section of the flange-plates should not exceed that of the angles unless the maximum size angles are employed. The flange-plates are in contact with the angles only and not with the edges of the web, which are always kept inside the faces of the horizontal legs of the angles.

At the supports and at points where the girder receives concentrated loads the web is stiffened by angles or tees which are riveted to it in pairs on its opposite sides. The ends of these *stiffeners* abut on the flanges.

The different parts of the girder are usually fastened together by $\frac{3}{4}$- or $\frac{7}{8}$-inch rivets; the larger size is used in heavy girders, and the smaller in light ones. The load of a girder may be placed on the upper flange, or upon brackets attached to the web.

In determining the stresses in a plate girder one of three methods may be utilized:

1. The girder may be treated as a solid beam. That is, the longitudinal stresses may be assumed to vary from the neutral axis to the surface, and the vertical shear to be uniformly distributed over the entire area of cross-section.

2. The flanges of the girder may be designed to resist the longitudinal stresses without the assistance of the web, and the web may be designed to resist the vertical shear without the assistance of the flanges. This method gives the greatest factor of safety and is the one usually employed.

3. The girder may be so designed that the longitudinal stresses are distributed over the flanges and web, approximately as in the first method, but the vertical shear is resisted by the web alone. This method is also employed in designing girders.

First Method.—The first method cannot well be employed in designing girders, but it may be employed in determining the load which may be placed on a girder whose section modulus is known. If the girder is of uniform section, we may determine M_m in terms of W from page 86 and substitute it in the formula

$$M_m = \frac{s''I}{y'}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (419)$$

in which M_m = the bending moment at the dangerous section in (inch-pounds);

s'' = allowable unit stress in pounds;

$\frac{I}{y'}$ = the section modulus of the beam in inches.

Second Method.—In the second method the girder is designed as follows: The depth of the web, which is practically that of the girder, is first assumed at $\frac{1}{8}$ to $\frac{1}{20}$ the span, depending on the load and the allowable depth. A girder is often placed in a position in which the most economical depth is not allowable.

The Flanges.—Since the flanges are designed to resist the entire longitudinal stresses, the moments of these stresses about the neutral axis, EE, Fig. 40, must, at every section, be equal to the bending moment. These stresses are assumed to be uniformly distributed over the area of the flanges, hence the center of stress will be at the center of gravity of each flange. The flange is therefore so designed that at every cross-section

$$s''A'd_1 = \text{or} > M, \quad \ldots \ldots \quad (420)$$

in which s'' = safe unit longitudinal stress in pounds;
A' = *net area* of cross-section of each flange in square inches;
d_1 = distance between centers of gravity of flanges in inches; (This is called the *effective depth* of the girder.)
M = the bending moment at the cross-section in (inch-pounds).

From this formula we can determine A' at any section when we know the values of s'', d_1, and M. The first and last can be readily determined, but the second must be assumed. In the tables in structural-metal handbooks are given the properties of the angles employed in making up plate girders.* From these tables it will be seen that the distance from the center of gravity of an angle to its back varies only from 0.60 to 2.41 inches for angles employed in plate-girder construction; therefore if d_1 is assumed from 0 to 2 inches less than the depth of the web, it will be sufficiently close to its true value to select angles for trial. Its true value, which must not be less than its assumed value, may be determined when the composition of the flange is definitely fixed or the angles and flange-plates are definitely selected.

Having determined the *true net area*, the *true gross area* of a flange is found by adding to the net area the area of the rivet-holes in a vertical plane through the center of a rivet-hole. The area added for this purpose in Fig. 40 is the area of cross-section of one of the web-rivet holes, AA, plus the area of cross-section of the two flange-rivet holes in the same cross-section. If

* Cambria Handbook, pp. 166–179.

the web- and flange-rivet holes are not in the same plane, they are nevertheless so considered unless the oblique plane through them has a net area 30 per cent greater than the net area of the vertical section.

The gross area of the compression-flange is made equal to that of the tension-flange, although this is not really essential if the rivets fully fill their holes.

The Web.—The web of a girder is usually made of plates* of the same depth and thickness, spliced end to end. Sometimes, when the load is heavy and the girder is long, the plates vary in thickness.

The web may yield because of insufficient bearing area at the flange- or stiffener-rivets, or because of insufficient thickness to resist the vertical shear or to prevent buckling. If at the point of maximum vertical shear, usually the supports, the web is thick enough to give sufficient bearing area on the stiffener-rivets, and thick enough to resist the vertical shear, buckling may be prevented by the use of stiffeners, and the yielding of the flange-rivets by increasing their number.

The thickness to resist the bearing of the stiffener-rivets is determined from the formula

$$t=\frac{(V_s)_m}{\text{bearing value of one rivet}\times nd}, \quad . \quad . \quad . \quad (421)$$

in which n=number of rivets in the stiffener; (The number of these rivets must be sufficient to resist the bearing and double shear of $(V_s)_m$.)

d=diameter of rivets in inches;

t=thickness of web in inches;

$(V_s)_m$=vertical shear at support or the maximum vertical shear in pounds.

The thickness of the web to resist shearing is determined from the formula

$$t=\frac{(V_s)_m}{S_s''d'}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (422)$$

* Cambria Handbook, p. 29.

in which s_s'' = allowable unit stress in shearing of the material;
d' = net depth of the web in inches;
t = thickness of the web in inches;
$(V_s)_m$ = vertical shear at the support or the maximum vertical shear in pounds.

The *net depth* of the web is the *gross or full depth* assumed, less the sum of the diameters of the rivets in a vertical section. The sum of the diameters of the rivets in the stiffener must be subtracted from the gross depth to determine the net depth.

The thickness of the web should be the larger of the values determined from the above formulas, and for ordinary girders should be from $\frac{3}{8}$ to $\frac{1}{2}$ inch.

The Web-rivets. — The web-rivets are the rivets which unite the angles and web. Their purpose is to so unite the web and flanges that the total flange stress from section to section shall vary as closely as possible with the bending moment.

Since the total flange stress can vary only at the web-rivets, the horizontal shear on any web-rivet must be equal to the difference of the flange stresses on either side of the rivet. This is equal to the difference between the bending moments at the given rivet and at the preceding rivet, divided by the effective depth of the girder. Hence

$$s_sA'''* = (s_1 - s_2)A' = \frac{M' - M''}{d_1}, \quad . \quad . \quad . \quad . \quad . \quad (423)$$

in which s_s = unit horizontal shear in any web-rivet;
A''' = area of cross-section of a web-rivet in square inches;
d_1 = effective depth of the girder;
s_1 = the unit flange stress on the side of the rivet towards the dangerous section;
s_2 = unit flange stress on the opposite side;
A' = net flange area;
M' = bending moment at preceding rivet on the side of the dangerous section;
M'' = bending moment at the rivet.

From the principle partially developed on page 90, that the bending moment at any section of a beam without weight is

* If bearing value of rivet is less than shearing value, s_bbt must be substituted for s_sA'''.

equal to the bending moment at any other section plus the moment of the vertical shear at the second section with respect to the first, we have

$$M' = M'' + V_s a, \quad \text{or} \quad M' - M'' = V_s a, \quad . \quad . \quad . \quad (424)$$

in which V_s = vertical shear at rivet where the moment is M'';
a = distance between this rivet and its adjacent one, or the pitch at this point of the girder.

Substituting this value in equation (423) we have

$$s_s A''' d_1 = V_s a, \quad \text{or} \quad a = \frac{s_s A''' d_1}{V_s}. \quad . \quad . \quad . \quad (425)$$

In this equation if we make A''' constant, that is, make all the rivets of the same size, and make $s_s = s_s''$, the safe unit stress in shear, it is evident that a, *the distance between rivets, must vary inversely with* V_s, *the vertical shear.* Where the vertical shear is greatest, as at the ends of a girder uniformly loaded and resting on end supports, the web-rivet spacing must therefore be least; and at the dangerous section, where the vertical shear is least, the rivet-spacing must be greatest.

The depth between the upper and lower web-rivets is frequently substituted in this formula for the effective depth; this increases slightly the factor of safety.

As it is not economical in practice to vary the rivet-spacing at every point, it is customary to divide the girder into panels and make the spacing the same throughout each panel, but to make the spacing in the different panels unequal. The maximum rivet-spacing is also governed by the rule that the maximum pitch in a girder must nowhere exceed six inches.

If a uniform load is placed on the upper flange, it may be transmitted to the web through the web-rivets. In that case the shearing and bearing resistances of these rivets in each panel should be sufficient to resist the resultant of the load on the panel and the horizontal shear in its web-rivets.

The Flange-rivets. — The flange-rivets are those connecting the flange-plates and angles. As the flange-plate stresses must also vary with the bending moment, the same conditions govern the horizontal stresses in both the flange- and web-rivets. The flange-plates, however, resist only a part of the total flange stress, hence the stress on the flange-rivets is less than the stress on the web-rivets; the flange-rivets are, however, in single shear, while those in the web are in double shear. The spacing of the flange-rivets is also governed by the condition that the maximum spacing shall not exceed six inches.

The flange-plates make the effective depth of the girder greater than it would be were they omitted. They also reduce the unit flange stress by distributing the stress over a larger area of cross-section. By varying the thickness of the flange-plates, making the thickness greatest where the bending moment is greatest, and the thickness least where the bending moment is least, the girder may be made approximately a beam of uniform strength. Each flange-plate is made longer at each end by twice the rivet-spacing than actually required by theory.

Stiffeners.—The web-stiffeners shown in Fig. 40 are small angles or tees employed in pairs on opposite sides of the web, and are riveted to the web and to each other. The stiffeners serve to transmit concentrated loads from the upper flange to the web, and also to prevent the web from buckling. For the former purpose they must be placed at the supports and wherever the load is concentrated; for the latter, they are placed at intervals equal to the depth of the girder, but not exceeding five feet.

To determine whether stiffeners are necessary to prevent the buckling of the web, the value of s_c is deduced from the following formula; if it exceeds the unit shearing stress in the web previously determined, it is assumed that the web requires stiffeners to prevent its buckling:

$$s_c = \frac{15.000}{1 + \dfrac{d''^2}{3000t^2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (426)$$

in which d'' = depth of web, and t = thickness of web.

Each pair of stiffeners is made strong enough to support the entire vertical shear at its section without exceeding the value of s_c in the following formula:

$$s_c = 12,000 - \frac{55l}{r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (427)$$

in which l = length of stiffener in inches;
r = its least radius of gyration in inches.

Lateral Stiffness.—The compression-flange of a girder resting on end supports is a long column supported along one side. It is evident that it requires lateral support to prevent it from buckling. This support is secured by connecting the compression-flanges of consecutive girders by suitable braces, whenever the length of the compression-flange exceeds sixteen times its width.

Third Method.—In the *third method* the flange area is not so great as in the second, since the web is assumed to bear part of the longitudinal stresses.

Having determined the area of the web as before, the area of each flange is deduced from the formula

$$s''d_1(A' + \tfrac{1}{6}A'') = M, \quad . \quad . \quad . \quad . \quad . \quad (428)$$

in which s'' = safe unit stress;
d_1 = effective depth of girder in inches;
A' = area of each flange in square inches;
A'' = area of web in square inches.

This formula is derived from the general formula

$$\frac{s''I}{y'} = M. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (429)$$

1st. By assuming that $2A'r^2$, the moment of inertia of the two flanges, is equal to $\frac{A'd_1{}^2}{2}$, or that *the effective depth of the girder* is equal to *twice the radius of gyration of the flanges about the neutral axis.*

2d. By assuming that $\frac{A''d''^2}{12}$, the moment of inertia of the web about the neutral axis, is equal to $\frac{A''d_1{}^2}{12}$, or that *the effective depth of the girder* is equal to *the gross depth of the web, d''.*

3d. By assuming that $d_1 = 2y'$, or that *the effective depth of the girder* is equal to *its gross depth.*

Each of these assumptions introduces a slight error, since $d_1 < 2r$, $d_1 < d''$ unless the flange-plates are exceptionally large, and $\frac{d_1}{2} < y'$.

Under these assumptions we have

$$\frac{I}{y'}=\frac{2A'r^2+\frac{A''d''^2}{12}}{y'}=\frac{\frac{A'd_1{}^2}{2}+\frac{A''d_1{}^2}{12}}{\frac{d_1}{2}}=\left(A'+\frac{1}{6}A''\right)d_1. \quad (430)$$

Since the web resists some of the longitudinal stress, the web-rivets are subjected to less horizontal shear than in the second method, and the rivet-spacing may be *increased* in the ratio of the total longitudinal stress to the longitudinal stress borne by the flanges.

The strength of the web and the web-splicing must be greater than in the second method in order to carry the horizontal stress in addition to the vertical shear.

Box Girders.—Box girders are formed by placing two or more rolled I beams or plate girders side by side and connecting them by flange-plates which extend over and under all the beams or girders. They are employed when great strength is required.

Application.—Let it be required to design, with the aid of a structural-metal handbook, a steel plate girder, 30 feet long between supports, which is to carry a load of 2000 pounds per linear foot, including the weight of the girder.

Specifications.—1° *Web and Stiffeners.*—Depth of the web equals $\frac{1}{15}$ of the span. Allowable unit stress in simple shear equals 12,000 pounds. The vertical shearing stress is to be borne entirely by the web-plate. The web must be stiffened at intervals not exceeding the depth of the girder, or a maximum of five feet wherever the unit shearing stress is greater than $12,500-90H$,* in which H equals the ratio of the depth of the web to the thickness. The edges of the web-plate must not project beyond the faces of the flange-angles, nor shall they be more than $\frac{1}{8}$ inch inside these faces at any point.

The web-plate must have stiffeners over bearing-points and at points of local concentrated loads. Such stiffeners must be fitted at their ends to the flange-angles over the bearing-points.

* A right-line form of equation (426).

The stiffeners are to be made of angles with equal legs. All stiffeners must be capable of carrying the maximum vertical shear without exceeding the unit stress in the formula $s_c = 12{,}000 - \frac{55l}{r}$, in which l=length of stiffener in inches, and r=its radius of gyration in inches about an axis parallel to the web of one angle. Each stiffener must be fastened to the web by enough rivets to transfer the total vertical shear at that point to or from the web.

2° *Rivets.* — Allowable unit stress in single shear = 6000 pounds. Unit bearing value for rivets = 12,000 pounds. The unsupported width or distance between rivets of flange-plates in compression shall not exceed thirty times the thickness of the plates. The rivets used shall be $\frac{3}{4}$ inch in diameter. The diameter of rivet-holes to be deducted in order to obtain net sections shall be assumed as $\frac{7}{8}$ inch.

3° *Flanges.*—Allowable unit flange stress = 13,000 pounds. The flanges shall be proportioned under the supposition that they carry the entire bending or longitudinal stress. The compression-flange shall be made of the same gross area as the tension-flange. All the joints in riveted flanges shall be fully and symmetrically spliced. At least one-half of the flange section shall be angles. Flange-plates must extend beyond their theoretical limits two rows of rivets at each end. The flange-plates must be limited in width so as not to extend beyond the outer lines of rivets connecting them with the angles more than five inches or more than eight times the thickness of the plate. The compression-flange must be stayed against transverse crippling whenever its length exceeds sixteen times its breadth.

Maximum Stresses.—The dangerous section will be at the middle point, where the maximum bending moment is

$$M_m = \frac{Wl}{8} = 2000 \times 30 \times 30 \times \frac{12}{8} = 2{,}700{,}000 \text{ (inch-pounds).}$$

The maximum shear equals $\frac{30 \times 2000}{2} = 30{,}000$ pounds.

Tension-flange.—Assume effective depth of girder$=d_1=23.5$ inches. Then

$$A's''d_1 = A' \times 13,000 \times 23.5 = 2,700,000;$$

hence $$A' = \frac{2,700,000}{13,000 \times 23.5} = 8.87 \text{ square inches.}$$

Try two angles $5'' \times 3'' \times \frac{3}{8}''$ *......	5.72	sq. inches gross area
one flange-plate $12'' \times \frac{3}{8}''$......	4.50	" " "
Total.	10.22	" " "
Deduct $\frac{7}{8}''$ rivet-holes connecting flange-plate and angles.	1.32	" " "
Net area..................	8.90	sq. inches, as required

To Find the True Value of the Effective Depth.—The moment of one flange about the neutral axis must be equal to the moment of its angles plus the moment of its flange-plate.

C. of gr. of flange-plate from neutral axis $\Big\}$ $= 12$ in. $+ \frac{1}{8}$ in. $+ \frac{3}{16}$ in. $= 12.31$ inches;

C. of gravity of angles from neutral axis $\Big\}$ $= 12$ in. $+ \frac{1}{8}$ in. $- 0.70$ in.† $= 11.43$ inches;

C. of gravity of rivet-holes from neutral axis $\Big\}$ $= 12$ in. $+ \frac{1}{8}$ in. $= 12.12$ inches.

12 inches = one half the depth of the web; $\frac{1}{8}$ inch = distance from edge of web to flange-plate; and 0.70 inch = the distance from center of gravity of angles to their backs. Hence

$$8.90 \times \frac{d_1}{2} = 4.50 \times 12.31 + 5.72 \times 11.43 - 1.32 \times 12.12.$$

Solving we find $d_1 = 23.54$ inches. This is greater than the assumed value, 23.5, and is therefore on the side of safety.

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* Cambria Handbook, p. 174, A 101.
† Ibid., p. 174, column 6.

Compression-flange.—This will be made of the same gross area as the tension-flange.

Design of Web.—1° *For Simple Shear.*—The web is 24 inches deep, and a thickness of $\frac{3}{8}$ inch will be assumed, as the girder is a light one, and this is a thickness commonly used. The stiffeners at the ends must carry a compressive force of 30,000 pounds from the points of support to the web. This will require nine $\frac{3}{4}$-inch rivets, since the bearing value of a single rivet in a $\frac{3}{8}$-inch plate is 3375 pounds.* The total number of rivet-holes to be deducted from the gross area of the web is therefore nine, in which are included the two rivets required to connect the flanges with the web at the same cross-section. The net section of the web will then be $(24-9\times\frac{7}{8})\frac{3}{8}=6.05$ square inches, and is amply strong to carry the maximum shear of 30,000 pounds, since $6.05\times 12,000$ is greater than 30,000. The seven rivets connecting the stiffeners with the web directly will have a pitch of $\frac{24-(2\times 3)}{7}=2.6$ inches, which lies between the allowable limits of three diameters and six inches, and may be adopted.

Each angle forming the stiffener carries a load of 15,000 pounds. Try two angles $2\frac{1}{2}\times 2\frac{1}{2}\times\frac{5}{16}$ inches,† for which $r=.76$. Hence $s_c=12,000-\frac{55l}{r}=10,300$. The area required $=\frac{15,000}{10,300}=1.46$; the angle selected has an area of 1.47 square inches and will be used. As the combined thickness of the angles is greater than the thickness of the web, the bearing area will be sufficient.

2° *To Resist Buckling.*—The allowable unit stress in the web is $12,500-90H=12,500-\frac{90\times 24}{\frac{3}{8}}=6740$ pounds per square inch. The actual stress is $\frac{30,000}{6.05}$, which is less than 6740, and hence no stiffeners are required except at the ends.

3° *Test for Bearing.*—The bearing value has already been found satisfactory for the rivets connecting the end stiffeners with the web, and the calculation for the number of rivets connecting the flange-angles with the web will indicate whether it is sufficient for them.

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* Cambria Handbook, p. 308, first table.

† Ibid., p. 166, A 17.

Rivets Connecting Flange-angles and Web.—The bearing value of a $\frac{3}{4}$-inch rivet in a $\frac{3}{8}$-inch plate is 3375 pounds, and is less than its value in double shear; the bearing value will therefore determine the rivet-spacing.

Since the maximum bending moment is 2,700,000 (inch-lbs.) and the effective depth is 23.5 inches, the maximum flange stress is $\frac{2{,}700{,}000}{23.5}$, or 115,000 pounds approximately. This is equal to the horizontal shear on all the web-rivets of each flange in each half of the girder. The total vertical shear on the web-rivets of each flange in one half the girder is 30,000 pounds. The resultant shear is $\sqrt{(115{,}000)^2+(30{,}000)^2}=118{,}000$. The total number of web-rivets required in each half of each flange is therefore $\frac{118{,}000}{3375}=35$.

To determine the rivet-spacing we have

$$a=\frac{3375d_1}{V_s}=2.6 \text{ inches,}$$

in which $d_1=23.5$;

$V_s=30{,}000$;

$a=$rivet-spacing at the supports.

To secure rivet-spacing in even inches, let the spacing be $2\frac{1}{2}$, 3, 4, 5, and 6 inches. If in the formula above we substitute for a, 3 inches, and solve with respect to V_s, we shall have $V_s=26{,}438$ pounds approximately. Hence from the point of the girder where the vertical shear is 26,438 pounds the rivet-spacing towards the middle point need only be 3 inches. Since the load is 2000 pounds per lineal foot, this is about 2 feet from the end of the girder.

Similarly the rivet-spacing need not exceed 4 inches from the point where $V_s=19{,}829$ pounds. This is about 5 feet from the end. For 5-inch pitch $V_s=15{,}683$ pounds, and for 6-inch pitch $V_s=13{,}219$.

Beginning at one end, the rivet-spacing will be $2\frac{1}{2}$ inches for a length of 2 feet; then 3 inches for a length of 3 feet additional, etc.; over the middle 13 feet of the girder the minimum allowable spacing for girders, or 6 inches, will be employed. In each half of the girder there will be $10+12+6+6+8\frac{1}{2}=42\frac{1}{2}$ web-rivets in each flange, which is more than required.

Rivets Connecting Flange-plates and Flange-angles.—These rivets are in single shear, but carry less stress than those connecting the flange-angles with the web. If, then, they are given the same pitch as the web-rivets, there being two flange-rivets for each web-rivet, they will safely transmit the stress.

Flange-plates.—These may be omitted between the supports and the section where s'' multiplied by the area of two angles multiplied by $d_1'=M$. In this expression d_1' is the distance between the centers of gravity of the flanges when the flange-plates are omitted, and equals $24+\frac{1}{4}-2\times.70=22.85$.

Substituting numerical values in $s''A'd_1'=M$ we have

$$13{,}000\times 5.72\times 22.85=30{,}000x-\frac{2000x^2}{12\times 2},$$

whence $x=70$ and 290, and the theoretical length of the flange-plates is 220 inches. This length must be increased so as to have two additional rows of rivets at each end, and so adjusted in length that the rivets connecting the plates with the angles will break joints with those connecting the angles with the web. This makes the length to the nearest full inch 240 inches or 20 feet.

Summary.—The girder will then be made up as follows:

Four angles $5''\times 3''\times\frac{3}{8}''$	1176.0	pounds *
One web $\frac{3}{8}''\times 24''\times 30'$	918.0	" †
Two flange-plates $\frac{3}{8}''\times 12''\times 20'$	612.0	" †
Four angles $24''\times 2\frac{1}{2}''\times\frac{5}{16}''$	40.0	" ‡
Heads of 375 rivets	124.0	" §
Total	2870.0	pounds

The weight of the girder is therefore about 5 per cent of the applied load.

* Cambria Handbook, p. 174, fourth column.

† Ibid., p. 399, last column.

‡ Ibid., p. 166, A 17, fourth column.

§ Ibid., p. 322, bottom of page.

Girder Considered as a Solid Beam.—To find the safe load under this supposition the moment of inertia of the cross-section about the neutral axis of the section may be calculated as follows and substituted in the formula $s''=13,000=\frac{My'}{I}$, in which $y'=\frac{24+\frac{1}{4}+\frac{3}{4}}{2}=12\frac{1}{2}''$. The moment of inertia of the angles and flange-plates is derived from the formula $Ar^2=Ad^2+Ar'^2$.

$Ar'^2=i$ of four angles about axes through center of gravity parallel to neutral axis of girder	8.16*
$Ar'^2=i$ of two flange-plates about axes through center of gravity parallel to neutral axes of girder	neglected
$Ar'^2=I$ of web about neutral axis of girder	432.00 †
$A\frac{d_1'^2}{4}=11.44\times(11.42)^2$	1493.00
$A\frac{d_1''^2}{4}=9\times(12.31)^2$	1364.0
I of four holes to be deducted $=2\frac{5}{8}\times(12\frac{1}{8})^2$.	386.0
Total about	2911.00

Therefore

$$13,000=\frac{My}{I}=\frac{Wly}{8I}=\frac{45Wy}{I}=\frac{45W\times 12.5}{2911.00},$$

whence $W=67,277$ app. Therefore the girder would be safe considered as a solid beam under a load $\frac{7277}{60,000}=12$ per cent greater than that for which it has been calculated.

* Cambria Handbook, p. 174, A 101, seventh column.
† Ibid., p. 184.

CHAPTER X.

DETERMINATION OF STRESSES BY ANALYTIC METHODS IN SIMPLE TRUSSES RESTING ON END SUPPORTS AND CARRYING UNIFORM DEAD LOADS.

A truss is a frame designed, like the simple beam and plate girder, to support weights and to transfer their effects to lateral supporting walls. A truss is therefore subjected to the bending moments and to the vertical and horizontal shears of the forces acting on it. (Fig. 41, p. 194.)

In the simple beam each fiber is subjected to a longitudinal and a shearing stress; in a girder the fibers of the flanges are assumed to be subjected to longitudinal stresses only, and the fibers of the web to shearing stresses only; in a truss all the fibers are subjected to longitudinal stresses only.

Trusses are used principally to support the floors and floor loads of bridges, and the floors, floor loads, and roofs of buildings when the supporting walls are widely separated. Floor-trusses are subjected to weights or vertical forces which may be either stationary or moving; the former are called *dead* and the latter *live* loads. Roof-trusses are subjected to weights or vertical forces, and to *wind pressures* or non-vertical forces.

Trusses may be divided into two general classes, those whose upper and lower pieces are parallel and horizontal, as the floor-trusses shown in Figs. 41, 43, and 45, and those in which the upper and lower pieces are not parallel, as in the roof-trusses shown in Figs. 44 and 46. The separate pieces of a truss are called the *members;* the upper and lower members are the *chord members*, and the intermediate ones the *web members* or *braces.* In a roof-truss the upper chord is also called the *principal rafter.* Web members subjected to tensile stress are called *ties*, those subjected to compressive stress, *struts.*

Trusses with Parallel Horizontal Chords.—A plate girder may be transformed into a truss with parallel horizontal chords by simply substituting for its web a system of web members which are fastened to each other and to the flanges by pin or riveted joints and divide the web area into a number of triangles, Fig. 41. The joints between the chords and web members are called the *panel points.*

In making the transformation, the flanges of the girder become the chords of the truss, which, like the flanges, are designed to resist the entire bending moment of the forces acting on the truss. The stresses in both flanges and chords are similar longitudinal stresses; the only difference between them being that the flange stress changes at every web-rivet and is uniform between rivets, while the chord stress changes at every panel point, and is uniform between panel points. The web members perform in the truss the same function as the web in the girder; they are designed to resist the entire vertical and horizontal shear of the forces which act on the truss. The horizontal component in each web member must therefore be equal to the horizontal shear, and the vertical component to the vertical shear, at the section of the truss. In the girder the loads and horizontal shears are transmitted from the rivets of the upper flange to the web-plate and produce shearing stresses in it; in the truss the loads and horizontal shears are transmitted from the panel points of the chords to the web members, and produce longitudinal stresses in them. The pins or rivets of the truss perform the same function as the web-rivets in the girder; the web-rivets connect the chord and web members and, by transferring the excess longitudinal chord stress to the web members, allow the cross-sections of the different members of the chords to vary with the bending moment. When the panel points in a chord are uniformly spaced the distance between them is called a *panel length.*

The simplest forms of trusses with parallel chords are the *Warren triangular truss*, Fig. 41, the *Pratt panel truss*, Fig. 43, and the *Howe panel truss*, Fig. 45. In the Pratt truss the vertical web members are struts, and in the Howe, ties. In the figures the tensile members are shown with light lines, and the compression members with heavy lines.

Trusses without Parallel Horizontal Chords.—Whenever the chord of a truss is inclined to the horizontal it must transmit a portion of the vertical shear and hence relieve the web members. The chords of such a truss are not therefore designed to resist the bending moment only, nor are the web members designed to resist the entire shear.

Methods of Determining the Stresses in the Members of a Truss.—As in the case of beams and girders, it is assumed that the loads or forces which are transmitted by a truss are coplanar forces which act in the plane containing the axes of the members. It is also assumed that the forces act only at the panel points of one or both of the chords.

There are two general methods of determining the stresses in the members of a truss, the *analytic* and the *graphic*. The analytic may be subdivided into the *analytic method of concurrent forces* and the *analytic method of moments.*

Analytic Method of Determining Reactions.—In all trusses resting on end supports the reactions at the supports are assumed to act along lines parallel to that of the resultant of the other external forces, unless the action lines of the reactions are fixed by special conditions. If the truss rests on a roller at either end, to allow for the expansion and contraction of its members due to changes of temperature, the reaction at that end is assumed to be normal to the surface on which the roller rests.

The two reactions with the resultant of the other forces will therefore always form a system either of parallel or concurrent forces in equilibrium. Having constructed this resultant, the intensities and directions of the reactions may always be determined from the general conditions of equilibrium of a system of coplanar forces, by taking moments with respect to the points of support.

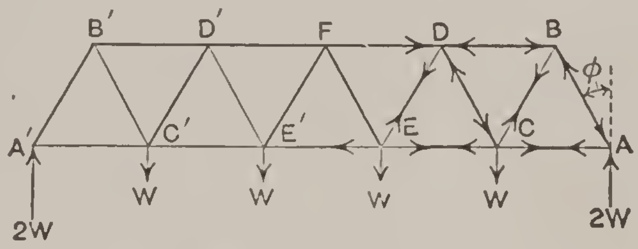
Analytic Method of Concurrent Forces.—This method depends upon the following principles:

If the truss is in a state of rest, the external forces acting on the truss as a whole, and the forces and stresses acting at *each panel point*, will form separate systems of forces in equilibrium. The system of forces at each panel point being a system of concurrent forces, the only requisites for equilibrium are that the algebraic sums of the horizontal and vertical components shall each be equal to zero.

In any system of concurrent forces in equilibrium in which the lines of direction of all the forces are given, the intensities of two and of not more than two unknown forces may be determined if all the others are given. This is evident since, if we find the resultant of the known forces and its components in the directions of the unknown forces, equilibrium is possible only when the unknown forces are equal in intensity and opposed in direction to these components.

In any system of concurrent stresses in a truss the stresses acting *towards* the common point of application are *compressive*, and those acting *away* from the point are *tensile;* and, conversely, a compressive stress in a piece must be assumed as acting towards the common point of application, and a tensile stress as acting away from it.

Stresses in Trusses with Parallel Chords.—Let these principles be applied to the problem of determining the stresses in the Warren truss shown in Fig. 41 due to a load of W at each panel point of the lower chord.



Warren Truss.

FIG. 41.

If it is assumed that there are stresses in all the members of this truss, it is seen that the only panel points where there are but *two* unknown stresses are at the points A and A'; hence we must begin the solution at one of these points.

Panel Point A.—At the panel point A if we resolve each force and each stress into horizontal and vertical components, and indicate the forces acting upwards and those acting to the right as positive, and those acting downwards and those acting towards the left as negative, we shall have the following table, in which the given stresses are in Roman and the required stresses in Italic characters.

In the first line we write the known force and its horizontal and vertical components. In the second line we write the unknown force whose *vertical component* must be equal to the

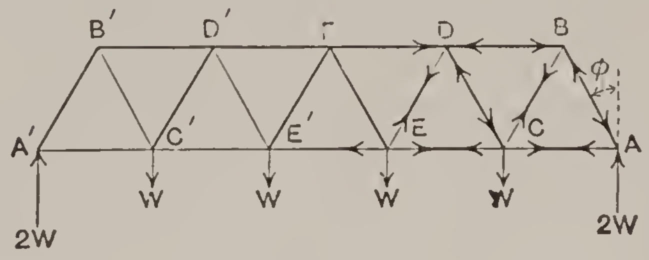
Force.	Vertical Components.	Horizontal Component.	Total Intensity.	Character.
2W	$+2W$	0	2W	Reaction
Stress BA	$-2W$	$+2W \tan\phi$	$\dfrac{2W}{\cos\phi}$	*Compressive*
" *AC*	0	$-2W \tan\phi$	$2W \tan\phi$	*Tensile*

vertical component of the known force. At the point A this must be the stress BA, since the stress CA, if resolved into horizontal and vertical components, will have a vertical component of zero. The vertical component must act downwards, hence the stress is compressive and its horizontal component acts towards the right or is positive. Knowing its vertical component and the angle ϕ, the stress BA is fully known in intensity and direction, and we can write out its horizontal component, its total intensity, and its character. In the third line we write the force whose intensity must be equal and contrary to the *horizontal component* of the now known forces. Since the horizontal component of AC is negative, it acts towards the left and away from A; the stress is therefore tensile. If the table is correct, the sum of the vertical and horizontal components of the forces or stresses will be separately equal to zero.

Panel Point B.—At the panel point B the compressive force AB is fully known and acts towards the point B or upwards and to the left. Its vertical component must therefore be positive and its horizontal component negative. Hence we may write out the following table:

Force.	Vertical Component.	Horizontal Component.	Total Intensity.	Character.
Stress AB	$+2W$	$-2W \tan\phi$	$\dfrac{2W}{\cos\phi}$	Compressive
Stress BC	$-2W$	$-2W \tan\phi$	$\dfrac{2W}{\cos\phi}$	*Tensile*
" *DB*	0	$+4W \tan\phi$	$4W \tan\phi$	*Compressive*

As the vertical component of BC must be negative to counteract the vertical component of AB, the stress BC acts downward and away from B, hence its horizontal component acts towards the left and is negative, and the stress itself is tensile. The intensity of the stress in DB must be equal to the sum of the horizontal components of the stresses AB and BC, and act in a contrary direction or towards B. It is therefore a compressive stress.



Warren Truss.

FIG. 41.

Panel Point C.—At this panel point we know both the tensile stresses BC and CA and the force W; hence they are placed in the first three lines of the table:

Force.	Vertical Component.	Horizontal Component.	Total Intensity.	Character.
Stress CB	$+2W$	$+2W \tan\phi$	$\frac{2W}{\cos\phi}$	Tensile
Stress CA	o	$+2W \tan\phi$	$2W \tan\phi$	"
W	$-W$	o	W	Applied load
" *DC*	$-W$	$+W \tan\phi$	$\frac{W}{\cos\phi}$	*Compressive*
" *CE*	*o*	$-5W \tan\phi$	$5W \tan\phi$	*Tensile*

The vertical component of DC must be equal to $-W$, hence the force acts towards C, is compressive, and has a positive horizontal component. The stress CE must be negative; it therefore acts away from the point C and is a tensile stress.

Panel Point D.—At this panel point we know the compressive stresses BD and CD.

Force.	Vertical Component.	Horizontal Component.	Total Intensity.	Character.
Stress BD	o	$-4W \tan\phi$	$4W \tan\phi$	Compressive
" CD	$+W$	$-W \tan\phi$	$\frac{W}{\cos\phi}$	"
Stress DE	$-W$	$-W \tan\phi$	$\frac{W}{\cos\phi}$	*Tensile*
" *FD*	*o*	$+6W \tan\phi$	$6W \tan\phi$	*Compressive*

The vertical component of DE must be equal to $-W$, hence the stress acts away from the point D, is tensile, and has a negative horizontal component. The stress FD must act towards D and is compressive.

Panel Point E.—At this point we know the tensile stresses ED and EC and the force W.

Force.	Vertical Component.	Horizontal Component.	Total Intensity.	Character.
Stress ED	$+W$	$+W \tan \phi$	$\frac{W}{\cos \phi}$	Tensile
" EC	o	$+5W \tan \phi$	$5W \tan \phi$	"
W	$-W$	o	W	Applied load
Stress EF	*o*	*o*	*o*	
" *EE'*	*o*	$-6W \tan \phi$	$6W \tan \phi$	*Tensile*

The only forces acting at the point E which have vertical components are the stresses ED and EF and the force W. Since the vertical component of the stress DE is exactly equal and opposed to the force W, the vertical component of the stress in EF must be equal to zero. If its vertical component is zero, the stress itself must be equal to zero, since its action line is not horizontal. The stress in EE' must be negative and equal to the stress in EC and the horizontal component of ED; the stress in EE' must therefore be tensile.

As the truss is symmetrical, the stresses in the members of the left half must be equal to the stresses in the corresponding members of the right half.

The stresses may then be tabulated as shown on next page.

If we examine the table it will be seen:

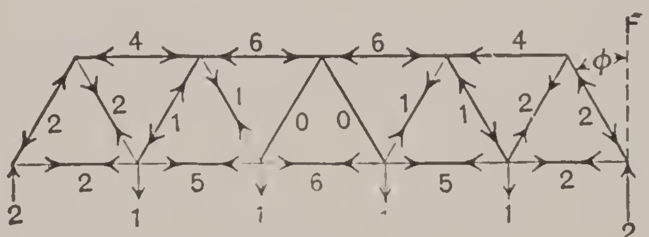
I. That the stresses in the web members are alternately compressive and tensile, and that between the points of application of the external forces the web members are subjected to the same amount of stress. This results from the fact that there is no change in the vertical or horizontal shear between these points.

II. That the stresses in the upper chord are all compressive and those in the lower chord all tensile, and that these stresses increase as we approach the middle of the span. This was to be expected, as the longitudinal fiber stresses in a beam or the

Stresses.	Compressive.	Tensile.
	UPPER CHORD.	
Stress BD	$4W \tan \phi$	....
" FD	$6W \tan \phi$	....
	LOWER CHORD.	
Stress AC	....	$2W \tan \phi$
" CE	....	$5W \tan \phi$
" EE′	....	$6W \tan \phi$
	WEB MEMBERS.	
Stress AB	$\frac{2W}{\cos \phi}$	....
" BC	....	$\frac{2W}{\cos \phi}$
" DC	$\frac{W}{\cos \phi}$	....
" DE	....	$\frac{W}{\cos \phi}$
" EF	0	0

fiber stresses in the flanges of a girder resting on two points of support are compressive above the neutral axis and tensile below, and the bending moment is a maximum at the middle point of a uniformly loaded beam resting on two end supports.

If the numerical coefficients of the forces and stresses are written on the diagram of the truss, we shall have the diagram shown in Fig. 42.

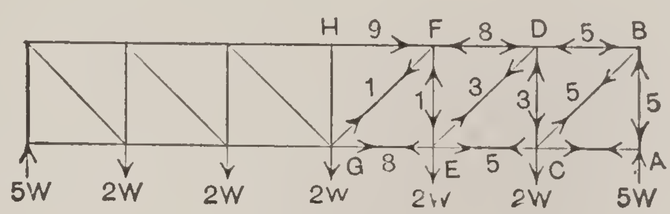


Warren Truss.
FIG. 42.

If this diagram is carefully examined, it will be seen that at every panel point the sum of the numerical coefficients of the stresses which tend to move the point upwards are exactly equal to the sum of the coefficients of the stresses which tend to move it downwards; and that the sum of the numerical coefficients of the stresses which tend to move it to the left is exactly equal to the sum of the coefficients of the stresses which tend to move it to the right.

This enables us to write out the numerical coefficients of the stresses of any truss with parallel chords and web members making the same angles with each other, provided we are careful to indicate by arrow-heads the direction of the stress in each piece as soon as determined. The arrow-heads will also indicate the character of the stress. A *compressive* stress must be indicated as acting *towards both extremities* of the member in which it is found, and a *tensile stress* as *acting away from both extremities* of the member in which it is found.

In the Pratt truss, Fig. 43, at the panel point A the reaction



Pratt Truss.

FIG. 43.

$5W$ tends to move the point A upwards; it is restrained only by the member AB; the stress in AB must therefore be compressive and its numerical coefficient must be 5.

The stress in AC must be zero, since there is no opposing force in a horizontal direction at A.

At the panel point B the known compressive stress in AB tends to move the point B upwards, but it is restrained by the piece BC; hence the numerical coefficient of the stress in BC is equal to that of AB, or is 5. The stress in BC must act downwards and be tensile. The stress tending to move B in a horizontal direction to the left is BC, but it is restrained by the stress BD; hence the stress in BD must be compressive and its numerical coefficient is 5.

At the panel point C the known stress tending to move the point C upwards is the tensile stress BC with a numerical coefficient of 5; the known force tending to move it downwards is the applied load $2W$; hence the stress in DC must act downwards or towards C, and be compressive and have a numerical coefficient of 3. The known stresses tending to move the point C to the right are the stresses CB and CA; it is restrained by the stress in CE, which must act towards the left. The stress in CA is zero; hence the stress in CE acts towards the left, is tensile, and has a numerical coefficient equal to that of CB, or 5.

In the same manner we may determine the numerical coefficients of all the other stresses.

To determine the actual stresses the

Coefficients in the chord members should be multiplied by $W \tan \phi$.

" " " diagonal web members should be multiplied by $\dfrac{W}{\cos \phi}$.

" " " vertical web members should be multiplied by W.

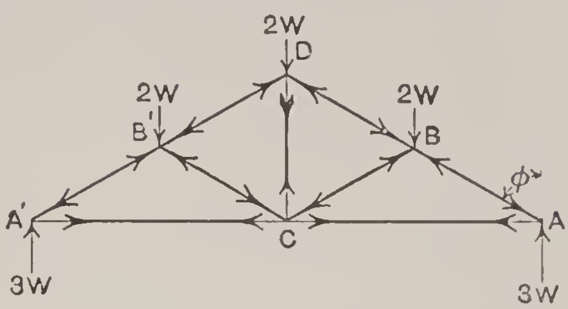
In these expressions ϕ is the angle CBA.

In writing out the stresses in a truss by this method, it is usually simpler to write out first the stresses in the web members, since the numerical coefficients of these stresses are the same as the numerical coefficients of the vertical shear. Having the stresses in the web members, those in the chords are easily determined. Thus in the Pratt truss, Fig. 43, the vertical shear between A and C is $5W$, hence the numerical coefficient of the stresses in AB and BC is also 5; the vertical shear between C and E is $5W-2W=3W$, hence the numerical coefficient of the stresses in CD and DE is also 3; the vertical shear between E and G is $5W-2W-2W=W$, hence the numerical coefficient of the stresses in EF and FG is also unity.

It will be observed that if in Figs. 41 or 42 we cut the truss by a plane which intersects one *diagonal* web member only, the sum of the numerical coefficients of the compressive stresses in the members cut will be equal to the sum of numerical coefficients of the tensile stresses. Thus if, in Fig. 43, we pass a vertical plane between E and C, cutting the members EC, ED, and DF, we have the sum of the numerical coefficients of the tensile stresses in CE and DE equal to the numerical coefficient of the compressive stress in FD. This affords a means of checking results obtained when the stresses are written out as described above.

Stresses in Trusses without Parallel Chords.—The solution by analysis of concurrent forces may also be applied to trusses without parallel chords.

Let it be required to determine the stresses in the truss shown in Fig. 44 by the analysis of forces.



King-post Truss.
FIG. 44.

It will be observed that we may determine the stresses in AB and AC in the same manner in which we determined the stresses in the corresponding pieces of the truss with parallel chords, by resolving the forces acting at the point A into horizontal and vertical components. However when we reach the panel point B we shall have both horizontal and vertical components in both of the unknown members CB and DB. The resolution of the stresses into horizontal and vertical components will not aid us in the solution of the unknown stresses at that point. The same method of determining the stresses may, however, be utilized if we take our components at A, C, and D vertical and horizontal, and at B normal and parallel to the chord AD. The angle between the chords is $90°-\phi$.

As before, the stresses acting upwards or to the right will be considered positive, those acting downwards or to the left will be considered negative.

PANEL POINT A.

Stress.	Vertical Component.	Horizontal Component.	Total Stress.	Character.
3W	$+3W$	0	3W	Reaction
AB	$-3W$	$+3W \tan\phi$	$\frac{3W}{\cos\phi}$	*Compressive*
AC	0	$-3W \tan\phi$	$3W \tan\phi$	*Tensile*

PANEL POINT B.

Stress.	Normal Component.	Parallel Component.	Total Stress.	Character.
2W	$-2W \sin\phi$	$+2W \cos\phi$	2W	Applied load
AB	0	$-\frac{3W}{\cos\phi}$	$\frac{3W}{\cos\phi}$	Compressive
BC	$+2W \sin\phi$	$\frac{+W \sin^2\phi - W \cos^2\phi}{\cos\phi}$	$\frac{W}{\cos\phi}$	*Compressive*
BD	0	$+\frac{2W}{\cos\phi}$	$\frac{2W}{\cos\phi}$	"

The normal component of the stress in BC must be equal in intensity and opposite in direction to the normal component of $2W$, which is $2W \sin \phi$. The parallel component of the stress in BC must therefore be equal to $2W \sin \phi \cot (180° - 2\phi)$, which is equal to $-2W \sin \phi \cot 2\phi$. $\text{Cot } 2\phi = \dfrac{\cot^2\phi - 1}{2 \cot \phi}$. Substituting for $\cot \phi$ its value $\dfrac{\cos \phi}{\sin \phi}$ we have

$$\cot 2\phi = \frac{\dfrac{\cos^2\phi - \sin^2\phi}{\sin^2 \phi}}{\dfrac{2 \cos \phi}{\sin \phi}} = \frac{\cos^2\phi - \sin^2\phi}{2 \cos \phi \sin \phi}.$$

Substituting this value of $\cot 2\phi$ in the term $-2W \sin \phi \cot 2\phi$, the parallel component becomes $\dfrac{W \sin^2 \phi - W \cos^2 \phi}{\cos \phi}$.

The stress in BC is equal to $\dfrac{2W \sin \phi}{\sin (180° - 2\phi)} = \dfrac{2W \sin \phi}{\sin 2\phi}$. $\sin 2\phi = 2 \sin \phi \cos \phi$, which substituted above makes the stress in BC equal to $\dfrac{W}{\cos \phi}$.

PANEL POINT C.

Stress.	Vertical Component.	Horizontal Component.	Total Stress.	Character.
BC	$-W$	$-W \tan \phi$	$\dfrac{W}{\cos \phi}$	Compressive
B′C	$-W$	$+W \tan \phi$	$\dfrac{W}{\cos \phi}$	"
AC	o	$+3W \tan \phi$	$3W \tan \phi$	Tensile
A′C	*o*	$-3W \tan \phi$	$3W \tan \phi$	*Tensile*
DC	$+2W$	*o*	$2W$	"

PANEL POINT D.

Stress.	Vertical Component.	Horizontal Component.	Total Stress.	Character.
2W	$-2W$	o	$2W$	Applied load
CD	$-2W$	o	$2W$	Tensile
BD	$+2W$	$-2W \tan \phi$	$\dfrac{2W}{\cos \phi}$	Compressive
B′D	$+2W$	$+2W \tan \phi$	$\dfrac{2W}{\cos \phi}$	*Compressive*

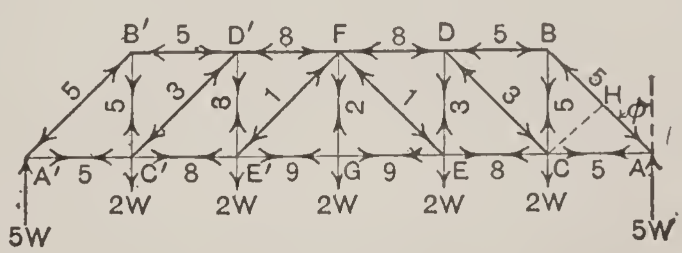
The following table gives the stress in each member:

Piece.	Compressive.	Tensile.
AB	$\frac{3W}{\cos\phi}$	....
BD	$\frac{2W}{\cos\phi}$	....
BC	$\frac{W}{\cos\phi}$	....
AC	......	$3W\tan\phi$
DC	......	2W

The Analysis of Moments.—The solution by analysis of moments depends upon the principle that, *since every member, as well as every combination of members, of a truss is in a state of rest, the algebraic sum of the moments of the forces acting on it, with respect to any point in the plane of the truss, must be equal to zero.* The forces acting on the member or combination of members may be either *external forces directly applied to it* or *stresses communicated to it through other members.*

It is usually better to take the panel points as the centers of moments. Clockwise moments are considered positive; stresses acting towards the part of the truss which is considered free to move are compressive; those acting away from it are tensile.

In the Howe truss shown in Fig. 45,



Howe Truss.
FIG. 45.

Let l = panel length AC;

d = panel depth $BC = \dfrac{l}{\tan\phi}$;

b = distance $CH = l\cos\phi$.

Let rotation be assumed about the panel points in succession.

Panel Point C.—Under the action of the force $5W$ the member AC would rotate about C were it not restrained by the member BA. The moment of the stress BA about the point C must therefore be equal and opposite to the moment of the force $5W$ about the same point, or

$$\text{stress } BA \times b - 5Wl = 0; \quad \therefore \text{ stress } BA = \frac{5Wl}{b} = \frac{5W}{\cos\phi}. \qquad (431)$$

As its moment is positive, it acts towards the vertex A and is a compressive stress.

Under the action of the force $5W$ the triangle ABC would rotate about the point C were it not restrained by the member DB. Hence the moment of the stress DB about C must be equal to the moment of the stress $5W$ about the same point, or

$$\text{stress } BD \times d - 5Wl = 0; \quad \therefore \text{ stress } BD = \frac{5Wl}{d} = 5W \tan\phi. \qquad (432)$$

As its moment is positive, the stress acts towards B and is compressive.

Panel Point B.—The reaction $5W$ would rotate the member AB about B were it not restrained by the member AC; hence we have

$$\text{stress } AC \times d - 5Wl = 0; \quad \therefore \text{ stress } AC = \frac{5Wl}{d} = 5W \tan\phi. \qquad (433)$$

As its moment is positive, this stress acts away from A and is tensile.

Panel Point D.—The reaction $5W$ would rotate the broken line ABD about the point D were it not restrained by the members AC and BC; hence we have

$$\text{stress } BC \times l + \frac{5Wl}{d} \times d - 5W \times 2l = 0;$$

$$\therefore \text{ stress } BC = \frac{10Wl - 5Wl}{l} = 5W. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (434)$$

As its moment is positive, it acts away from the point B and is a tensile stress.

The parallelogram $ABCD$ is held in a state of rest with respect to D by the forces $5W$, $2W$, and the stress in the member CE; hence we have

$$\text{stress } CE \times d - 5W \times 2l + 2Wl = 0,$$

$$\text{stress } CE = \frac{8Wl}{d} = 8W \tan \phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad (435)$$

As its moment is positive, it acts away from C and is a tensile stress.

Panel Point E.—The triangle ABC is held in a state of rest with respect to the center E by the forces $5W$ and $2W$ and the stresses in BD and CD. Hence we have

$$\text{stress } BD \times d + \text{stress } DC \times b - 5W \times 2l + 2Wl = 0.$$

But stress $BD = \dfrac{5Wl}{d}$, hence

$$\text{stress } DC \times b = 10Wl - 2Wl - 5Wl = 3Wl,$$

or $$\text{stress } DC = \frac{3Wl}{b} = \frac{3W}{\cos \phi}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (436)$$

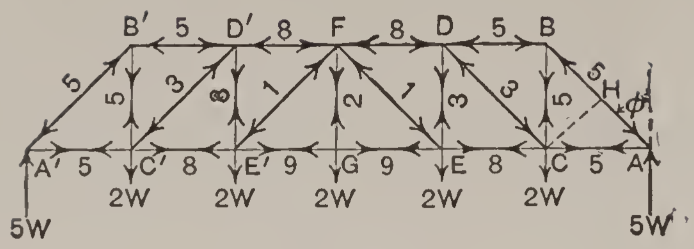
As its moment is positive, it acts towards C and is a compressive stress.

The trapezoid $ABDE$ is held in a state of rest with respect to the center E by the forces $5W$ and $2W$ and the stress in the member FD. Hence

$$\text{stress } FD \times d - 5W \times 2l + 2Wl = 0,$$

$$\text{stress } FD = \frac{8Wl}{d} = 8W \tan \phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad (437)$$

As its moment is positive, it acts towards D and is a compressive stress.



Howe Truss.
FIG. 45.

Panel Point F.—The figure $FBACDF$ is held in a state of rest with respect to the center F by the forces $5W$, $2W$, and the stresses in CE and DE. Hence

$$\text{stress } DE\times l+\text{stress } CE\times d-5W\times 3l+2W\times 2l=0.$$

But stress $CE=\dfrac{8Wl}{d}$. Hence

$$\text{stress } DE=11Wl-8Wl=3Wl. \quad . \quad . \quad . \quad (438)$$

As its moment is positive, it acts away from D and is a tensile stress.

The parallelogram $FBAE$ is held in a state of rest with respect to the center F by the forces $5W$, $2W$, $2W$, and the stress in EG; hence

$$\text{stress } EG\times d-5W\times 3l+2W\times 2l+2W\times l=0,$$

$$\text{stress } EG=\frac{9Wl}{d}=9W\tan\phi: \quad . \quad . \quad . \quad . \quad (439)$$

As its moment is positive, it acts away from E and is a tensile stress.

Panel Point G.—The trapezoid $DBAE$ is held in a state of rest with respect to the center G by the forces $5W$, $2W$, $2W$, and the stresses FD and FE.
Hence

$$\text{stress } FE\times b+\text{stress } FD\times d-5W\times 3l+2W\times 2l+2Wl=0.$$

Substituting for FD its value $\dfrac{8Wl}{d}$ we have

$$\text{stress } FE=\frac{Wl}{b}=\frac{W}{\cos\phi}. \quad . \quad . \quad . \quad . \quad . \quad (440)$$

As its moment is positive, it acts towards E and is a compressive stress.

Panel Point D′.—The figure $D'BAEFD'$ is held in a state of rest with respect to center D' by the forces $5W$, $2W$, and $2W$, and the stresses FG, EG, and $E'F$. The stress in $E'F$ may be assumed as equal to EF on account of the symmetrical loading, or may be determined by solving the truss from the point A'. Assuming it to be equal to EF we have

$$\text{stress } FG\times l-\text{stress } FE'\times b+\text{stress } GE\times d-5W\times 4l$$
$$+2W\times 3l+2W\times 2l=0,$$

$$\text{stress } FG\times l=20Wl+Wl-9Wl-6Wl-4Wl=2Wl, \quad (441)$$

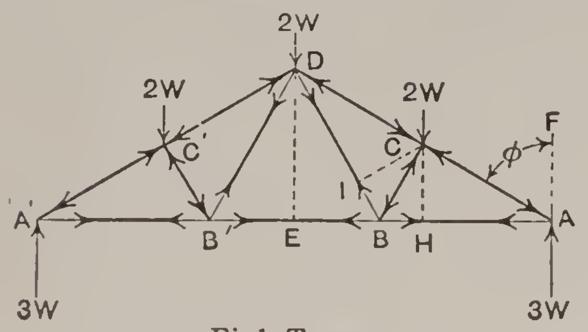
$$\text{stress } FG=2W.$$

As its moment is positive, it acts away from F and is a tensile stress.

The following table gives the stress in each member:

Members.	Compressive.	Tensile.
	UPPER CHORD.	
BD	$5W \tan \phi$	......
DF	$8W \tan \phi$	......
	LOWER CHORD.	
AC	......	$5W \tan \phi$
EC	......	$8W \tan \phi$
EG	......	$9W \tan \phi$
	DIAGONAL WEB.	
AB	$\dfrac{5W}{\cos \phi}$	......
CD	$\dfrac{3W}{\cos \phi}$	......
EF	$\dfrac{W}{\cos \phi}$	......
	VERTICAL WEB.	
BC	......	$5W$
DE	......	$3W$
FG	......	$2W$

Method of Moments in Roof-trusses.—To apply the method of moments to the determination of the stresses in the members of a truss without parallel chords, let it be required to determine the stresses in the truss shown in Fig. 46.



Fink Truss.
FIG. 46.

Let $l = AD$ = length of principal rafter or upper chord;

ϕ = angle CAF. Then

$l \sin \phi = AE$ = one-half the span AA';

$l \cos \phi = DE$ = rise of the truss.

Panel Point C.—To find the stress in AB, take the center of moments at C. The reaction $3W$ will rotate the member AC about C unless restrained by the member AB. Hence we have

$$\text{stress } AB \times CH - 3W \times AH = 0,$$

$$\text{stress } AB \times \frac{l \cos \phi}{2} = 3W \times \frac{l \sin \phi}{2},$$

$$\text{stress } AB = 3W \tan \phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (442)$$

As its moment is positive, the stress acts away from A and is a tensile stress.

Panel Point D.—To find the stress in BB', take the center of moments at D. The reaction $3W$ and the force $2W$ will rotate the triangle ABD about D unless restrained by the member BB'. Hence we have

$$\text{stress } BB' \times DE - 3W \times AE + 2W \times HE = 0,$$

$$\text{stress } BB' \times l \cos \phi = 3W \times l \sin \phi - 2W \times \frac{l \sin \phi}{2}$$

$$= 2Wl \sin \phi,$$

$$\text{stress } BB' = 2W \tan \phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (443)$$

As its moment is positive, the stress acts away from B and is a tensile stress.

Panel Point B.—To find the stress in AC, take the center of moments at B. The reaction $3W$ will rotate the member AB about B unless restrained by the member CA. Hence we have

$$\text{stress } CA\times BC-3W\times AB=0,$$

$$BC=\frac{l}{2\tan\phi},\quad AB=\frac{l}{2\sin\phi},$$

$$\text{stress } CA\times\frac{l}{2\tan\phi}=\frac{3Wl}{2\sin\phi},$$

$$\text{stress } CA=\frac{3W}{\cos\phi}. \qquad (444)$$

Since its moment is positive, the stress acts towards A and is a compressive stress.

To find the stress in DC, take the center of moments at B. The reaction $3W$ and the force $2W$ will rotate the triangle ABC about B unless restrained by the member DC. Hence we have

$$\text{stress } DC\times BC-3W\times AB+2W\times BH=0,$$

$$BH=BC\cos\phi=\frac{l\cos\phi}{2\tan\phi}=\frac{l\cos^2\phi}{2\sin\phi},$$

$$\text{stress } DC\times\frac{l}{2}\times\frac{\cos\phi}{\sin\phi}=\frac{3Wl}{2\sin\phi}-\frac{2Wl\cos^2\phi}{2\sin\phi},$$

$$\text{stress } DC=\frac{3W}{\cos\phi}-2W\cos\phi. \qquad (445)$$

As its moment is positive, this stress acts towards C and is a compressive stress.

Panel Point D.—To find the stress CB, take the center of moments at D. The reaction $3W$ and the force $2W$ will rotate the member ACD about D unless restrained by the members BC and AB. Hence we have

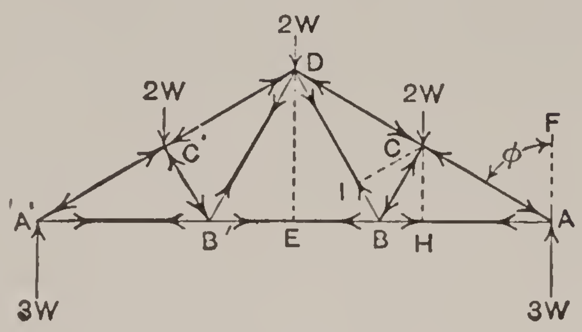
$$\text{stress } BC\times CD-3W\times AE+2W\times HE+\text{stress } AB\times DE=0,$$

$$\text{stress } BC\times\frac{l}{2}=3Wl\times\sin\phi-\frac{2Wl\sin\phi}{2}-3W\tan\phi\times l\cos\phi$$
$$=-Wl\sin\phi,$$

$$\text{stress } BC=-2W\sin\phi. \qquad (446)$$

As its moment is negative, this stress acts towards C and is a compressive stress.

Panel Point C.—To find the stress BD, take the center of moments at C. The reaction will rotate the triangle ABC about



Fink Truss.
FIG. 46.

C unless restrained by the members BB' and BD. Hence we have

$$\text{stress } BD \times CI - 3W \times AH + \text{stress } BB' \times CH = 0,$$

$$IC = CH = \frac{l \cos \phi}{2},$$

$$\text{stress } BD \times \frac{l \cos \phi}{2} = \frac{3Wl \sin \phi}{2} - 2W \tan \phi \times \frac{l \cos \phi}{2} = \frac{Wl \sin \phi}{2},$$

$$\text{stress } BD = W \tan \phi. \quad \ldots \ldots \quad (447)$$

As its moment is positive, this stress acts away from B and is a tensile stress.

The stresses in all the members may be tabulated thus:

Members.	Compressive.	Tensile.
AC	$\frac{3W}{\cos \phi}$	......
CD	$\frac{3W}{\cos \phi} - 2W \cos \phi$	......
BC	$2W \sin \phi$	......
AB	......	$3W \tan \phi$
BB′	......	$2W \tan \phi$
BD	......	$W \tan \phi$

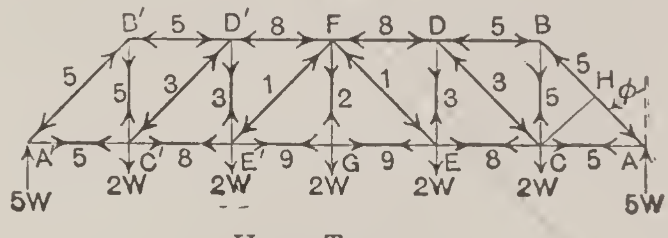
Method of Sections.—The method of moments is often called the method of sections, since, if we pass a plane through a truss cutting its members, *the algebraic sum of the moments of the stresses in the pieces cut, and of the external forces which act on the truss on one side of the plane about any point in the plane of the truss, will be equal to zero.* The resulting equation may be solved when it contains but one unknown intensity. This will be the case when *only two members are cut and the center of* *moments is taken on one of the members;* when *three members which are not all parallel to each other are cut and the center of moments is taken at the intersection of two of them;* when *the intensities of all the members cut, save one, are known.*

Thus in Fig. 47 we may determine the stresses in all the members of the panel $FDGE$ in the following manner:

Let $l = AC$;

$$d = BC = \frac{l}{\tan\phi};$$

$$b = CH = l\cos\phi.$$



Howe Truss.
FIG. 47.

Pass a plane cutting the members FD, FE, and GE; let F, the intersection of the members FD and FE, be the assumed center of moments. If we consider the part of the truss $A'B'FGA'$, to the left of the section, to be at rest, the part $ABDE$, to the right of the section, would rotate about F due to the extraneous forces acting at A, C, and E were it not restrained by the stress in GE. Hence we have

$$\text{stress } GE \times d - 5W \times 3l + 2W \times 2l + 2Wl = 0,$$

$$\therefore \text{ stress } GE = \frac{9Wl}{d} = 9W\tan\phi. \quad \ldots\ldots \quad (448)$$

As the moment of the stress GE is positive, it must act away from E and be a tensile stress.

If E, the intersection of FE and GE, is taken as the center of moments, we have

$$\text{stress } FD \times d - 5W \times 2l + 2Wl = 0,$$

$$\therefore \text{ stress } FD = \frac{8Wl}{d} = 8W\tan\phi. \quad \ldots\ldots \quad (449)$$

As its moment is positive, it must act towards D and be a compressive stress.

If we take the center of moments at G, we have

$$\text{stress } FE \times b + \text{stress } FD \times d - 5W \times 3l + 2W \times 2l + 2Wl = 0;$$

but $$FD \times d = 8Wl, \text{ hence}$$

$$\text{stress } FE \times b = 9Wl - 8Wl = Wl,$$

$$\text{stress } FE = \frac{W}{\cos \phi}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (450)$$

As its moment is positive, the stress must act towards E and be compressive.

Pass a plane cutting $E'G$, FG, FE, and FD, and take the center of moments at E'. Then

$$\text{stress } FG \times l + \text{stress } FE \times 2b + FD \times d - 5W \times 4l + 2W \times 3l + 2W \times 2l + 2Wl = 0,$$

$$\text{stress } FG \times l = -2Wl - 8Wl + 20Wl - 6Wl - 4Wl - 2Wl,$$

$$\text{stress } FG = -2W. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (451)$$

Since its moment is negative, it must act away from G and be a tensile stress.

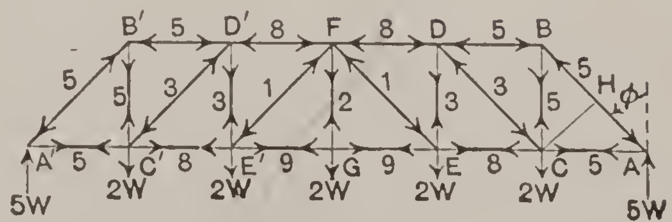
Pass a plane cutting EC, DE, and FD, and take C as the center of moments. Then

$$\text{stress } ED \times l + FD \times d - 5Wl = 0,$$

$$\text{stress } ED \times l = 5Wl - 8Wl = -3Wl,$$

$$\text{stress } ED = -3W. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (452)$$

Since its moment is negative, it must act away from D and be a tensile stress.



Howe Truss.
FIG. 47.

CHAPTER XI.

EFFECT OF MOVING LOADS UPON TRUSSES WITH PARALLEL CHORDS.

SINCE a truss with parallel chords is simply a beam in which the chords must resist the bending moment and the web members must resist the shear, the same principles must govern them both.

Concentrated Live Load.—As applied to trusses proposition **I**, page 119, should read: If a single concentrated load is moved over a truss without weight resting on end supports, *the stress in any chord member will be numerically greatest when the load is between the middle point of the member and the middle point of the truss, and as near to the former point as the method of loading will permit.* Thus in the Warren truss, Fig. 48, if it is assumed that the load can be placed on either the upper or lower chord, the greatest stress in the member *FD* is produced by placing the load at *E*, and the greatest stress in the member *GE* is produced by placing the load at *F*.

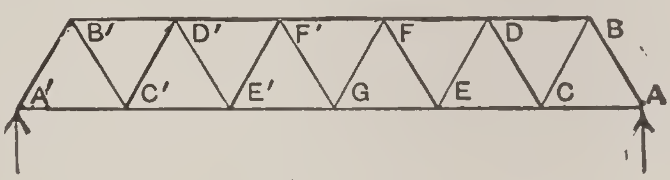


FIG. 48.

If the load is confined to the panel points of a single chord, as the lower, the greatest stress in a member of that chord is produced by placing the concentrated load at that end of the member which is nearest the middle point of the truss. Thus if the load is applied at *G*, it will produce a greater stress in *GE* than if placed at *E*.

As applied to a truss, proposition II, page 121, should read: *The chord member having the maximum stress is the one at the middle point of the truss.*

The maximum chord stress is in the middle member of the upper chord FF' when the concentrated load is at G; the maximum chord stress in the lower chord is in the middle members $E'G$ and EG also when the load is at G.

Proposition III, page 121 should read: *Propositions I and II are equally true if the truss is of uniform weight per lineal foot and its weight is considered, or if we combine the live with a uniform dead load.*

Proposition VIII, page 129, should read: If a concentrated load is moved over a truss without weight resting on end supports, *the stress in any web member will be numerically greatest when the load is between the middle point of the truss and the member, and as near the member as the method of loading will permit.*

Thus in Fig. 48 the stress in DC will be a maximum when the concentrated load is at E.

The final conclusion on page 130, under the same proposition, should read: *The maximum stress in the web members of the end panels is greater numerically than the maximum stress in any other web members.*

Thus in Fig. 48 the maximum stress in the web members of the panels ABC and $A'B'C'$ is greater than the maximum stress in any other web members.

From equations (350) and (351) it follows that the shear on one side of the concentrated load is positive and on the other negative. In a truss this must be interpreted as showing that *in any system of parallel web members, those on one side of the load will be under tension, and those on the other under compression.*

Thus, in the truss Fig. 48, if the concentrated load is at E, the stress in the members ED and BC is tension, but the stress in the parallel members FG, $F'E'$, $D'C'$, and $A'B'$ is compression.

Proposition IX, page 130, should read: If a concentrated load moves over a truss of uniform weight, *the live- and dead-load web stresses will have the same signs between the load and the nearer support, as well as between the middle point of the beam and the farther support, but will have unlike signs between the load and the middle point.*

Thus, in the truss Fig. 48, if the truss has a concentrated load at E and a uniform dead load, the stress in any member between E and A or between G and A' will be the numerical sum of the live- and dead-load stresses; but in the members EF and FG the resultant stress is the numerical difference between the stresses due to the live and dead loads.

It follows that *there will be a point of no shear in the truss as in the beam, and if this point is at least one panel length from the middle point, the stress in some of the web members will be reversed as the live load crosses the truss.*

As in beams, page 132, *the resultant stress in any web member is greatest when its live-load component stress is greatest.*

Uniformly Distributed Live Loads. — Proposition V should read: If a live load of uniform weight moves over a truss without weight, resting on end supports separated by a distance less than the length of the live load, *the stress in any chord member will be greatest when the live load entirely covers the truss.*

Proposition VI should read: Proposition V is equally true *if the truss upon which the load is moved is of uniform weight and its weight is considered, or if we combine the live and dead loads.*

Proposition X should read: If a uniformly distributed live load is moved over a truss of uniform weight resting on end supports separated by a distance less than the length of the load, *the stress in any web member will be algebraically greatest when the live load covers the greater segment into which its panel divides the truss, and algebraically least when the live load covers the smaller segment.*

To illustrate these statements, let the Warren truss, shown in Fig. 48, be subjected to a concentrated load $6W$ which acts in succession at each panel point of the lower chord. Determine the numerical coefficients by any of the methods heretofore explained, and tabulate them in columns 1 to 6 as shown. To determine the actual stresses, chord coefficients must be multiplied by $\tan \phi$, and web coefficients by $\frac{1}{\cos \phi}$.

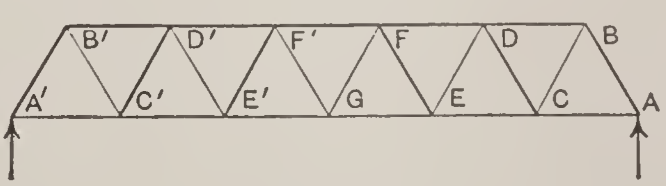


Fig. 48.

1	2	3	4	5	6	7
	Moving Load of 6W at					
	C	E	G	E′	C′	Uniform Distributed Load of 30W.
			UPPER CHORD.			
BD	−10W	− 8W	− 6W	− 4W	− 2W	−30W
DF	− 8W	−16W	−12W	− 8W	− 4W	−48W
FF′	− 6W	−12W	−18W	−12W	− 6W	−54W
F′D′	− 4W	− 8W	−12W	−16W	− 8W	−48W
D′B′	− 2W	− 4W	− 6W	− 8W	−10W	−30W
			LOWER CHORD.			
AC	+ 5W	+ 4W	+ 3W	+ 2W	+ 1W	+15W
CE	+ 9W	+12W	+ 9W	+ 6W	+ 3W	+39W
EG	+ 7W	+14W	+15W	+10W	+ 5W	+51W
GE′	+ 5W	+10W	+15W	+14W	+ 7W	+51W
E′C′	+ 3W	+ 6W	+ 9W	+12W	+ 9W	+39W
C′A′	+ 1W	+ 2W	+ 3W	+ 4W	+ 5W	+15W
			WEB MEMBERS PARALLEL TO AB.			
AB	− 5W	− 4W	− 3W	− 2W	− 1W	−15W
CD	+ 1W	− 4W	− 3W	− 2W	− 1W	− 9W
EF	+ 1W	+ 2W	− 3W	− 2W	− 1W	− 3W
GF′	+ 1W	+ 2W	+ 3W	− 2W	− 1W	+ 3W
E′D′	+ 1W	+ 2W	+ 3W	+ 4W	− 1W	+ 9W
C′B′	+ 1W	+ 2W	+ 3W	+ 4W	+ 5W	+15W
			WEB MEMBERS PARALLEL TO A′B′.			
BC	+ 5W	+ 4W	+ 3W	+ 2W	+ 1W	+15W
DE	− 1W	+ 4W	+ 3W	+ 2W	+ 1W	+ 9W
FG	− 1W	− 2W	+ 3W	+ 2W	+ 1W	+ 3W
F′E′	− 1W	− 2W	− 3W	+ 2W	+ 1W	− 3W
D′C′	− 1W	− 2W	− 3W	− 4W	+ 1W	− 9W
B′A′	− 1W	− 2W	− 3W	− 4W	− 5W	−15W

EXERCISE.—Construct a similar table for a five-panel Warren truss whose panel load is $5W$.

Concentrated Load Alone.—If this table is examined, it will be seen that, as stated above, *the greatest stress occurs in any given member of the upper chord when the concentrated load acts at the panel point beneath it.* Thus the greatest stress, $-10W \tan \phi$, occurs in the member BD when the concentrated load is at C.

The greatest stress occurs in any given member of the lower chord when the concentrated load acts at the adjacent panel point nearest the middle of the truss; thus the greatest stress, $+12W \tan \phi$, occurs in CE when the concentrated load is at E.

The maximum stress in either chord occurs in the middle member or members when the load is at the middle panel point. Thus the maximum stress in the upper chord is $-18W \tan \phi$ in the member FF', and in the lower chord $+15W \tan \phi$ in EG and $E'G$, when the concentrated load is at G.

In any web member the stress is a maximum when the concentrated load is between the member and the middle point of the truss and as near to the member as the method of loading will permit. Thus the greatest stress in FE is $-\dfrac{3W}{\cos}$ when the concentrated load is at G. It is a compressive stress.

In any web member the stress is a minimum when the concentrated load is between the member and the nearer support and as near to the member as the method of loading will permit. Thus the minimum stress in EF is $+\dfrac{2W}{\cos}$ when the load is at E. It is a tensile stress.

By the *maximum stress* in any member is meant the *greatest stress* which is of the *same character* as that produced by the *dead load.*

If the stress in the member is not reversed, its *minimum stress* is the *least stress* of the *same character as the maximum stress;* if the stress in the member is reversed, the *minimum stress* is the *greatest stress* of an *opposite character from the maximum.*

The maximum web stresses are $-5W$ in AB and $B'A'$ and $+5W$ in $C'B$ and BC.

In a series of parallel web members the stresses are all negative, or compressive, on one side of the concentrated load and positive, or tensile, on the other side of the load. Thus when the concentrated load acts at E the stress in AB and in CD is negative, and in EF, GF', $E'D'$, and $C'B'$ positive.

Dead and Concentrated Moving Loads Combined.—If we add together the coefficients given for each member in the columns 2, 3, 4, 5, and 6, the result, given in column 7, will be the coefficients in all the members due to a uniform dead load whose panel length weighs $6W$.

If we now assume that members of the truss are subjected to this dead load and at the same time are subjected to a moving load as before, we may determine the combined effects of the single concentrated load and the uniform dead load by adding the stresses taken from column 7 to those given in columns 2, 3, 4, 5, or 6, as the concentrated load moves from A to C, E, G, E', and C'.

The greatest stress occurs in any given member of the upper chord when the concentrated load is at the panel point beneath it; thus the stress in DF is a maximum, $-64W \tan \phi$, when the concentrated load is at E.

The greatest stress occurs in any given member of the lower chord when the concentrated load is at the adjacent panel point toward the center; thus the stress in EG is a maximum, $+66W \tan \phi$, when the concentrated load is at G.

The maximum stress in either chord is in the middle member when the concentrated load is at the middle panel point. Thus the maximum chord stress is $-72W \tan \phi$ in the member FF' when the concentrated load is at G.

It will be observed that *the stress in every member of the upper chord is compression and in the lower chord tension, whatever be the method of loading.*

In any web member the stress is a maximum when the concentrated load is between the member and the middle point of the truss and as near to the member as the method of loading will permit; the stress is a minimum when the load is between the middle point and the nearer support and as near to the member as the method of loading will permit. Thus the stress in EF is a maximum, $-\dfrac{6W}{\cos \phi}$, when the load is at G, and a minimum, $-\dfrac{W}{\cos \phi}$, when the load is at E.

As these stresses are both compressive there can be no reversal of stress in the web members if the panel weight of the dead load is equal to that of the moving load, as we have assumed.

The web members of the panels on either side of the middle point, which have the least stress under the dead load alone, are the ones in which the stresses will be the first reversed; therefore, if we find that the maximum and minimum stresses in FG and GF' are of the same character, we may conclude that the relative weights of the concentrated and dead loads are such that the limiting points of zero shear do not move as far from G as E

or E'. From the table we see that the maximum stress in FG and GF' is $+\frac{6W}{\cos\phi}$, and the minimum is $+\frac{W}{\cos\phi}$.

To find the value of concentrated load which placed at E' will reduce the stress in GF' to zero, we shall have, from previous principles,

$$R_1 - W - 2w = 0, \quad \text{or} \quad 2\tfrac{1}{2}w + \tfrac{2}{3}W - W - 2w = 0,$$

in which W = weight of moving load;
w = " " panel length of dead load;
R_1 = reaction at A'.

Hence $W = \frac{3}{2}w$, placed at E', will reduce the stress in GF' and $E'F'$ to zero. A greater value of W would reverse the stress in these members. In a similar manner we may determine the value of W which, placed at C', will reverse the stress in GF', and the value of W which will reverse the stress in $E'D'$ and $D'C'$.

The maximum stress in the web members occurs in the end panels when the concentrated load rests at the adjacent panel point; thus the maximum stress is $-\frac{20W}{\cos\phi}$, which is the stress in AB and $+\frac{20W}{\cos\phi}$ in BC when the load is at C.

Moving Load of Uniform Weight per Unit of Length, whose Length Equals that of the Truss.—The effect of such a load upon a truss is similar to its effect on a simple beam.

Whether the live load acts alone or in conjunction with the uniformly distributed weight of the truss, the stress in any member of EITHER CHORD *is a maximum when the moving load covers the entire truss; it is a mininum when the moving load is entirely off the truss.* Thus, in Fig. 48, if the load is confined to the lower chord, the maximum stress in CE occurs when the moving load acts at the panel points C, E, G, E', and C'; it is a minimum when it acts at none of these panel points.

The maximum chord stress occurs in the middle members when the load covers the entire truss: in FF' of the upper chord, and in GE and GE' of the lower chord.

The stress in any WEB MEMBER *is a maximum when only the panel points of the longer segment into which the member divides the truss are acted upon by the moving load, and a minimum when* *only the panel points of the shorter segment are acted upon.* Thus, in Fig. 48, the stress in GF is a maximum when the moving load acts only at C', E', and G, and a minimum when it acts only at E and C.

The maximum web stress of the truss occurs in the members of the end panels when all the panel points are covered.

To illustrate these statements, if in the table above given we add to the stress given in column 7 the stresses given in 2, in 2 and 3, in 2, 3, and 4, in 2, 3, 4, and 5, in 2, 3, 4, 5, and 6, in 3, 4, 5, and 6, in 4, 5, and 6, in 5 and 6, and in 6, we may determine the effect of a moving load whose panel length weighs $6W$ upon a truss whose panel length also weighs $6W$.

The maximum stress in the chord member CE, $+78W \tan \phi$, is obtained by adding the stresses due to loads of $6W$ at C, E, G, E', and C' to the stress in that member due to the dead load. The minimum stress in that member is $+39W \tan \phi$, due to the dead load alone. The maximum chord stress is $-108W \tan \phi$ in FF' of the upper chord, when all the panel points are loaded; the maximum stress in the lower chord is $+102W \tan \phi$ in EG and GE'.

The maximum stress in the web member EF occurs when the panel points in the longer segment, C', E', and G, are loaded; and is $-\frac{9W}{\cos \phi}$. The minimum stress 0 in the web member EF occurs when both the panel points C and E, in the shorter segment, are loaded.

The maximum stress in the web members is the stress $\frac{30W}{\cos \phi}$, which occurs in the members of the end panels when the truss is fully loaded.

To determine whether there is any reversal of stress in the web members, we first determine the maximum and minimum stresses in the middle members GF and GF'. The maximum stress in these members is $+\frac{9W}{\cos \phi}$, and the minimum stress zero; there is therefore no reversal when the panel lengths of the dead and moving loads are of equal weight, as we have assumed.

Any increase in the weight of the moving load would, however, reverse the stresses in GF and GF'.

Counterbraces.—It is evident, from the discussion above given, that the stress in *any web member* of a truss, due to a uniform dead load, may be reversed or changed from a tensile to a compressive stress or the converse, by placing an additional load upon *one or more* of the panel points between the web member and the nearer support. *The intensity of this load must be such that its reaction at the farther support will be greater than the vertical component of the stress due to the dead load alone in the member considered.*

If the live load is a concentrated one, the *least intensity* of such a load which will cause reversal of stress in any web member is ascertained by placing it at the adjacent panel point on the side of the nearer support and making its reaction at the farther support equal to the numerical coefficient of the dead-load stress in the member considered. The least intensity of a concentrated load which will produce a reversal of stress in $E'F'$, Fig. 48, is the load placed at E', whose reaction at A is equal to the numerical coefficient of the dead-load stress in $E'F'$. A concentrated load at C' may also reverse the stress in $E'F'$, but its weight must be greater than that of the load placed at E'.

If the live load is a distributed one, its *least weight* per panel length which will reverse the dead-load stress in any member is determined by placing it so as to cover all the panel points between the member and the nearer support. In this position its reaction at the farther support must be greater than the numerical coefficient of the dead-load stress in the given member. The least weight per panel length of the distributed live load which will produce a reversal of the dead-load stress in $E'F'$ may be ascertained by placing the load at C' and E' simultaneously.

The reversal of stress in web members of bridge-trusses is produced by the passing of heavy wagons and trains. To provide for this reversal, the truss must be *counterbraced;* that is, the web members which are subject to reversed stress must either be so arranged that they can safely bear the greatest tensile and compressive stresses to which they may be subjected, or, in addition to the *main braces*, which are the web members designed to carry the dead load, there must be additional members or braces to carry the reversed stresses. These are called *counterbraces.*

In a Warren truss the counterbracing consists in making the web members themselves of such forms and materials as to resist both compression and tension, and in connecting them with the chords by joints which will also resist tension and compression. In a metal Warren truss the web members are usually angles riveted to the chords.

In the Pratt, Howe, and similar trusses with vertical web members the counterbraces are diagonal members crossing the main braces. In Fig. 43 the counterbraces would be tie-rods connecting H and E, F and C, etc., and in Fig. 45 they would be struts connecting D and G, B and E, etc.

By this method of counterbracing each web member of the truss is subjected only to a stress of compression or to one of tension. This makes it possible to design a simpler and more economical truss than one in which the web members are subject to reversed stresses.

If the live load is a concentrated one, the maximum reversed or counterbrace stress in any web member is ascertained by placing the concentrated load at the adjacent panel point on the side of the nearer support.

If the live load is a distributed one, the maximum counterbrace stress is ascertained in a similar manner by placing the live load on all the panel points between the member and the nearer support.

The counterbraces must be placed in all panels in which reversal of stress in the web members may occur under the heaviest moving load for which the truss is designed. They are strongest in the middle panels of the truss and decrease in strength towards the supports.

Queen-post Truss.—The function of counterbraces may be seen from a discussion of the simple queen-post truss shown in Fig. 49, in which the angle between AB and the vertical is assumed to be ϕ.

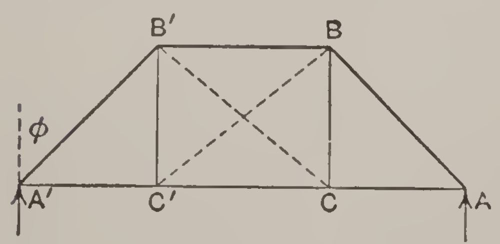


Fig. 49.

The verticals BC and $B'C'$, although *ties*, are called queen-posts. If this truss is uniformly loaded on its lower chord, that is equal weights, as $3W$, are attached at C and C', the truss will be in equilibrium and will support these weights without the diagonals $B'C$ and BC'. The stress in AB

and $A'B'$ will be $-\frac{3W}{\cos\phi}$; in AC, CC', and AC', $+3W\tan\phi$; in BC and $B'C'$, $+3W$; and in BB', $-3W\tan\phi$. There is therefore equilibrium at every point.

Now conceive the joints to be pin-joints, and the truss to be acted upon by the single weight $3W$ at C; the reactions would be $2W$ at A, and W at A', since $A'C'=C'C=AC$. As it is impossible to have equilibrium at every panel point under the action of these three forces, the truss will be deformed by the rectangle $B'BCC'$ becoming a rhomboid, and finally the top chord will fall out. The weight will then be transferred to the lower chord and the weight will be supported by a catenary instead of a truss, if the end pins at A and A' are firmly fixed.

If we place a tie-rod from C to B' or a strut from B to C', the movement of the panel points of the truss will be prevented. In either case the reaction at A will be $2W$, and at A', W.

If the tie CB' is used, the stresses in the different member of the truss will be

$$AB=-\frac{2W}{\cos\phi}, \qquad A'B'=-\frac{W}{\cos\phi},$$

$$AC=+2W\tan\phi, \qquad B'C'=0,$$

$$BC=+2W, \qquad B'C=+\frac{W}{\cos\phi},$$

$$BB'=-2W\tan\phi, \qquad A'C'=+W\tan\phi,$$

$$CC'=+W\tan\phi,$$

and there is equilibrium at every point.

If a strut BC' had been used instead of the tie $B'C$, the stresses would have been

$$AB=-\frac{2W}{\cos\phi}, \qquad A'B'=-\frac{W}{\cos\phi},$$

$$AC=+2W\tan\phi, \qquad B'C'=+W,$$

$$BB'=-W\tan\phi, \qquad A'C'=+W\tan\phi,$$

$$BC=+3W, \qquad BC'=-\frac{W}{\cos\phi},$$

$$CC'=+2W\tan\phi,$$

and there is also equilibrium at every panel point.

Truss Loading.—In the conventional method of determining the stresses due to a moving load as above described, we assume that one panel point bears its maximum load, while the next is unloaded. This would not be true in practice; the load on each panel point could only be a maximum when the load extended to the panel points on either side. If the maximum and minimum stresses in the web members are desired with great accuracy, the stresses due to the true loading of the truss must be determined instead of the stresses due to the conventional loading above described. However, as the conventional method of loading produces slightly greater maximum and minimum stresses, its errors are on the side of safety, and, being simpler, it is the loading usually assumed.

In determining the maximum and minimum stresses in a truss due to a moving load, it will be observed that we need only determine and tabulate the chord stresses when the moving load covers the entire truss and when it is entirely off the truss. We must determine the stresses in each web member when the longer and the shorter segments into which it divides the truss are covered by the live load; and we must place counterbraces in every panel containing a web member subject to reversed stress. If the counterbrace makes the *same angle* with the vertical as the main brace, the amount of stress in it will be equal to the amount of reversed stress in the main brace which it replaces.

PROBLEMS.

50. In the Pratt truss, Fig. 43, what is the least concentrated load which, placed at C, will reverse the stress in FG? What is the least concentrated load which, placed at E, will produce the same result? *Ans.* $6W$ and $3W$.

51. What is the weight per panel length of the least uniform load which, moving over the Pratt truss, Fig. 43, will reverse the stress in FG? *Ans.* $2W+$.

52. A concentrated load of $4W$ is moved over the lower chord of the Howe truss, Fig. 45. Insert counterbraces where necessary in the truss, and find counterbrace stresses. *Ans.* GD and GD'; $\frac{W}{3}$.

53. A uniformly distributed live load whose weight per panel length is $3W$ moves over the same truss; insert the necessary counterbraces and determine the stresses in them.

Ans. GD and GD'; $\frac{W}{2}$.

CHAPTER XII.

GRAPHIC METHOD OF DETERMINING THE STRESSES IN A TRUSS.

The Simple Truss Problem.—In the solution of a simple truss by the graphic method, the same problems arise as in its solution by the analytic methods. The system of forces or stresses acting at each panel point is a concurrent coplanar system in equilibrium, of which the line of direction and a point of application of each force are given. Such a system can be solved graphically as well as analytically when the intensities of all the forces except two are known. The system of forces acting upon the truss as a whole is usually a non-concurrent system, in which all the forces are known save the two reactions; of the reactions, the points of application are known, and also one or both of the action lines. Such a system can be solved graphically as well as analytically when the intensities of the reactions and one of the action lines are unknown.

Representation of Forces. — In the graphic method the intensity of a force in terms of the unit of force is not expressed in figures, as in the analytic method, but it is expressed by the length of a right line drawn to a scale whose unit of length represents a unit of force. By means of the scale, the intensity of a force given in figures may be expressed in graphic units, or the converse.

The direction of a force is indicated by an arrow-head written on its action line; if the force acts towards a material point, the arrow-head is directed towards the point; if the force acts away from the point, the arrow-head is directed away from the point.

Concurrent System—Problem I.—*To find the intensity and action line of the resultant of any concurrent system of coplanar forces.*

In Fig. 50 let AB, BC, and CD be the action lines of three forces forming a concurrent system whose resultant is desired. Let the intensity of each force be represented by the length of its action line. Let the unit of length of the scale represent an intensity of one pound.

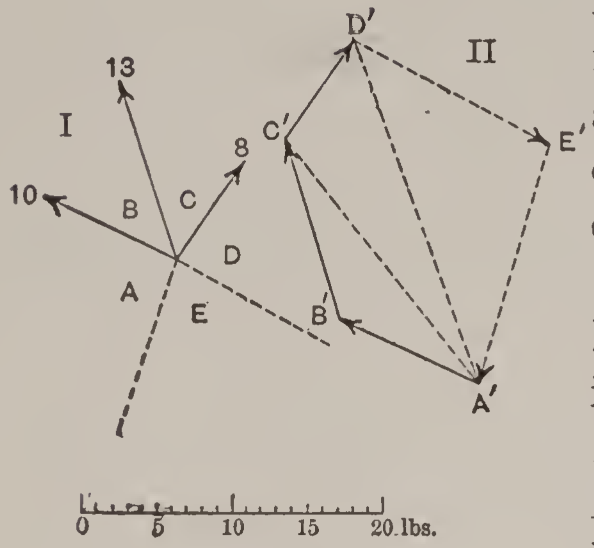


FIG. 50.

From any point, as A', draw a line $A'B'$ parallel to AB and make it equal to ten linear units; from B' draw a line $B'C'$ parallel to BC and make it equal to thirteen linear units; from C' draw the line $C'D'$ parallel to CD and make it equal to eight linear units; complete the polygon by drawing the line $A'D'$. If through the common vertex we draw a line parallel to $A'D'$, it will be the action line of the resultant of AB, BC, and CD, and its intensity will be the number of linear units in the length $A'D'$. This results from the principle of the parallelogram of forces, since by construction $A'C'$ must be equal and parallel to the resultant of AB and BC, and $A'D'$ must be equal and parallel to the resultant of $A'C'$ and CD, or of the three forces given.

Graphic Nomenclature. — In Fig. 50 the diagram $ABCD$ shows the actual positions as well as the directions of the known forces in the plane of construction; it is therefore called a *position diagram.* In this diagram the forces are indicated by the letters written between the action lines.

The diagram $A'B'C'D'$ is called the *force polygon,* and always gives the intensities of the forces it represents. It will also give the directions of the forces, if care is taken to draw all the known forces in the directions in which they act; thus the force AB acts in the direction $A'B'$. Each force in the force polygon is indicated by the same letters as in the position diagram, but in the force polygon the letters are written at the ends of the forces and are distinguished by accents. In constructing the force polygon $A'B'C'D'$ the forces were drawn in the direction of the arrow-heads, and $A'D'$ was found to be the direction and intensity of the resultant. As a similar result would have been obtained in solving any other system of concurrent forces, we may conclude that:

If the forces in the force polygon are drawn in the direction in which they act, the closing line, drawn from the origin to the extremity of the last force, will represent the intensity and direction of the resultant of the forces.

Problem II.—*To find the unknown intensities of two forces of a concurrent system in equilibrium when the action lines of all the forces, and the intensities of all the other forces, are known.*

In Fig. 50 let AB, BC, and CD be the action lines of the known forces, and DE and EA be the action lines of the unknown forces whose intensities are desired. As before, construct the force polygon $A'B'C'D'$; from D' draw the line $D'E'$ parallel to DE, and from A' the line $A'E'$ parallel to AE. Then the number of linear units in $D'E'$ and $E'A'$ will be the intensities in pounds of the unknown forces. According to Problem I, $A'D'$ is equal and parallel to the resultant of AB, BC, and CD, and $D'A'$ is equal and parallel to the resultant of DE and EA. Since $A'D'$ and $D'A'$ are equal and contrary, the five forces AB, BC, CD, DE, and EA must form a concurrent system in equilibrium, and $D'E'$ and $E'A'$ must be the intensities of the unknown forces.

Other Uses of the Force Polygon.—The force polygon may be utilized to determine whether a system of concurrent forces is in equilibrium or has a resultant. If the force polygon closes, the sum of the horizontal and the sum of the vertical components of the system of forces must each be equal to zero, and the concurrent system must be in equilibrium. Conversely, if the concurrent system of forces is in equilibrium, its force polygon must close.

Any force of the system may be resolved into two components acting along any desired lines of direction. Thus the force DA, Fig. 50, if drawn through the common vertex, may be resolved into two components parallel to DE and AE, by simply drawing through the extremities of $D'A'$ the lines $D'E'$ and $A'E'$, parallel respectively to DE and AE.

The force polygon may be utilized in determining two unknown directions in a concurrent system of forces when all the intensities are known. If the intensities instead of the directions of

AE and ED were given, by describing arcs with A' and D' as centers and the intensities as radii the point E' could be determined. In the general case the arcs will intersect in two points and give two solutions of the problem.

The force polygon may also be utilized in determining one unknown intensity and one unknown action line. Thus if the action line of DE and the intensity of AE were given, the point E' could be determined by drawing $D'E'$ parallel to DE, and from A' as a center with the intensity of EA as a radius describing an arc intersecting $D'E'$. Under these conditions there would also be two solutions in the general case. If both the intensity and the action line of either DE or EA were given, there would be but one solution.

Non-concurrent Forces—Problem III.—*To determine the resultant of a system of non-concurrent coplanar forces.*

In Fig. 51 let AB, BC, and CD be the action lines of the three forces whose intensities are given by the force polygon $A'B'C'D'$; required the resultant.

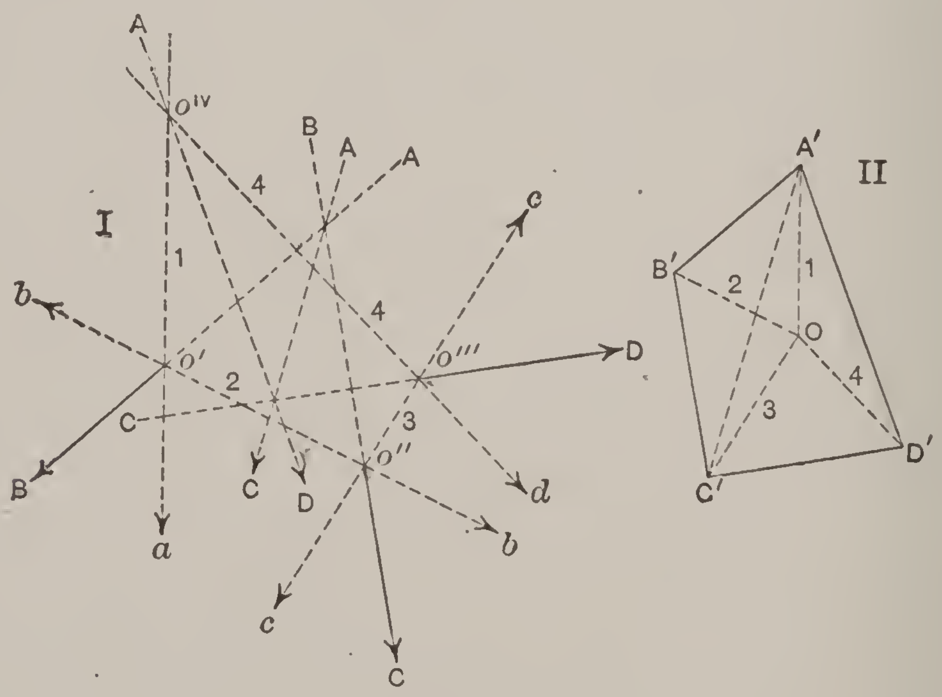


FIG. 51.

The direction and intensity of the resultant is given by the closing line of the force polygon $A'D'$. This results as before from the principle of the parallelogram of forces, since by construction $A'C'$ must be equal and parallel to the resultant of AB and BC, and $A'D'$ must be equal and parallel to the resultant of $A'C'$ and CD.

The resultant will be fully known, therefore, whenever we know the position of one point of its action line.

When the action lines of the forces intersect within the limits of the drawing we may employ the following method based on Problem I. Produce the action lines of AB and BC until they intersect. Through the point of intersection draw their resultant which will be parallel to $A'C'$. Produce this resultant until it intersects the action line of CD; their point of intersection will be one point on the action line of the resultant, and the resultant itself will be AD, drawn through this point parallel to $A'D'$, as shown in Fig. 51, diagram I.

When the action lines of the forces do not intersect within the limits of the drawing we must employ the following method:

Assume a point O, Diagram II, called the *pole* of the force polygon, at any convenient point in the construction plane, and draw the lines $A'O$, $B'O$, $C'O$, and $D'O$. These lines may be taken as the intensities and directions of a new system of forces which are components of the original system. $A'O$ and OB' will be the intensities and directions of the components of AB, and $B'O$ and OC' will be the intensities and directions of the components of BC, etc.

If we assume any point on the action line of the force AB, as o', and draw the lines $o'a$ and $o'b$ equal and parallel to $A'O$ and OB', the original force AB may be replaced by its components $o'a$ and $o'b$. If we find the point o'' where the line bo' intersects the action line of the force BC, we may replace the force BC by its components $o''b$ and $o''c$, equal and parallel to $B'O$ and OC'. If we find the point o''' where the line $o''c$ intersects the action line of the force CD, we may also replace the force CD by its components $o'''c$ and $o'''d$, equal and parallel to $C'O$ and OD'. The resultant of these six components, or the resultant of the three resultants arising from taking these components in pairs, must also be equal to the original resultant.

Let the pairs be $o'b$ and $o''b$, $o''c$ and $o'''c$, and $o'a$ and $o'''d$. The forces $o'b$ and $o''b$ are equal and directly opposed, hence their resultant is zero; for the same reason the resultant of $o''c$ and $o'''c$ is zero; hence the resultant of $o'a$ and $o'''d$ must be the resultant of the system. The resultant of the system, AD, therefore acts through the point o^{IV}, the point of intersection of $o'a$

and $o'''d$, and is parallel to $A'D'$. This is evident from the force polygon, since $A'O$ and OD' are the components of $A'D'$ in that polygon. The lines $o'a$ and $o'''d$ are in this case called the *closing lines* of the polygon $o'o''o'''o^{IV}$.

The polygon $o'o''o'''o^{IV}$ is called the *equilibrium polygon.*

From the construction it follows that the point of the resultant of any system of non-concurrent forces not in equilibrium may be determined by finding the point of intersection of the two closing lines of the equilibrium polygon.

Problem IV.—*To determine the intensities of two unknown parallel forces in a system of non-concurrent forces in equilibrium when only the points of application of the unknown forces are given.*

In Fig. 52 let the forces AB, BC, and CD whose intensities are given by the force polygon $A'B'C'D'$ form a non-concurrent system of forces in equilibrium with two unknown parallel forces

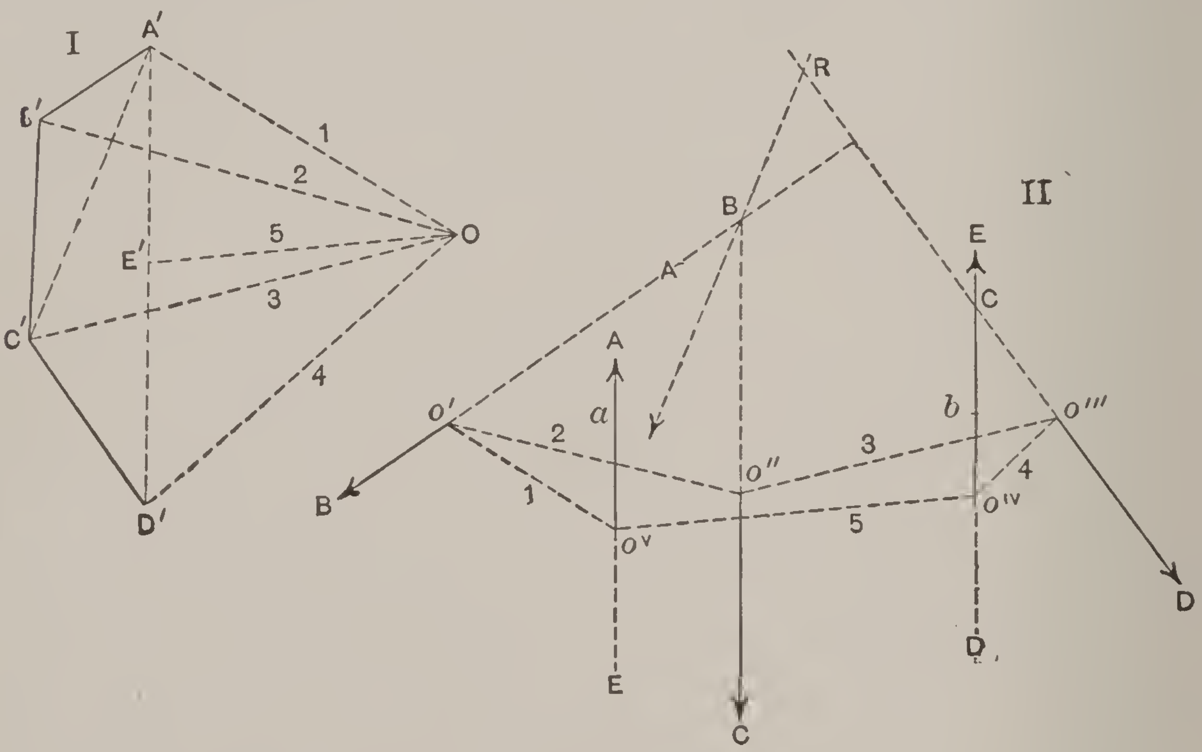


FIG. 52.

which act through the points a and b. If the unknown forces are parallel, they must also be parallel to the resultant of the forces AB, BC, and CD in order that the sum of the horizontal and the sum of the vertical components of the system shall each be equal to zero.

In the force polgyon draw the line $A'D'$; it will be equal and parallel to the resultant of the forces AB, BC, and CD, and must be equal to the sum of the intensities of the two unknown forces. By hypothesis the lines DE and EA drawn through the points a and b parallel to $A'D'$ must be the action lines of the unknown forces.

When the position of the resultant of the known forces can be determined without constructing an equilibrium polygon we may employ the following method to determine the intensities of the unknown forces:

Let AB (Fig. 53) be the action line of the resultant of the known forces, and BC and AC be the action lines of the two unknown parallel forces. Let $A'B'$ be the intensity of the force AB. From any point on AB, as e, draw def perpendicular to

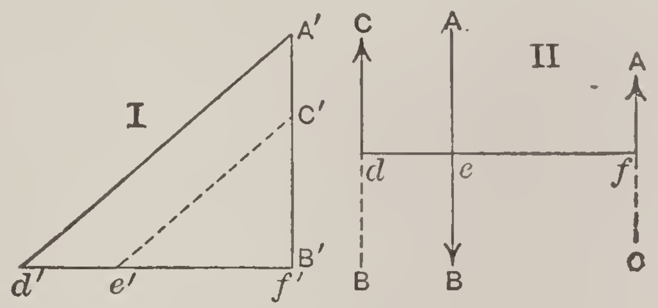


FIG. 53.

AB. Then will ef and de be the lever-arms of the forces AC and BC with respect to any point on AB. Since for equilibrium the resultant of BC and CA must be equal and opposed to AB, we have, from the principle of parallel forces,

$$\text{Intensity } BC \times ed = \text{intensity } CA \times ef. \quad . \quad . \quad (453)$$

To determine the unknown intensities of BC and CA, from B' draw $f'd'$ making any convenient angle with $A'B'$. Lay off $f'e' = fe$, and $e'd' = ed$, and draw $d'A'$ and parallel to it $e'C'$. Then will $B'C'$ be the intensity of the force BC, and $C'A'$ the intensity of the force CA; since from the similar triangles we have

$$B'C' : C'A' = e'f' : e'd' : : ef : ed,$$

or

$$B'C' \times ed : : A'C' \times ef. \quad . \quad . \quad . \quad . \quad . \quad . \quad (454)$$

When the position of the resultant of the known forces can be determined only by constructing an equilibrium polygon we must employ the following method:

In Fig. 52 assume O as the pole and draw the lines OA', OB', OC', and OD'. Then from any point on the action line of the force AB, as o', construct the equilibrium polygon $o'o''o'''o^{\text{IV}}o^{\text{V}}$.

Since the action line of the force EA, acting through a, intersects the line $o'o^{\text{V}}$ at o^{V}, we may lay off from the point o^{V}, on the line $o^{\text{V}}o'$, an intensity equal to OA' and consider this as one of the components of the unknown force EA. In the same manner we may lay off from o^{IV} on $o^{\text{IV}}o'''$ an intensity equal to $D'O$, and consider it one of the components of the unknown force DE acting through b.

In the concurrent system of forces acting at o^{V} one of the angles of the equilibrium polygon $o'o''o'''o^{\text{IV}}o^{\text{V}}$, we have the component of EA acting along $o^{\text{V}}o'$ fully given, and the action line of the component acting along $o^{\text{V}}o^{\text{IV}}$, and the action line of the force itself, EA; hence we may determine the intensities of the unknown component and the force EA by drawing lines through the points O and A' of the force polygon, parallel to $o^{\text{V}}o^{\text{IV}}$ and EA. In a similar manner we may determine the intensity of the force DE and of its component acting at o^{IV} along $o^{\text{IV}}o^{\text{V}}$ by drawing lines in the force polygon through O and D' parallel to $o^{\text{IV}}o^{\text{V}}$ and ED. We thus determine the point E' and the intensities of the two unknown forces; $E'A'$ is the intensity of the force acting through a, and $D'E'$ is the intensity of the force acting through b. In this case the line $o^{\text{IV}}o^{\text{V}}$ is the closing line of the equilibrium polygon.

Hence we see that to find the intensities of two unknown parallel forces of a non-concurrent system in equilibrium we need only draw, in the force polygon, a line through the pole parallel to the closing line of the equilibrium polygon. It will divide the resultant into two parts, each of which represents the intensity of one of the unknown forces.

Problem V.—*To determine the intensities of two unknown forces in a system of non-concurrent forces in equilibrium when only the points of application of both unknown forces and the action line of one of them are given.*

Let the full lines AB, BC, and CD (Fig. 54) be the action lines of the known forces whose intensities are given by the force polygon $A'B'C'D'$; let the full line DE be the action line of one of the unknown forces, and let o' be a point on the action line of the other unknown force.

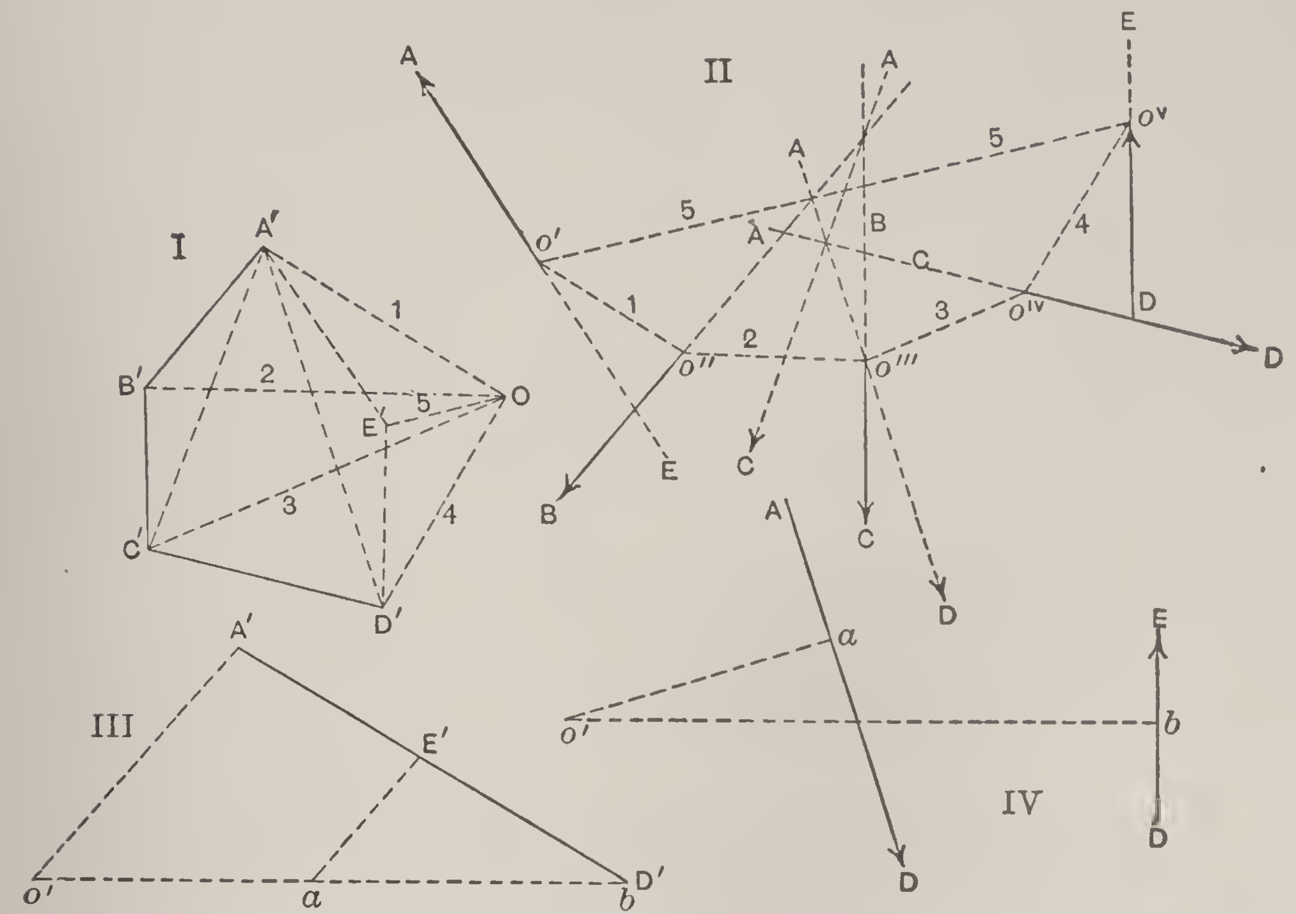


FIG. 54.

I. When the action line of the resultant of the known forces can be determined without the aid of the equilibrium polygon we may use the following methods:

1st. When the action line of the resultant and that of one of the unknown forces intersect within the limits of the drawing. In this case the action line of the other force may be determined by connecting its given point of application with the point of intersection. Thus, in Fig. 54, we construct AD parallel to $A'D'$, the resultant of the known forces, and find its point of intersection with DE; the action line of AE will pass through o' and this point of intersection. This construction results from the fact that three forces in equilibrium must form either a parallel or a concurrent system of forces. Having given the action lines of the unknown forces, their intensities may be found from the force polygon.

2d. When the action line of the resultant and that of one of the unknown forces do not intersect on the drawing. Since the moment of the resultant with respect to o' must be equal to moment of DE with respect to the same point, we may employ a method similar to that shown in Fig. 53.

In diagram IV, Fig. 54, let o' represent the point o' of diagram II, AD represent the action line of the resultant AD, and DE the action line of the unknown force DE; then will $o'a$ be the lever-arm of the resultant, and $o'b$ be the lever-arm of the force DE with respect to the point o'. In diagram III, Fig. 54, make $A'D'$ equal $A'D'$ in diagram I, and make $o'b$ and $o'a$ equal to $o'b$ and $o'a$ in diagram IV. Connect o' with A' and draw parallel to it the line aE'; then will $E'A'$ be the intensity of the force DE. Since from the similar triangles $o'A'D'$ and $aE'D'$ we have

$$A'D' : A'E' :: o'D' : o'a :: o'b : o'a,$$

hence

$$A'D' \times o'a = A'E' \times o'b. \quad . \quad . \quad . \quad . \quad . \quad (455)$$

Having determined the intensity DE, in I lay off $D'E$ parallel to DE and equal to $E'A'$ (III); $A'E'$ will be parallel to the action line of EA and will represent its intensity.

II. When the action line of the resultant cannot be determined without the use of the equilibrium polygon we must employ the following method:

Assume the pole O and draw the lines OA', OB', OC', and OD'; then from the preceding problem OA' may be taken as one of the components of the force EA whose direction and intensity are required, and $D'O$ as one of the components of the force DE. From o', the known point on the action line of the force EA, as an origin, construct the equilibrium polygon $o'o''o'''o^{\text{IV}}o^{\text{V}}$. The closing line $o^{\text{V}}o'$ will be the action line of the other component of the forces DE and EA. Hence if in the force polygon we draw OE' parallel to $o^{\text{V}}o'$, and $D'E'$ parallel to DE, $E'A'$ will be the intensity and direction of the force EA, and $D'E'$ will be the intensity of the force DE. To complete the solution the line EA should be drawn through o' parallel to $E'A'$.

It will be observed that when only one point on the action line of an unknown force is given, the equilibrium polygon must begin at that point.

Uses of the Equilibrium Polygon.—In a system of non-concurrent forces, to determine whether the forces are in equilibrium or have a resultant, we must utilize both the force and equilibrium polygons. If the force polygon closes, the sum of the horizontal and the sum of the vertical components of the system must each be equal to zero. If the equilibrium polygon closes, the sum of the moments of the forces with respect to any point in the plane of the forces must be equal to zero. Hence if the force and equilibrium polygons of a non-concurrent system of forces both close, the system is in equilibrium, and conversely the force and equilibrium polygons of a non-concurrent system of forces in equilibrium must close.

If the resultant of a system of forces is a single couple, the force polygon will close, but the equilibrium polygon will not. Hence we may assume that a non-concurrent system whose force polygon closes but whose equilibrium polygon does not may be reduced to a single couple.

The Solution of a Truss.—Let it be required to determine by the graphic method the maximum and minimum stresses in the truss shown in Fig. 55. The truss is fixed at the right-

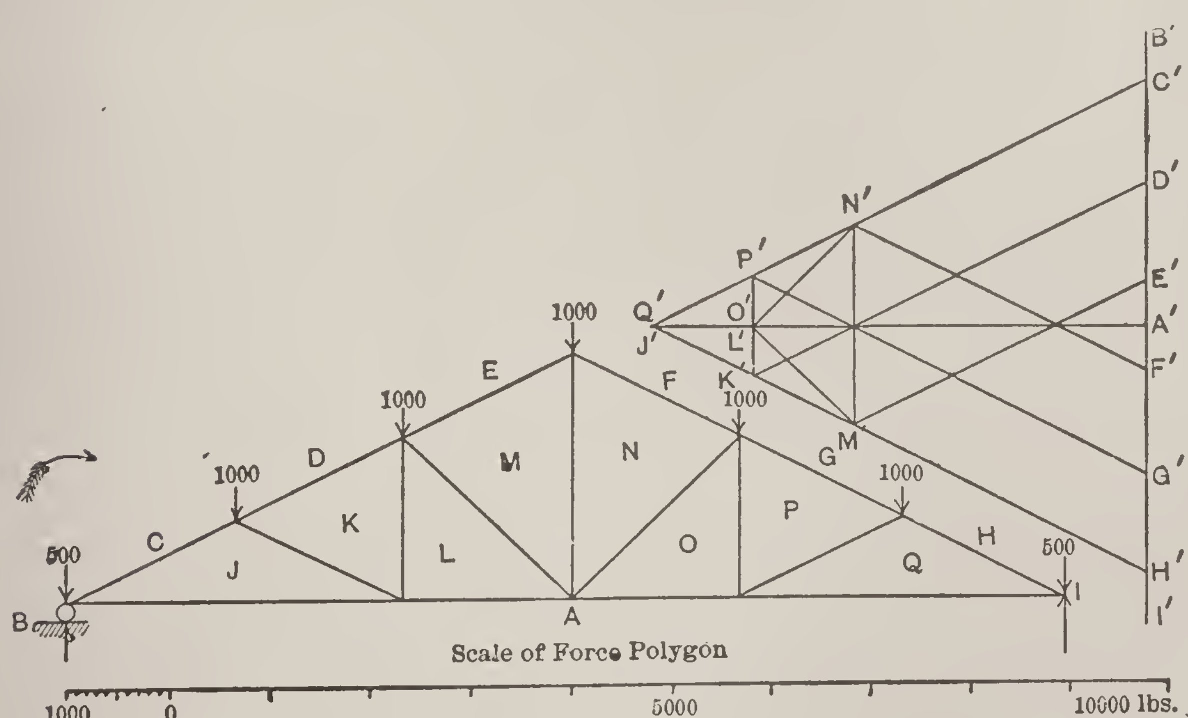


FIG. 55.

hand support, rests on a roller at the left support, and is subjected to a vertical load of 500 pounds at the end panel points and a vertical load of 1000 pounds at each of the other panel points of the upper chord, and to an intermittent wind pressure equivalent to a normal load of 500 pounds at the ridge and eave panel points, and of 1000 pounds at the intermediate points.

Case I.—*When the truss is acted upon by vertical weights alone.*

In Fig. 55 the reactions are each equal to 3000 pounds, and act vertically. The position diagram is lettered according to the method employed for concurrent forces known as Bow's notation, and the force polygons are constructed for each panel point by taking the forces and stresses in order clockwise, as indicated by the arrow.

Force Polygon of External Forces.—Beginning at A' construct the force polygon of the external forces, $A'B'C'D'E'F'G'H'I'A'$; in this polygon the distance $C'D'$ represents 1000 pounds.

Panel Point ABCJ.—At this panel point the forces AB and BC are known, and the action lines of the stresses CJ and JA; the intensities and directions of the stresses CJ and JA are unknown. Construct the force polygon $A'B'C'J'A'$. Then $C'J'$ is the direction and intensity of the stress CJ, and $J'A'$ is the intensity and direction of the stress JA. As CJ acts towards the panel point it is compressive, and as JA acts away from the panel point it is tensile.

Panel Point CDKJ.—At this point we know the compressive stress JC and the force CD; the intensity and direction of the stresses DK and KJ are unknown. Since the stress JC is compressive, it must act towards the panel point; hence we construct the force polygon $J'C'D'K'J'$. Then $D'K'$ and $K'J'$ are the intensities and directions of the stresses in DK and KJ. As they both act towards the panel point they are both compressive stresses.

Panel Point JKLA.—At this panel point the tensile stress AJ and the compressive stress JK are known. Hence we construct the force polygon $A'J'K'L'A'$ and find the intensity and direction of the stresses KL and LA. As both act away from the panel point they are both tensile.

Panel Point DEMLK.—At this panel point we know the tensile stress LK, the compressive stress DK, and the force DE; hence we construct the force polygon $L'K'D'E'M'L'$ and find the intensity and direction of the stresses EM and ML. As both act towards the panel point, they are both compressive.

Panel Point MEFN.—At this point we know the compressive stress ME and the force EF; hence we construct the force polygon $M'E'F'N'M'$, and find the intensity and direction of the stresses FN and NM. The stress FN acts towards the panel point and is compressive, the stress NM acts away from it and is tensile.

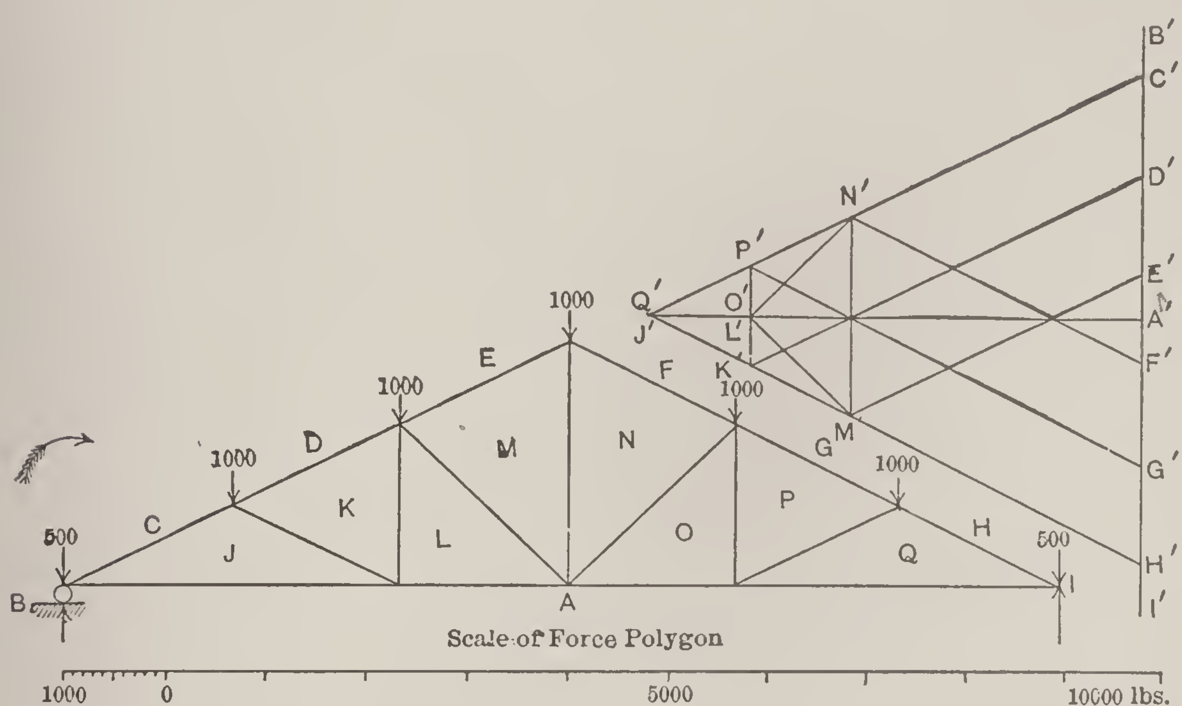


FIG. 55.

Panel Point ALMNO.—At this point we know the tensile stresses AL and MN and the compressive stress LM; hence we construct the force polygon $A'L'M'N'O'A'$ and find the compressive stress NO and the tensile stress OA.

Panel Point ONFGP.—At this point we know the compressive stresses ON and NF and the force FG; hence we construct the force polygon $O'N'F'G'P'O'$ and find the compressive stress GP and the tensile stress PO.

Panel Point AOPQ.—At this point we know the tensile stresses AO and PO; hence we construct the force polygon $A'O'P'Q'A'$ and find the compressive stress PQ and the tensile stress QA.

Panel Point QPGH.—At this point we know the compressive stresses QP and PG and the force GH; hence we construct the force polygon $Q'P'G'H'Q'$ and find the compressive stress

HQ. If the solution is correct, the line $H'Q'$ must connect the points Q' and H', and be parallel to HQ.

In the position diagram, compressive stresses should be indicated by heavy lines as soon as the character of the stress in

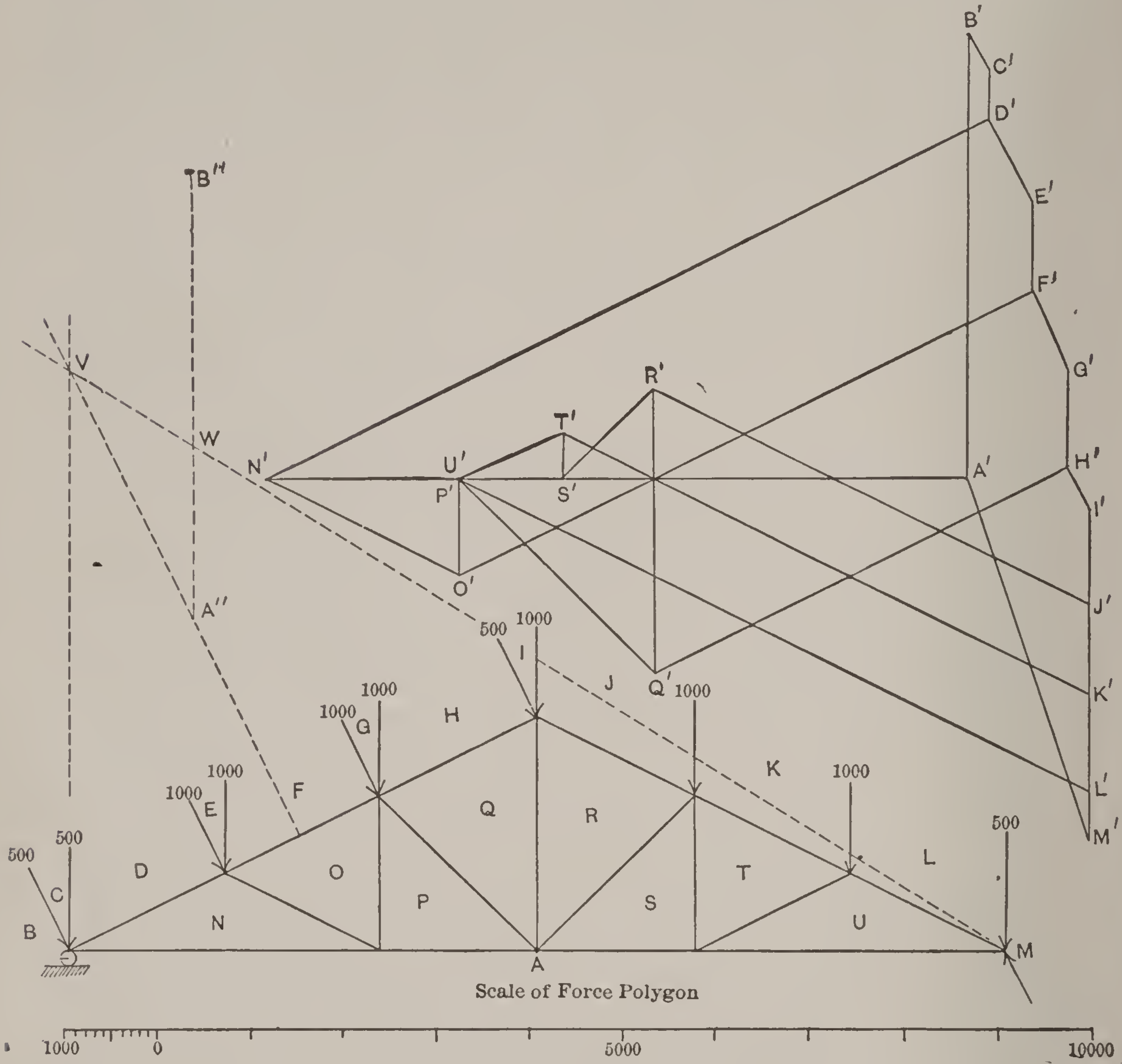


FIG. 56.

a member is determined. The accuracy of the results will increase with the scale of the drawing and with the skill of the draughtsman.

Case II.—*When the truss is subjected to vertical loads and to wind pressure acting on the left side.*

Beginning at the force BC (Fig. 56), construct the lines of the force polygon of the external known forces, $B'C'D'E'F'G'H'I'J'K'L'M'$. As the truss rests on a roller at the left end, we know that a vertical line through B' will represent the direction of the reaction at the panel point $ABCDN$, and that the point A' lies somewhere on this line. To determine A', we might make use of Problem V. A simpler method, however, is the following, in which the reactions at the panel point due to the vertical and oblique loads are considered separately. From inspection it is evident that the resultant of the wind load is perpendicular to and bisects the member FO. This resultant with the reactions at the supports due to the wind pressures alone must form a concurrent system of forces, since the resultant of the reactions must be equal and directly opposed to the resultant of the forces producing them. Since, by hypothesis, the reaction at the panel point $ABCDN$ is vertical, the point V must be the point where the action lines of the resultant and the reactions due to wind pressure intersect. If we lay off the distance VA'' by the scale equal to 3000 pounds, the total wind pressure, $A''W$ and WV must represent graphically the intensities of the reactions due to wind pressure at the left and right supports respectively. If we lay off to scale WB'' equal to 3000 pounds, the reaction at the panel point $ABCDN$ due to the weights alone, then $A''B''$ must be the total reaction at the panel point $ABCDN$. Making $B'A'$ of the force polygon equal to $B''A''$, we determine the point A' and the reaction $M'A'$.

We then determine the intensity and the character of the stress in each piece, by constructing in succession the force polygons $A'B'C'D'N'A'$, $N'D'E'F'O'N'$, $A'N'O'P'A'$, $P'O'F'G'H'Q'P'$, $Q'H'I'J'R'Q'$, $A'P'Q'R'S'A'$, $S'R'J'K'T'S'$, $A'S'T'U'A'$, and $U'T'K'L'U'$.

Case III.—*When the truss is subjected to vertical loads and to wind pressure acting on the right side, as shown in Fig.* 57.

As before construct the lines of the force polygon of the known external forces $B'C'D'E'F'G'H'I'J'K'L'M'$; the point A' must be on the vertical line through B'.

The reactions may be determined by constructing an equilibrium polygon as in Problem V, with the panel point $AUKLM$ as the initial vertex. However, it will be simpler to find the vertical reaction through the panel point $ABCN$ in the following manner: From F'' draw the force polygon $F''H''J''L''M''$ of the wind pressures FG, HI, JK, and LM; the reaction at the panel point $ABCN$ due to the wind pressure will be the vertical line $A''F''$, whose length must be determined. Assume O as a pole and draw the lines OF'', OH'', OJ'', OL'', and OM''. Prolong the lines of wind pressure, and beginning at o^1 construct the equilibrium polygon $o^1o^2o^3o^4o^5$. Draw the closing line o^5o^1, and parallel to it the line OA'' of the force polygon. Then $A''F''$ will be the vertical reaction at the panel point

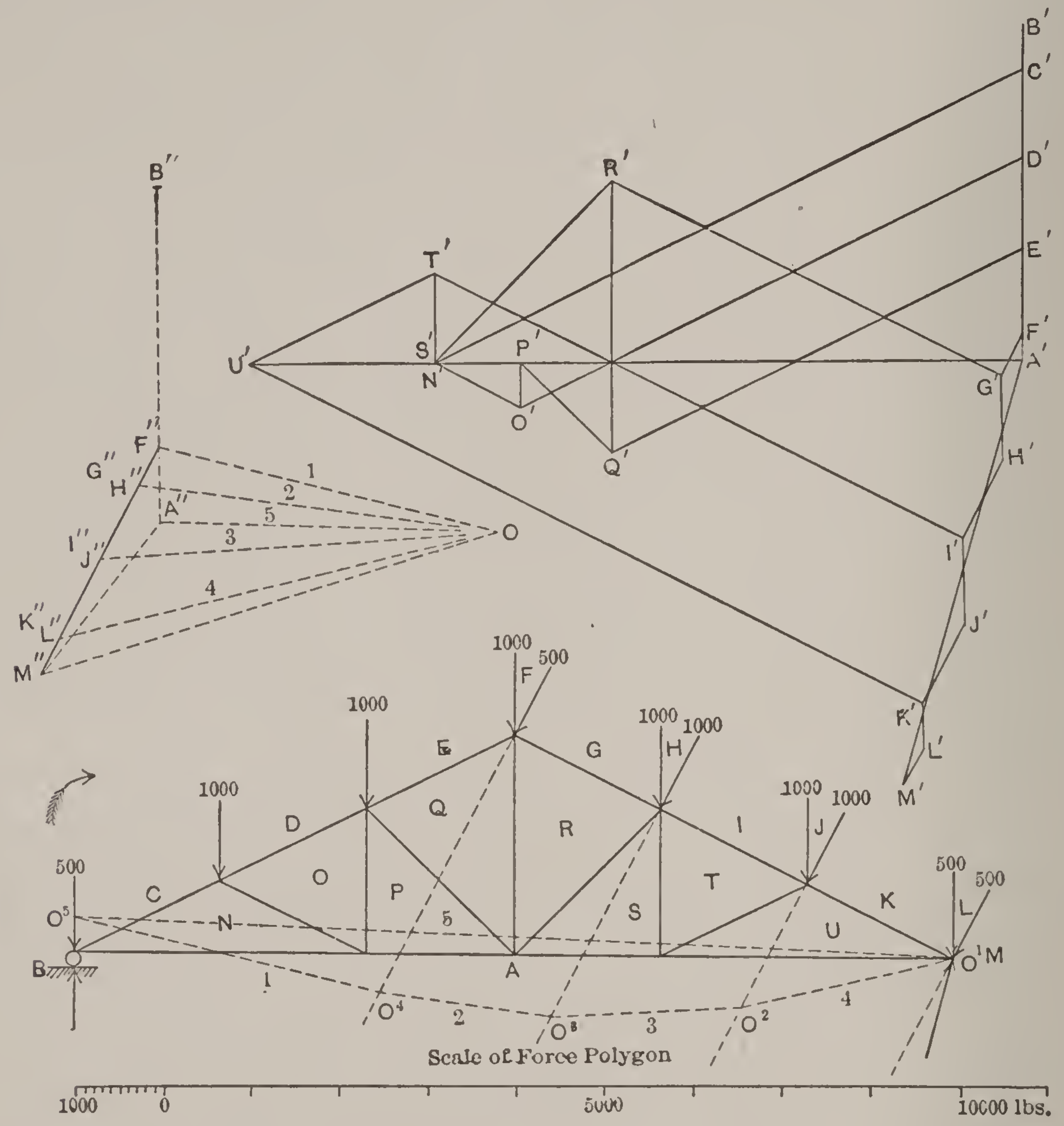


FIG. 57.

$ABCN$ due to the wind pressure. The vertical reaction at this panel point due to the weight alone is 3000 pounds. Laying off $F''B''$ equal to 3000 pounds, the total reaction at the panel point $ABCN$ is $A''E''$. Making $B'A'$ equal to $A''E''$ and drawing the line $M'A'$, the force polygon of the external forces is fully given.

The intensity and character of the stresses may be determined by constructing in succession the force polygons $A'B'C'N'A'$,

$N'C'D'O'N'$, $A'N'O'P'A'$, $P'O'D'E'Q'P'$, $Q'E'F'G'R'Q'$, $A'P'Q'R'S'A'$, $S'R'G'H'I'T'S'$, $A'S'T'U'A'$, and $U'T'I'J'K'U'$.

The three force polygons, Figs. 55, 56, and 57, give us the greatest and least stresses in the different members of the truss under the varying conditions of the loading, and hence all the data necessary for the designing of the members.

In solving a truss by the graphical method the following points should be noted:

1st. The external forces should be indicated as acting in their proper directions and at their proper points of application, outside the perimeter of the truss.

2d. In constructing force polygons for the different panel points, always take the forces and stresses in the same order clockwise or counter-clockwise, beginning with the known forces and stresses.

3d. In a graphic solution all redundant or unnecessary members of a truss should be omitted.

4th. If the truss is fixed at both ends, the reactions are both parallel to the resultant of the known external forces and can be determined by the methods explained under Problem IV.

CHAPTER XIII.

THE GRAPHIC METHOD APPLIED TO PARALLEL FORCES.

Besides the solution of simple trusses without parallel chords, there are several other engineering problems which can be solved more easily by the graphic than by the analytic method.

I. *To find the center of gravity of an area.* According to the theory of flexure the neutral axis of the cross-section of a beam passes through its center of gravity, and the stress in the extreme fibers of the cross-section depends upon the distance of these fibers from that axis. To determine the strength of a beam to resist flexure we must therefore be able to determine the position of the neutral axis or the center of gravity.

If the cross-section is symmetrical with respect to two rectangular axes, as a rectangle, circle, ellipse, an I section, etc., the center of gravity must lie at the intersection of these axes.

If the cross-section is symmetrical with respect to only one axis, as a T section or an angle with equal legs, we know that its center of gravity must lie on that axis, but its actual position is not known. It may be determined graphically in the following manner, providing the cross-section can be divided into smaller areas, the positions of whose centers of gravity are known, and whose areas may be easily computed.

Let it be required to find the center of gravity of a T beam whose flange is $3\frac{1}{2}'' \times \frac{5}{8}''$ and whose web is $3'' \times \frac{5}{8}''$. The cross-section is shown in Fig. 58, is symmetrical with respect to AB, and may be divided into two rectangles whose areas are $\frac{35}{16}$ and $\frac{30}{16}$ square inches respectively.

Through the center of gravity of the first rectangle draw the action line of its resultant force of gravity Co_1; through the center of gravity of the second rectangle draw the action line of its resultant force of gravity EF. Then will the point where the resultant of Co_1 and EF intersects AB be the center of gravity of the cross-section.

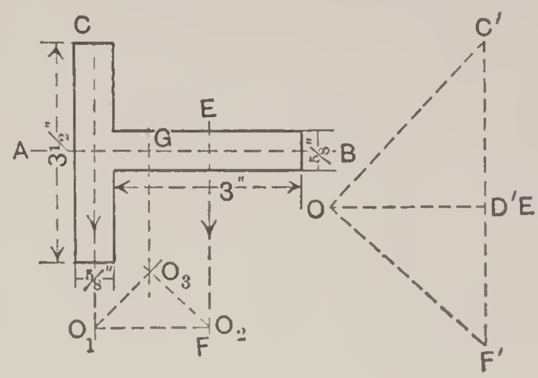


FIG. 58.

Construct the force polygon $C'D'E'F'$, in which $C'D'$ is 35 units and $E'F'$ 30 units in length. Assume the pole O, and draw the forces OC', OD', and OF'. From any point on the action line of Co_1, as o_1, construct the equilibrium polygon $o_1o_2o_3$. The resultant of Co_1 and EF will pass through o_3, be parallel to Co_1 and EF, and intersect AB at the center of gravity of the area.

If the cross-section is not symmetrical with respect to any axis, as an angle with unequal legs, the above operation may be repeated. We may first find the resultant force of gravity perpendicular to the axis of one of the legs, and then find the resultant force of gravity perpendicular to the other; the two resultants will intersect at the center of gravity of the cross-section.

The center of gravity of any irregular area used in the design of beams may be found by dividing the total area into rectangles, triangles, or circular sections. The center of gravity of any two areas must lie on the line joining their centers of gravity. The center of gravity of a triangle lies on the line connecting any vertex with the middle point of the opposite side, and two-thirds of the distance from the vertex. The center of gravity of a circular sector lies on the bisecting radius and at a distance from the center $= \dfrac{\frac{2}{3}(\text{radius} \times \text{chord})}{\text{arc}}$.

II. *To find the bending moment at any section of a beam.* In the force polygon of a system of parallel forces the perpendicular distance from the pole to the line representing the original forces is called the *polar force.* Thus in Fig. 58 the perpendicular distance from O to $C'F'$ is the polar force. The graphical determination of the bending moment depends on the following theorem:

The moment of any force with respect to any point in its plane is equal to the polar force multiplied by the distance between the components of the force in the equilibrium polygon measured on a line through the point parallel to the action line of the force.

Let AB, Fig. 59, be the action line of any assumed force, and a be the point about which the moment is taken. Let $A'B'$ be the intensity of the force, OA' and OB' its two components, and OH the *polar force*. From a draw ab perpendicular to AB; from b draw bc parallel to OB', and bd parallel to OA'; through a draw cd parallel to AB. Then, according to the theorem,

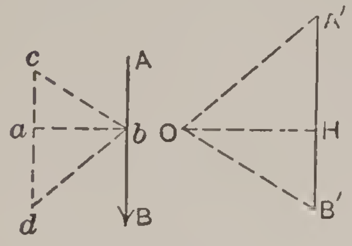


FIG. 59.

$$A'B' \times ab = OH \times cd.$$

From the similar triangles $OA'B'$ and bcd we have

$$A'B' : OH :: cd : ab \text{ or } A'B' \times ab = OH \times cd, \quad . \quad (456)$$

which was to be proved.

Let it be required to find the bending moment at any section of the beam shown in Fig. 60 resting on end supports and acted

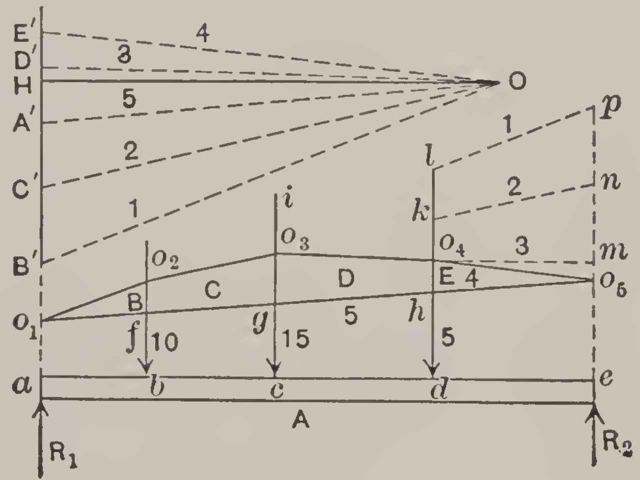


FIG. 60.

upon by the parallel forces ED, DC, and CB, whose intensities are 5, 15, and 10 pounds. The reactions will be vertical.

Construct the force polygon of the known vertical forces $E'D'C'B'$. Assume a pole O and draw the forces OB', OC', OD', and OE'. From any point on the action line of either reaction, as o_1, construct the equilibrium polygon $o_1o_2o_3o_4o_5$. The closing line o_5o_1 will determine the direction of OA' of the force polygon, and will determine the intensities of the reactions $B'A'$ and $A'E'$. The bending moment at b is

$$M = B'A' \times ab, \quad \ldots \ldots \quad (457)$$

but by the theorem above

$$B'A' \times ab = HO \times o_2f. \quad \ldots \ldots \quad (458)$$

The bending moment at c is

$$M'' = B'A' \times ac - C'B' \times bc, \quad \ldots \ldots \quad (459)$$

but according to the theorem above

$$B'A' \times ac = HO \times ig \quad \text{and} \quad C'B' \times bc = HO \times io_3. \quad (460)$$

Hence

$$B'A' \times ac - C'B' \times bc = HO \times ig - HO \times io_3 = HO \times o_3g. \quad (461)$$

The bending moment at d is

$$M''' = B'A' \times ad - C'B' \times bd - D'C' \times cd. \quad (462)$$

But

$$B'A' \times ad = HO \times lh, \quad \ldots \ldots \quad (463)$$

$$C'B' \times bd = HO \times lk, \quad \ldots \ldots \quad (464)$$

$$D'C' \times cd = HO \times ko_4, \quad \ldots \ldots \quad (465)$$

and

$$lh - (lk + ko_4) = o_4h. \quad \ldots \ldots \quad (466)$$

Hence

$$B'A' \times ad - C'B' \times bd - D'C' \times cd = HO \times o_4h. \quad (467)$$

Hence at f, g, and h the bending moment is equal to the *polar force* multiplied by the distance between the opposite sides of the equilibrium polygon measured on the action lines of the forces. In a similar manner, it could be shown that at any other section of the beam the bending moment will be equal to the polar force multiplied by the distance between the opposite sides of the equilibrium polygon, measured on a line drawn through the section considered and parallel to the action lines of the parallel forces. We may therefore always determine the bending moment at any section of a beam by the graphic method when all the normal forces acting upon it can be determined.

III. *To determine the bending moment on a pin.* If we know the stresses acting in the members of a truss which are connected by a pin, as Fig. 61*a*, we can resolve them into horizontal and vertical components and determine the bending moment at any cross-section due to either system of components as described above. AC is at the middle point of the pin; AF is made of two bars each $1\frac{1}{2}$ inches from AC; AE is made of two struts each $2\frac{1}{8}$ inches from AC; AD is made of two bars each $4\frac{1}{2}$ inches from AC; AB is made of two bars each $5\frac{1}{4}$ inches from AC. The angle EAF is 45 degrees.

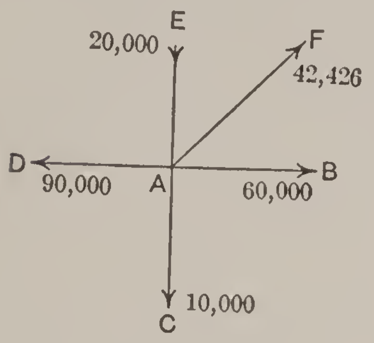


FIG. 61*a*.

From the principles of Mechanics we know that if at any section the horizontal moment be represented by M_h and the vertical moment by M_v, the resultant moment will be $\sqrt{M_h^2+M_v^2}$. If, in constructing the force and equilibrium polygons of the horizontal and vertical components, we employ the same polar force, we shall have for a cross-section at a distance x from the end of the pin

$$M_h = \text{polar force} \times h,$$

in which h = distance between sides of the equilibrium polygon of the horizontal components at x;

$$M_v = \text{polar force} \times v,$$

in which v = distance between sides of the equilibrium polygon of the vertical components at x.

Hence $$\sqrt{M_h^2+M_v^2} = \text{polar force } \sqrt{h^2+v^2}.$$

The maximum resulting moment will be at the section where $\sqrt{h^2+v^2}$ is a maximum.

Let these principles be applied to the determination of the maximum bending moment in the pin represented in Fig. 61.

The total length of the pin between the exterior eye-bars is $10\frac{1}{2}$ inches, as shown in I, Fig. 61. The distances between the axes of the different pieces is shown in I and III, Fig. 61. The force polygon of the horizontal components is II, Fig. 61, and the equilibrium polygon is I, Fig. 61.

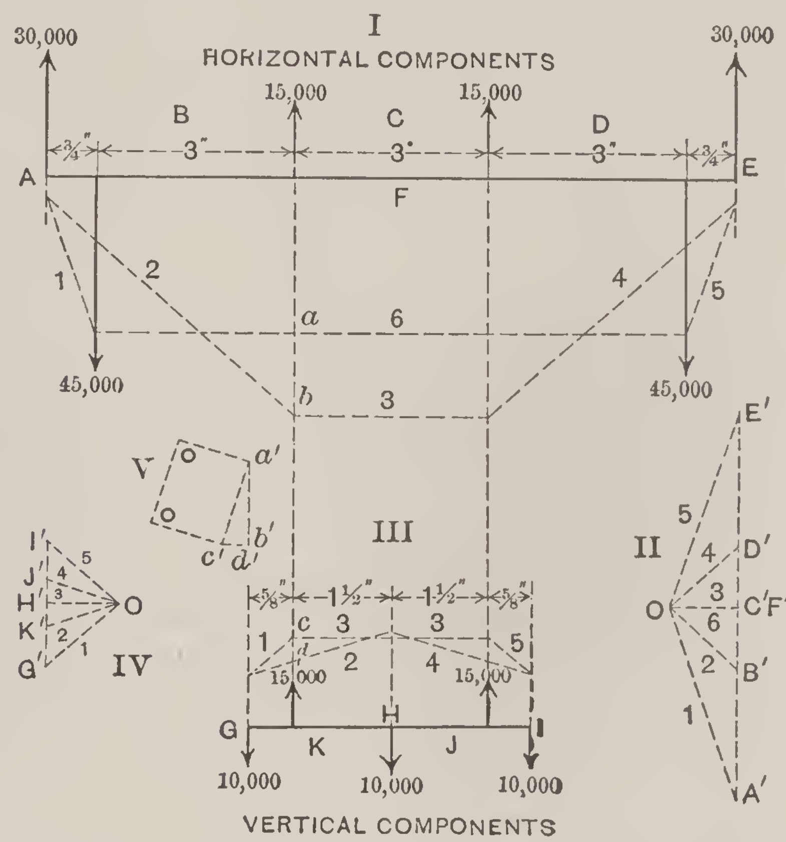


Fig. 61.

The force polygon of the vertical components is IV, Fig. 61, and the equilibrium polygon is III, Fig. 61.

To obtain symmetrical equilibrium polygons, the pole of the force polygon must be opposite the middle points of the lines $E'A'$ and $I'G'$.

Then at any point of the pin, as the point of application of one of the bars AF, the horizontal moment will be $OC' \times ab$, and the vertical moment will be $OH' \times cd$; or, since $OC' = OH'$ by construction, the resultant bending moment at this point of the pin will be $OC' \sqrt{(ab)^2 + (cd)^2}$.

This may be represented graphically by drawing $a'b'$ in V, Fig. 61, equal to ab, perpendicular to it $c'd'$ equal to cd, and oc' equal to OC' in diagram II. The area $ooa'c'$ will be the resulting bending moment in the pin at the action line of the force

BC. In the same manner we can determine the bending moment at any other point.

IV. *To find the moment of inertia of an area approximately.* Let it be required to find the moment of inertia of the angle represented in Fig. 62 about a vertical axis through its center of gravity G.

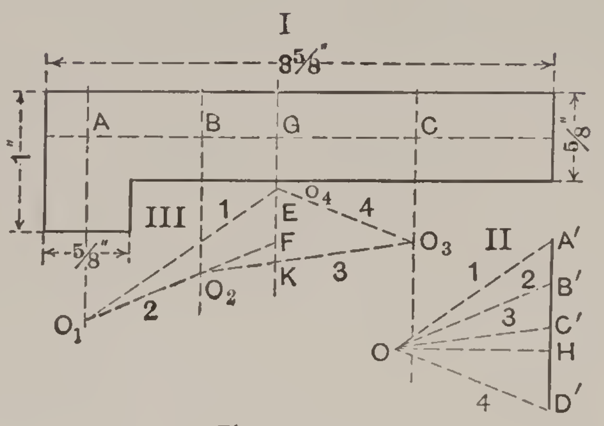


FIG. 62.

Let the area be subdivided into three rectangles whose centers of gravity are on the vertical lines through A, B, and C. In the force polgyon $A'B'C'D'$ let $A'B'$ represent the intensity of the gravity force acting on the line through A, $B'C'$ the intensity of force acting at B, and $C'D'$ the intensity of force acting at C. Let I be the moment of inertia of the angle about the axis GE. Then we shall have

$$I = A'B' \times (AG)^2 + B'C' \times (BG)^2 + C'D' \times (CG)^2, \text{ approximately.}$$

Assume the pole O, and draw the rays OA', OB', OC', and OD'. Construct the equilibrium polygon whose sides are 1, 2, 3, and 4. Then from the preceding theorem we have

$$A'B' \times AG = EF \times OH, \quad \therefore \; A'B' \times (AG)^2 = EF \times AG \times OH; \tag{468}$$

$$B'C' \times BG = FK \times OH, \quad \therefore \; B'C' \times (BG)^2 = FK \times BG \times OH; \tag{469}$$

$$C'D' \times CG = EK \times OH, \quad \therefore \; C'D' \times (CG)^2 = EK \times CG \times OH. \tag{470}$$

But

$EF \times AG =$ twice the area of the triangle o_1EF,
$FK \times BG =$ " " " " " " o_2FK,
$EK \times CG =$ " " " " " " o_3EK.

Hence the moment of inertia of the cross-section

$=[EF\times AG\times OH]+[FK\times BG\times OH]+[EK\times CG\times OH]$
$=$twice the area of the equilibrium polygon$\times OH$.

The error introduced in this method is that of assuming that the radius of gyration of each rectangle about GE is equal to the distance of its center of gravity from the axis, or $R=d$, when in fact $R=\sqrt{d^2+r^2}$. For the area whose center of gravity is on Ao_1,

$R=$radius of gyration of area about GE,
$r=$radius of gyration of area about Ao_1,
$d=$distance AG.

In theory the line $o_1o_2o_3o_4$ is a curve which is obtained by making the width of each of the elementary parts into which the section is divided equal to δl, an elementary part of the length of the cross-section. It is evident that we will approach this limit as we increase the number of parts into which we divide the cross-section; we thus decrease the radius of gyration r.

This affords a method of determining very closely the moment of inertia of irregular sections. By drawing the cross-section to a large scale, dividing it into many parts, usually of equal width, and measuring the area of the equilibrium polygon with a planimeter, the moment of inertia may be determined.

V. *To construct an equilibrium polygon which shall pass through two or three given points.* Having given the three parallel forces BC, CD, and DE in Fig. 63, let it be required to construct an equilibrium polygon passing through the points a and b. The assumed forces being parallel, by means of the force polygon $A'B'C'D'E'A'$ and the equilibrium polygon 12345 we may determine the intensities of the parallel forces $A'B'$ and $E'A'$ which, acting at the points a and b, will hold the given forces in equilibrium. If through A' the line $A'X'$ is drawn parallel to the line connecting a and b, and a pole is taken anywhere on $A'X'$ for a new equilibrium polygon beginning at a, this new polygon will have vertices at a and b, since its closing line, 5, must pass through a, the origin of the equilibrium polygon, and be parallel to $A'X'$, which by construction is parallel to a line connecting a and b.

To construct an equilibrium polygon passing through three points a, b, and c not in a straight line, the force polygon for all the known forces is constructed. Selecting a pole and constructing an equilibrium polygon, the line $A'X$ corresponding to the points a and b may be determined. Selecting a second pole and

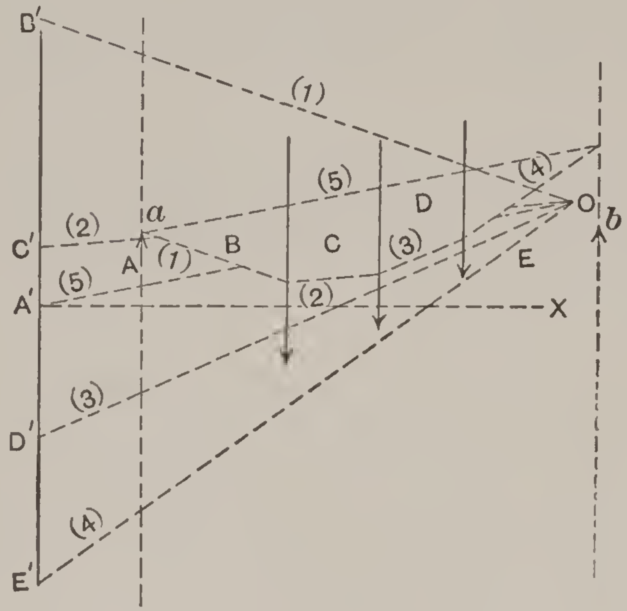


Fig. 63.

constructing an equilibrium polygon the line $A''X$ for the points b and c may be constructed; the point where the lines $A'X$ and $A''X$ intersect will be the pole, by the use of which the desired equilibrium polygon may be constructed.

The methods above explained also apply to a system of non-parallel forces.

CHAPTER XIV.

SOLUTION OF TRUSSES CONTINUED.

Modified Pratt Truss.—In the modified form of this truss, Fig. 64, the end vertical is omitted, and the end diagonal becomes the *batter-brace*. This modification makes the last vertical a tie, which supports only the load suspended from its lower end.

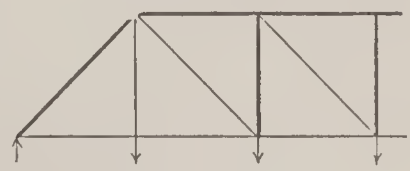


Fig. 64.

The *Baltimore truss* (Fig. 65) is a Pratt truss of great height and span which has been modified by increasing the number of panel points of the lower chord by inserting half-verticals between the main ones. If the upper chord is not straight, but of the bowstring type, it is a *Petit truss*.

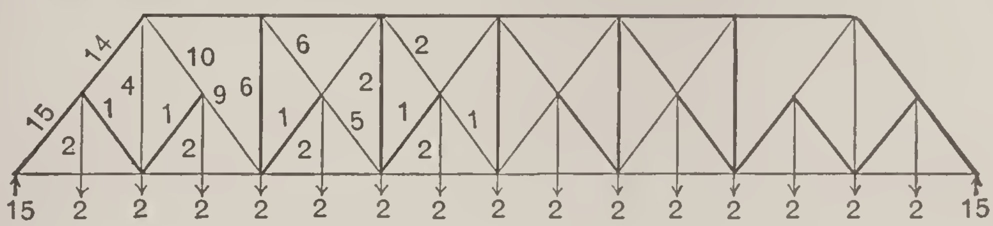


Fig. 65.

The Cantilever Truss.—The simple cantilever truss (Fig. 66) is a truss which has a single support at one of its ends, to which its chords are fastened. A double cantilever rests on a single central support; as the two overhanging parts balance each other, the chords are not fastened to the support. The cantilever truss may be solved by the analytic or graphic methods in the same manner as the truss resting on two supports. The solution must, however, be begun at the free end of the cantilever. If the cantilever supports only weights, the stress in the upper chord will be tension, and in the lower chord compression.

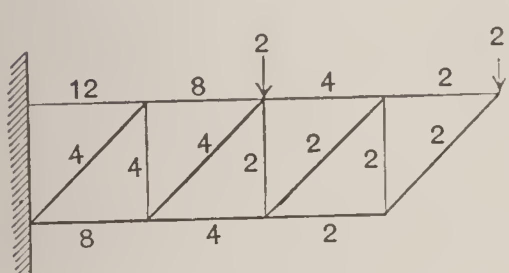


Fig. 66.

Combined Cantilever and Simple Truss.—A truss resting on two supports may have one or both of its ends prolonged so as to form cantilevers, Fig. 67. In solving this truss, the first step

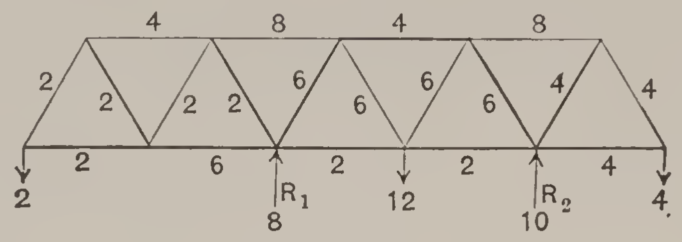


FIG. 67.

is to determine the reactions at the supports by either the method of moments or by the graphic method. When the reactions are known the solution of the truss is begun at one of the free ends. If a truss of this description supports only weights, the chord stresses in the members between the supports may be either compressive or tensile, depending on the system of loading.

Continuous Truss.—A continuous truss, Fig. 68, is one which rests upon three or more supports; it is usually a riveted truss. In determining the reactions at the supports, if uniformly loaded, such a truss may be treated as a beam resting on three or more

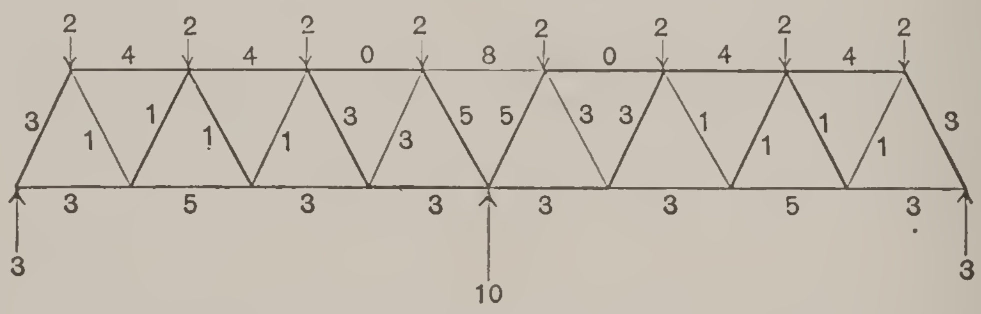


FIG. 68.

supports. Thus if it rests on three supports equally spaced, and its load and dead weight are uniformly distributed, the reaction at each end support will be $\frac{3}{16}$, and the reaction at the middle support will be $\frac{10}{16}$, of the total weight and load. If uniformly loaded it may be treated as made up of separate trusses resting on end supports.

Bowstring Truss.—A bowstring truss (Fig. 69) is one in which one of the chords is straight and the other is a circle or parabola or made up of the chords of these curves. A double bowstring truss is one in which both chords are thus bent and are concave to each other.

In a bowstring girder the shear is divided between the inclined chord and the web members; the total stress in the former is therefore increased, and in the latter decreased. In a uniformly loaded truss the curvature of the chord may be so adjusted as to reduce the stress in the inclined web members to zero. If there is no stress in the inclined web members, the

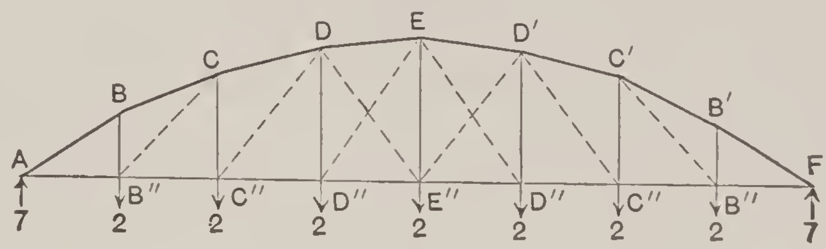


FIG. 69.

stress in each of the vertical web members must be the same, and the stress in all the members of the straight chord must also be the same.

If in Fig. 69 we assume that there is no stress in the diagonal web members, equilibrium will result if the equations of moments below are satisfied.

Let l = length of truss AF;
l' = panel length AB'';
h = height of truss EE'';
$h' = DD''$;
$h'' = CC''$;
$h''' = BB''$.

Cutting the lower chord between D'' and E'' and taking moments about E of the upper chord,

$$7W \times 4l' - 2W \times 3l' - 2W \times 2l' - 2W \times l' + \text{stress } D''E'' \text{ (or } AB'') \times h = 0;$$

therefore
$$h = -16 \frac{Wl'}{\text{stress } AB''}. \qquad (471)$$

Cutting the lower chord between C'' and D'' and taking moments about D of the upper chord,

$$7W \times 3l' - 2W \times 2l' - 2Wl' + \text{stress } C''D'' \text{(or } AB'') \times h' = 0;$$

therefore
$$h' = -15 \frac{Wl'}{\text{stress } AB''}. \qquad (472)$$

Cutting the lower chord between B'' and C'' and taking moments about C of the upper chord,

$$7W \times 2l' - 2W \times l' + \text{stress } B''C'' \text{ (or } AB'') \times h'' = 0;$$

therefore $$h'' = -12\frac{Wl'}{\text{stress } AB''}. \qquad (473)$$

Cutting the lower chord between A and B'' and taking moments about B of the upper chord,

$$7Wl' + \text{stress } AB'' \times h''' = 0;$$

therefore $$h''' = -7\frac{Wl'}{\text{stress } AB}. \qquad (474)$$

Hence to fulfil the conditions that there shall be no stress in the diagonal web members, the relative values of h, h', h'', and h''', in the truss shown in Fig. 69, must be as 16, 15, 12, and 7.

To support a moving load such a truss would require diagonal main braces and counterbraces. The bowstring girder is more easily solved by the graphic than by the analytic method.

The Pegram truss (Fig. 70), which is a modification of the bowstring truss, is formed by drawing the arc of a circle through

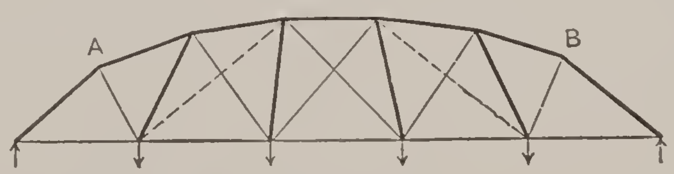


FIG. 70.

the points A and B of the batter-braces, and making the intermediate upper chord members equal to each other and equal in number to those of the lower chord.

Compound Truss.—A compound truss (Fig. 71) is one which

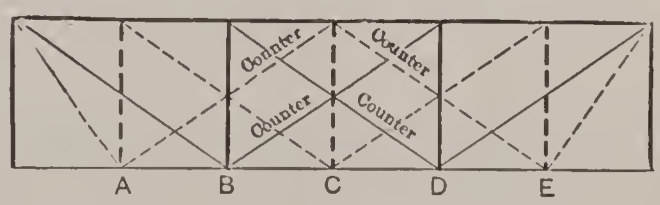


FIG. 71.

has two or more systems of web-bracing. In computing the stresses in the members of such a truss, resting on end supports, it is to be assumed that the loads at B and D are carried by a truss composed of the full-lined web members with the chords and end-posts, and that the loads at A, C, and E are carried by a truss composed of the dotted web members with the chords and end-posts. The stresses in the chord members and end-posts are therefore the sum of the stresses deduced for the separate trusses.

If the web system is a riveted lattice-work, it becomes a lattice truss and may be treated as a plate girder.

Three-hinged Truss.—A three-hinged truss (Fig. 72) is a truss formed of two separate trusses which rotate about fixed axes at the supports, and abut against a common pin at their free ends. This construction allows the truss to rotate about the

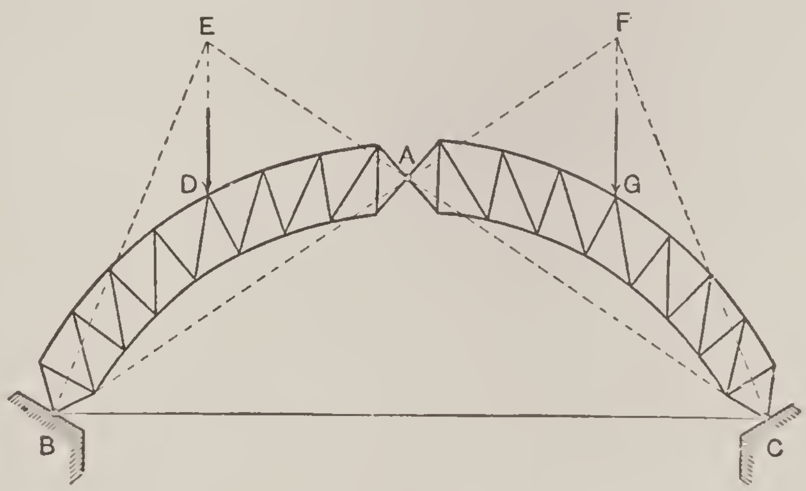


FIG. 72.

axes and thus accommodate itself to changes of length of its component parts due to variations in temperature. The fixed axes are connected by a tie-rod, or are fixed by strong abutments.

If a concentrated load acts on one side of the truss only, as at D, the reaction at C must pass through A if we neglect the friction of the joints and the weight of the truss. The action lines of the reactions will therefore be BE and CE. The action lines of the reactions will have the same directions if ED is the action line of the resultant of uniformly or non-uniformly distributed load resting on the left half of the truss. Knowing the intensity of the force acting at D, we may readily determine the intensities of the reactions.

Similarly if the load is placed on the right half only, as at G, we may find the reactions whose action lines are BF and CF.

If both loads act simultaneously, the reaction at A will be the resultant of reactions acting along FA and CA, or along BA and EA. Having thus determined the direction of the reaction at A due to both loads, the action lines of the reaction at B due to both loads is obtained by prolonging the action line of the resultant reaction at A until it intersects ED and joining the point of intersection with B.

Having determined the reactions, the stresses in the different members may be determined as in any other truss. If employed as a roof-truss, the maximum and minimum stress due to dead and wind loads in each member must be determined; if employed as a bridge-truss, the maximum and minimum stress due to dead and live load in each member must be determined.

The Boom Derrick.—The boom derrick (Fig. 73) is a machine employed in lifting weights and distributing them over a horizon-

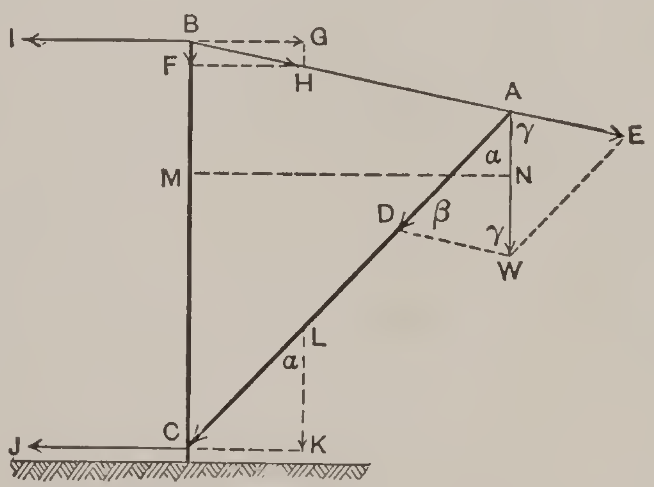


FIG. 73.

tal area whose extreme radius is the length of the boom. The vertical member BC is the *mast;* it is a wooden or steel column which rotates about a pin resting in a socket at C. The inclined member AC is the *boom;* it is a wooden or steel column which rotates about a horizontal axis at C. The member AB is the *topping-lift, boom-hoist,* or *boom-fall;* it is a rope attached to the boom at A, passes through a pulley at B, and is fastened to a cleat attached to the mast at C, or passes through a pulley at the same point and runs thence to the hoisting-engine at J. The member IB is one of four or more *guys* which are employed to hold the mast in a vertical position. The weight to be lifted is attached to a rope called the *main hoist* or *main fall* at W; this rope passes through a pulley at A, through another at C, and runs thence to the hoisting-engine at J. To reduce the tension in the topping-lift rope, pulley-sheaves may be introduced at

A and B, and to reduce that in the main fall, CJ, pulley-sheaves may be introduced at A and W.

Let angle $BCA = DAW = CLK = \alpha$;
" $BAC = ADW = \beta$;
" $CBA = DWA = \gamma$;
" $HBG = 90° - \gamma = \phi$;
W = weight whose intensity is represented by AW.

Let it be required to determine the maximum stress in AC, AB, BC, and IB while the boom rotates from a horizontal to a vertical position.

To simplify the discussion we shall first assume that the main fall is attached at A, and the topping-lift at B. The derrick is now a simple truss with one exterior force and two stresses acting at each vertex. Since the forces IB, AW, and the reaction at C are the only exterior forces acting on the derrick and are in equilibrium, they must form a concurrent system, and the reaction at C must pass through the intersection of IB and AW.

The Boom.—If we resolve AW into its components in the direction of AC and AB, we have

$$AD = W\frac{\sin \gamma}{\sin \beta}.$$

From the triangle ABC we have

$$AC : BC :: \sin \gamma : \sin \beta,$$

$$\therefore \frac{AC}{BC} = \frac{\sin \gamma}{\sin \beta} \quad \text{and} \quad AD = W\frac{AC}{BC}. \quad . \quad . \quad . \quad (475)$$

In any derrick AC and BC are fixed lengths, hence $\dfrac{\sin \gamma}{\sin \beta}$ is a constant ratio, and the compression in the boom is constant. If $AC = BC$, $AD = W$.

The Topping-lift.—The component of AW in the direction of AB is

$$AE = W\frac{\sin \alpha}{\sin \beta}.$$

From the triangle ABC we have

$$BA : BC :: \sin\alpha : \sin\beta,$$

$$\therefore \frac{BA}{BC} = \frac{\sin\alpha}{\sin\beta}, \quad \text{or} \quad AE = W\frac{BA}{BC}. \quad . \quad . \quad . \quad (476)$$

BC being fixed, the ratio $\frac{BA}{BC}$ increases as BA increases and is a maximum, under the condition assumed, when the boom is horizontal.

The Mast.—If the stress in the topping-lift at B be resolved into its components, the stress in the mast will be

$$FB = \frac{W \sin\phi \sin\alpha}{\sin\beta} = \frac{W \cos\gamma \sin\alpha}{\sin\beta} = W\frac{BA \cos\gamma}{BC}. \quad (477)$$

Since BC, the length of the mast, is constant, the stress in the mast due to the topping-lift will vary with $AB \cos\gamma$, which is the projection of the topping-lift on the mast. It will be compressive when A is below B, and will increase as the vertical distance between A and B increases; it will be tensile if A is above B, and will increase as the vertical distance between A and B increases.

The Guy.—The stress in the guy due to topping-lift will be equal to BG:

$$BG = \frac{W \cos\phi \sin\alpha}{\sin\beta} = \frac{W \sin\gamma \sin\alpha}{\sin\beta} = W\frac{AC \sin\alpha}{BC}. \quad (478)$$

Since AC and BC are constant, the stress in the guy will vary with the sine of α, and will be a maximum when α equals 90 degrees or the boom is horizontal.

If the guy is not horizontal, $\frac{W \times AC \sin\alpha}{BC}$ will be the horizontal component of the stress in the guy. The stress in the guy will therefore be

$$W\frac{AC \sin\alpha}{BC \cos\phi'}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (479)$$

which increases as ϕ' increases. The vertical component of this stress which must be borne by the mast is

$$W\frac{AC \sin\alpha \tan\phi'}{BC}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (480)$$

in which ϕ' is the angle which the guy makes with the horizontal.

If the main fall is fastened at C, the stress in the boom will be increased by the tensile stress in the fall, which must be W. If the boom-fall is fastened at C, the stress in the mast will be increased by a compressive force equal to the stress in the fall between B and C. If a single pulley is employed at B, this compressive stress will be AE or $\frac{W \sin\alpha}{\sin\beta}$. If pulley-blocks are employed at A and B, the stress in the boom-fall between B and C, and the compression due to it, will be diminished.

If the boom is fastened to the mast above C, the mast will also be a vertical beam resting on end supports, and subject to a horizontal bending force at the junction of mast and boom equal to the horizontal component of the stress in the boom. The mast will then have a shear and bending moment at every section.

Reaction.—The derrick when at rest is acted on by the weight which is being lifted, the stress in the guy, and the stresses in the hoists if the hoisting-engine or other power is placed to one side of the derrick. The reaction at the bottom of the mast is a force equal and opposed to the resultant of these forces. If the hoisting-apparatus is attached to the mast, the derrick will be acted on by the weight and guy only, and the reaction will be opposed to their resultant.

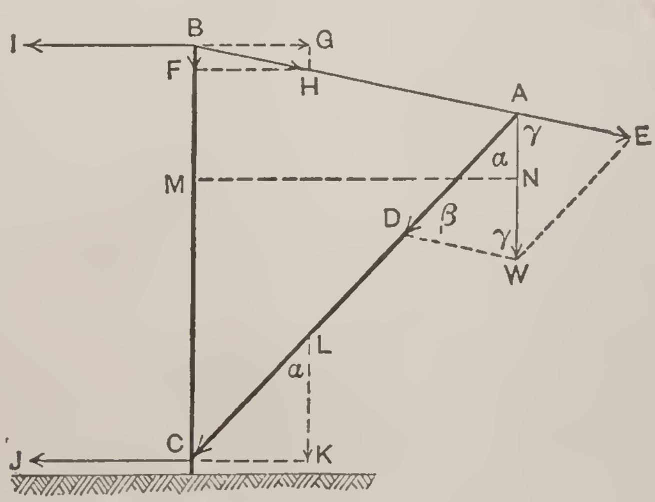


FIG. 73.

CHAPTER XV.

MASONRY DAMS AND RETAINING-WALLS.

A MASONRY dam is a wall designed to resist the lateral pressure of the water upon one of its sides. If the wall is straight, it must resist the water pressure by its weight alone; if curved, it may also resist as an arch. Only straight walls are here considered.

The side exposed to the water pressure is the *back*, the opposite side is the *face*, and the bottom surface is the *base*. The intersection of the back and the base is the *heel*, the intersection of the base and the face is the *toe*, and the intersection of the face and the top is the *crest* of the dam. The face and back may be either vertical or inclined, and if inclined, may be either planes or curved surfaces. The degree of inclination is called the *batter;* its measure is the tangent of the angle between the inclined and horizontal planes. The *profile* of a dam is a vertical section normal to the face.

Water Pressure. — From Mechanics of Liquids we know that:

I. The pressure of the water on the back of the dam is at all points normal to the surface.

II. The intensity of the pressure upon any submerged plane surface is equal to the weight of a column of water whose base is the surface pressed, and whose height is the depth of the center of gravity of the surface below the surface of the water.

III. The resultant pressure upon any immersed rectangular plane surface one of whose sides coincides with the water line, acts in a normal vertical plane through its center of gravity, and at two-thirds the depth of the lower edge below the water surface.

Let Fig. 74 represent the cross-sections of two walls of equal height, and let the surface of the water be on a level with the top of each wall.

Let d = depth of the water = height of the wall;

RS = line parallel to AB and $\frac{2}{3}d$ below it;

α = angle IFH;

w = weight of a cubic foot of water = 62.5 pounds;

P' = resultant pressure on a linear foot of the back of the wall $ABCD$;

P'' = resultant pressure on a linear foot of the back of the wall $EFGH$.

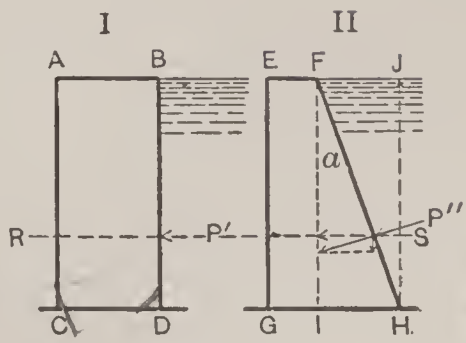


FIG. 74.

From the principles above given, P', the resultant pressure on the back of a linear foot of the rectangular wall $ABCD$, will be $w \times BD \times \frac{BD}{2} = \frac{wd^2}{2}$. This resultant must act normal to the surface BD, and at a distance $\frac{2}{3}BD$ from B, or $\frac{1}{3}BD$ from D.

P'', the resultant pressure on a linear foot of the back of the trapezoidal wall $EFGH$, must be $w \times FH \times \frac{d}{2} = \frac{wd^2}{2 \cos \alpha}$. This resultant must act normal to the surface FH and at a distance $\frac{2}{3}FH$ from F or $\frac{2}{3}d$ from the surface of the water.

The horizontal component of P'' is

$$P''' = \frac{wd^2}{2 \cos \alpha} \times \cos \alpha = \frac{wd^2}{2} = P'. \quad . \quad . \quad . \quad (481)$$

Hence, whatever be the slope of the back of the wall, the horizontal component of the water pressure will be the same as the pressure on a wall of equal height with a vertical back, and will depend only upon the depth of the water.

The vertical component of P'' is

$$P^{\mathrm{IV}} = \frac{wd^2 \sin \alpha}{2 \cos \alpha} = \frac{wd^2 \tan \alpha}{2}, \quad . \quad . \quad . \quad . \quad (482)$$

in which $\frac{d^2 \tan \alpha}{2}$ is the area of the triangle FJH, and $\frac{wd^2 \tan \alpha}{2}$ is the weight of a linear foot of the prism of water whose end is FJH. Its action line intersects GH at a distance $\frac{1}{3}IH$ from

H. As the action line of the vertical component acts within the base of the wall, it increases the total pressure upon that base.

The Profile.—The profile of the wall should fulfil the following requirements:

I. When the reservoir is full, the resistance of the profile should be sufficient to prevent the wall from shearing off in a horizontal plane, and the weight of the wall should be sufficient to prevent the wall from sliding on its base or on any horizontal joint.

II. The resultant of the weight and the water pressure above any horizontal section, should pierce that section within the middle third, whether the reservoir be full or empty. This keeps the entire area of every horizontal joint in compression.

III. The maximum pressure at the toe and heel of any joint should not exceed the allowable crushing value of the material of which the dam is constructed, nor should the maximum pressure at the toe and heel of the dam exceed the allowable bearing value of the material upon which the dam is built.

In the discussion which follows it is assumed that the surface of the water is on a level with the top of the dam.

Resistance to Shearing.—Resistance to shearing in masonry dams is secured by avoiding continuous horizontal joints and by bonding the wall with its foundation. Such a wall cannot rupture by simple sliding.

Resistance to Sliding.—If the dam is constructed of horizontal layers of masonry, depending only upon the friction between the layers to prevent sliding, stability requires that the friction at each joint shall be greater than the horizontal component of the water pressure upon the wall above the joint when the reservoir is full.

If the dam has a vertical back and

w' = weight of masonry per cubic foot = 144 pounds for ordinary masonry,
w = weight of water per cubic foot = 62.5 pounds,
h = height of wall and depth of water above the joint,
b = mean thickness of wall,
f = coefficient of friction of masonry on masonry = $\frac{2}{3}$,

for stability we must have

$$w'bh \times f > \frac{wh^2}{2}, \therefore b > \frac{wh}{2w'f} > 0.32\ h. \quad . \quad . \quad . \quad (483)$$

Since b varies with h, a masonry dam of triangular profile, as shown in I and III, Fig. 75, is a dam equally strong at every horizontal joint to resist rupture by sliding. If in I, Fig. 75, $BC > 0.64AB$, or $b > 0.32h$, the wall is stable at every horizontal joint. This is therefore the most economical profile for a dam with a vertical back to resist sliding, since the unit frictional resistance in every horizontal plane is the same.

If the dam is constructed on an earth foundation, the base must be enlarged, because the coefficient of friction between masonry and earth is less than $\frac{2}{3}$.

Masonry dams rarely fail because of weakness in horizontal planes, but sometimes fail due to the insufficient frictional resistance at the foundation.

The Middle Third. — In order that the action line of the weight shall intersect each horizontal section within its middle third *when the reservoir is empty*, the limiting forms of the profile must be a right-angled triangle with a vertical back, and a similar triangle with a vertical face, as in I and III, Fig. 75.

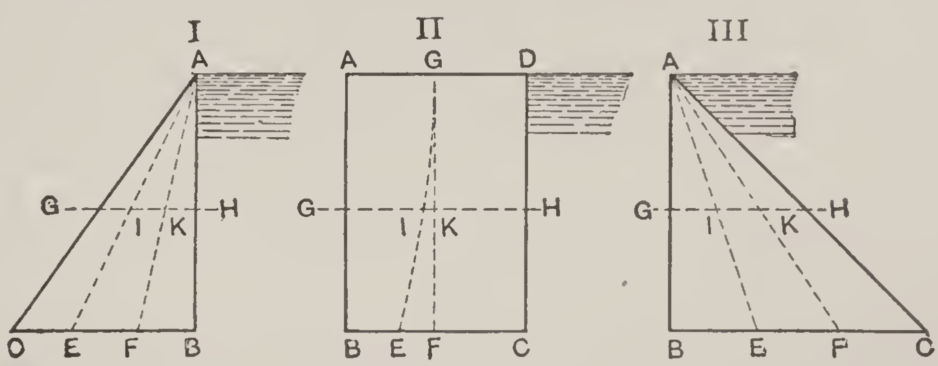


FIG. 75.

In I the action line of the weight alone passes through F, and in III through E. In the rectangular wall II the action line of the weight passes through F, the middle point of BC.

Let it be required to compare the amount of masonry in the three walls of equal height, Fig. 75, under the requirement that the resultant of the maximum water pressure and the weight shall not intersect any horizontal section outside the middle third. This will be true in the sections shown when the resultant pierces *the base* within the middle third.

Let h = height of dam = depth of water = AB;
b = width of base = CB;
w' = weight of masonry per cubic foot = 144 pounds, assuming specific gravity as 2.3;
w = weight of water per cubic foot = 62.5 pounds.

The effect of the water pressure in cases I and II is to cause the resultant to pierce the plane CB to the left of the point where it is intersected by the action line of the weight alone. In I and II the resultant may move from F, when the reservoir is empty, to E, when the reservoir is full; equating the moments of the weight and the water pressure about E, we shall have,

$$\text{in I,}\quad \frac{w'hb}{2}\times\frac{b}{3}=\frac{1}{2}wh^2\times\frac{1}{3}h,\quad \therefore\ b=h\sqrt{\frac{w}{w'}}=0.66h=\frac{2}{3}h; \qquad (484)$$

$$\text{in II,}\quad w'hb\times\frac{b}{6}=\frac{1}{2}wh^2\times\frac{1}{3}h,\quad \therefore\ b=h\sqrt{\frac{w}{w'}}=0.66h=\frac{2}{3}h. \qquad (485)$$

In III, when the reservoir is full, the resultant must pass through E, where the action line of the weight intersects BC when the reservoir is empty; hence the moments of the vertical and horizontal components of the water pressure about this point must be equal, or

$$\frac{whb}{2}\times\frac{b}{3}=\frac{1}{2}wh^2\times\frac{1}{3}h,\quad \therefore\ b=h.\quad .\ .\ .\ . \qquad (486)$$

Under the above assumption, that the specific gravity of masonry is 2.3, the area of the profile of wall I will be $\frac{1}{3}h^2$; of wall II, $\frac{2}{3}h^2$; of III, $\frac{1}{2}h^2$; or in the proportion 2:4:3 approximately.

The wall I is therefore the most economical form of the profile under the second requirement, page 262, since in any trapezoidal profile between I and II the areas would lie between those of I and II, and in any trapezoidal form between II and III the areas would lie between those of II and III. Its factor of safety against overturning about the toe is 2, or the moment of the weight about that edge is twice the moment of the water pressure.

Distribution of Pressure on the Horizontal Joints.—Under the subject of eccentric loading it was shown that:

(1) If the center of pressure due to the external forces, at any horizontal joint, is at the center of gravity, the pressure is uniformly distributed;

(2) If the center of pressure is at the extremity of the middle third, the pressure at the nearer edge will be twice the mean pressure, and at the farther edge zero;

(3) If the center of pressure is without the middle third, the entire joint will not be under pressure, and the pressure at the nearer edge will exceed twice the mean pressure.

Masonry dams should be so designed that the entire surface of each joint is under compression; hence the center of pressure should always be within the middle third.

In applying these principles to the three profiles shown, let the common height of the dams be assumed as 60 feet, and the width of the base of I and II as $\frac{2}{3}h$, or 40 feet. The weight of the dams per linear inch will be: I, 14,400 pounds; II, 28,800 pounds; III, 21,600 pounds. The mean pressure on the base per square inch when the reservoirs are empty will be: 30 pounds in I and III and 60 pounds in II. In I and III, since the center of pressure is at the extremity of the middle third, the maximum unit pressure will be 60 pounds and the minimum zero. In II the center of pressure is at the center of gravity and the pressure is uniform. Hence the maximum pressure due to weight of the dam alone is the same in the three profiles.

When the reservoirs are full, the vertical component of the resultant pressure on dam I is the weight of the dam; it acts through E. The mean vertical pressure is therefore, as before, 30 pounds; the maximum pressure at C is 60 pounds, and the minimum at B is zero. In dam II the vertical component of the resultant pressure is the weight of the dam; it acts through E, the extremity of the middle third. The mean pressure is therefore, as before, 60 pounds; the maximum at B is 120 pounds, and the minimum at C is zero. In dam III the vertical component is the weight of the wall and the vertical component of the water pressure, both of which act through E. The mean pressure is $30+\frac{1}{2\cdot 3}\,30=43$ pounds; the maximum pressure at B is 86 pounds, and the minimum pressure at C is zero. Hence profile I produces the least maximum pressure at the toe when the reservoir is full, and is the most economical profile under the third requirement, page 262.

Modification of the Theoretical Profile.—Profile I, Fig. 75, has been shown to be the most economical profile under each of the three requirements of masonry dams; it should therefore be the basis of the designs for such structures.

The upper part of dam I, Fig. 75, is designed to withstand static water pressure alone. Dams of large reservoirs are subject to wave action and ice pressure, and overflow-dams to the pressure of the current and the shock of floating bodies. Upon the top of all except overflow-dams a footway at least three feet wide is desirable for communication, and upon all long dams a driveway at least 10 feet wide. The thickness of dams at the crest is therefore made from 3 to 20 feet, and the upper part of the profile is approximately rectangular.

If a dam constructed as in I, Fig. 75, were 120 instead of 60 feet high, since its weight increases with the square of the height, and its base as the first power of the height, the mean unit pressure at the base would be 60 pounds, and the maximum unit pressure would be 120 pounds; if 180 feet high, the mean unit pressure would be 90 pounds, and the maximum 180 pounds. It is not considered desirable in dams of moderate height to have the maximum unit pressure exceed 100 pounds per square inch, or about 7 tons per square foot; nor in very high dams is it considered safe to have a maximum unit pressure exceeding 200 pounds per square inch or 14 tons per square foot. To limit the maximum pressures to the intensities above given it is necessary to increase the width at the base of all high dams beyond that given in I, Fig. 75. From this increase at the top and base of high dams, there results the form shown in Fig. 76, which is the profile of the new Croton Dam. In determining the resultant pressure upon such a dam the vertical component of the water pressure is usually omitted; as this component would tend to move the center of pressure towards the center of gravity when the reservoir is full, it is an error on the side of safety. The face of the dam is subjected to a back pressure due to the earth and water below the surface of the lower pool.

Curve of Pressure.—The curve of pressure is the curve which connects the centers of pressure of the horizontal joints. To find the curve of pressure of a dam, divide the profile by a number of horizontal planes. When the reservoir is empty, the center of pressure at each horizontal joint is determined by dropping a perpendicular through the center of gravity of the section

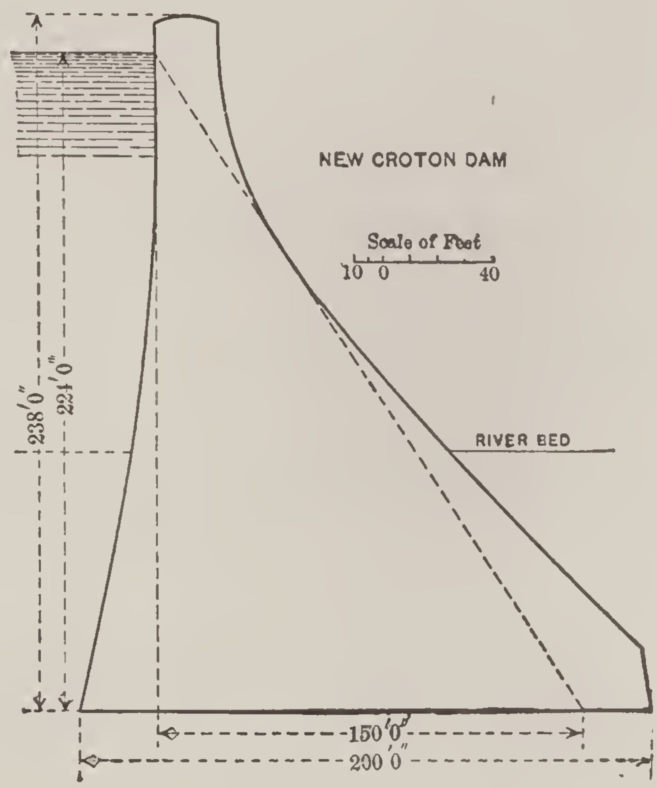


FIG. 76.

of the wall above it. When the reservoir is full, the center of pressure at each horizontal joint is the point where the resultant of the weight and the water pressure above the joint pierces that joint.

To construct the curve of pressure of dam I, Fig. 75, let

A = origin of coordinates;
axis of $X = AB$;
axis of Y = line through A parallel to BC;
$AH = x$;
$GH = x \tan \phi$, in which ϕ is the angle BAC;
$IH = y$ = distance of center of pressure from AB when the reservoir is full;
w' = weight of unit volume of masonry;
w = " " " " " water.

Equating moments of weight of wall and pressure of water about I, we have

$$\frac{w'x^2 \tan\phi}{2}\left(y-\frac{x\tan\phi}{3}\right)=\frac{wx^3}{6},$$

or

$$y=\frac{wx}{3w'\tan\phi}+\frac{x\tan\phi}{3}=IK+KH. \quad . \quad . \quad . \quad (a)$$

Similarly for dam II, Fig. 75, we have

$$(w'xb)\left(y-\frac{b}{2}\right)=\frac{wx^3}{6},$$

in which b is the breadth of the dam, or

$$y=\frac{wx^2}{6w'b}+\frac{b}{2}=IK+KH. \quad . \quad . \quad . \quad . \quad . \quad (b)$$

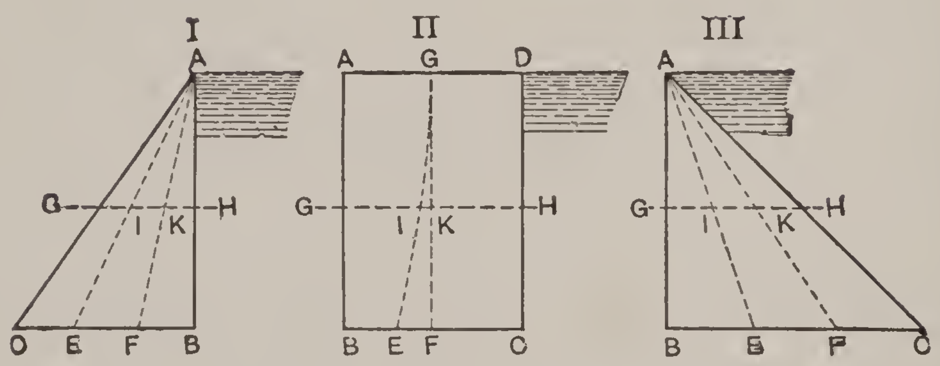


FIG. 75.

In dam III, Fig. 75, the sum of the moments of the weight of the wall above GH, of the water resting on the wall above GH, and the horizontal water pressure on AH must be equal to zero. Hence

$$\frac{w'x^2\tan\phi}{2}\left(y-\frac{x\tan\phi}{3}\right)-\frac{wx^2\tan\phi}{2}\left(\frac{2x\tan\phi}{3}-y\right)+\frac{wx^3}{6}=0,$$

or

$$y=\frac{(w'+2w)x\tan^2\phi-wx}{3(w'+w)\tan\phi}. \quad . \quad . \quad . \quad . \quad . \quad (c)$$

In the formulas (a), (b), and (c) the value of y for the empty reservoir may be obtained by making $w=0$.

The curves of pressure resulting from the formulas (a), (b), and (c) are shown in dotted lines, Fig. 75.

If a dam rests on a fissured or a permeable foundation, or the connection between the dam and foundation is not water-tight, there will be an upward pressure on the base of the dam which will greatly reduce the resultant vertical component of the pressure of the dam on its base. The upward pressure is avoided by constructing the dam upon solid rock, and by making the joints between the dam and its foundation water-tight.

PROBLEMS.

53. What should be the thickness of a rectangular masonry dam whose height is 30 feet if the level of the reservoir is 2 feet below that of the dam? Specific gravity of masonry is 2.3. Maximum pressure must not exceed twice mean pressure.

Ans. 17.8 feet.

54. What will be the maximum and mean pressure upon the base of the dam in problem 53?

Ans. Maximum 8640 pounds per square foot.
Mean 4320 " " " "

55. A masonry dam with a vertical back is 48 feet high, 14 feet wide at the top, and 28 at the base. Determine the position of the center of pressure of base when the dam is full.

Ans. 9.16 feet from toe.

Retaining-walls.

A masonry retaining-wall is one designed to resist the lateral pressure of the earth upon one of its sides. The terms back, face, base, batter, etc., are applied as in masonry dams. If the earth is higher than the wall, the volume above the top of the wall is called the *surcharge.* In Fig. 77 the prism of earth *MBE* is the surcharge.

Pressure of Earth.—The pressure of earth against the back of the wall differs from that of water in that the particles of earth cohere to each other with considerable tenacity, especially when in a moist condition, and free movement of the mass is resisted by friction. The cohesion is a very uncertain factor in determining the pressure of earth, as it varies from the tenacity of compact clay, which when dry will stand at any angle, to that of dry sand, which is practically without cohesion. It varies in the same material with the degree of moisture and compactness, and is not uniform through any large mass of earth.

To avoid these irregularities, formulas for earth pressure are based on the following hypothesis:

I. The cohesion of the material has been largely destroyed and the earth consists of sandlike grains which are free to move, subject to the force of friction. It is evident that such a hypothesis is on the side of safety.

If earth, in this condition, is poured out of a vessel on a smooth surface, it will form a cone the inclination of whose surface with the horizontal will vary slightly with the percentage of moisture. The inclination of the surface to the horizontal is called the *natural slope* of the material; the angle of inclination is called the *angle of repose.* Since a particle is held in equilibrium on the surface by the component of its weight acting parallel to the surface, and by an equal force of friction developed by the normal component, the angle of repose is also the *angle of friction*, and the tangent of the angle of repose is the *coefficient of friction* of the material. The angle of repose of wet earth is about 30°, of very dry earth 38°, of moist earth 45°. In formulas it is usually assumed to be about 34°; its natural tangent being $\frac{2}{3}$.

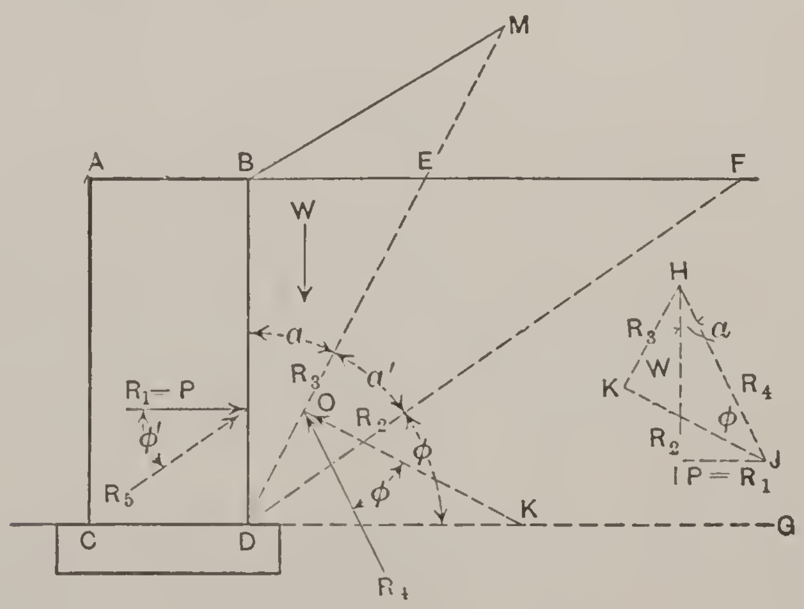


FIG. 77.

In Fig. 77 *ABCD* is a section of a straight retaining-wall; *BF* is the upper surface of the earth in its rear, which is assumed to be horizontal; *DF* is the natural slope of the earth. To deduce the formula for this pressure it is necessary to make further hypotheses. The two hypotheses ordinarily accepted are:

II. If the wall is suddenly removed, a portion of the volume BDF will at once fall; the remainder will assume its natural slope gradually. The pressure upon the back of the wall is assumed to be due to the volume which will at once fall, and this volume is assumed to be separated from the remainder by a plane surface DE, called the *plane of rupture*, along which it slides in the form of a compact prism. The angle BDE is called the *angle of rupture.* The less cohesion there is in the material, the more closely will the results of the experiment agree with this hypothesis.

III. The pressure due to the volume BDE is distributed over the back of the wall BD, in the same manner as water pressure; the unit pressure at any point varies with its distance from B, and the center of pressure on a rectangular area whose height is BD is $\frac{2}{3}BD$ from B.

Coulomb's Formula.—This formula is based upon the three hypotheses above given, and upon the additional one:

IV. The action line of the resultant pressure upon the back of the wall is normal to the surface, as in water pressure, or the friction on the surface BD may be omitted.

In Fig. 77 let

h = height of the wall = BD;
angle ϕ = angle of repose of the earth = FDG;
angle α = angle of rupture of the earth = BDE;
angle α' = EDF;
w = weight of a cubic foot of earth;
P = resultant pressure on BD;
W = weight of prism BDE per foot of length;
R_1 = normal reaction of surface $BD = P$;
R_2 = normal reaction of surface DE;
R_3 = frictional resistance of surface DE;
R_4 = resultant of R_2 and R_3.

Under the hypotheses above given there must be equilibrium between the weight of the prism BDE, the normal reaction of the surface BD, the normal reaction of the surface DE, and the friction on the surface DE caused by the weight of the prism. The weight of the prism BDE per foot of length is $\frac{1}{2}wh^2 \tan \alpha$; its action line is W, through its center of gravity and O. Of the other forces we know the action lines of R_1, R_2, and R_3, but we do not know their intensities. We do know, however, that the friction on DE or R_3 must be equal to the normal pressure or normal resistance R_2 multiplied by the coefficient of friction, or $R_2 \tan \phi$. Hence if we draw R_4 making an angle ϕ with R_2, it must be the action line of the resultant of the friction R_3 and the normal reaction R_2, whatever be their intensities. There must therefore be equilibrium between W, R_1, and R_4, and our system is reduced to a system of three concurrent forces of which we know the intensity of one, and the action lines of all three. We may therefore determine their intensities by constructing the force polygon HIJ, in which $HI=W$, $IJ=R_1=P$, and $JH=R_4$. We may also determine the intensity of R_2 and R_3 by drawing the lines JK and KH. Since the angle which W makes with $R_2=EDG$, and the angle $R_2OR_4=FDG$, the angle $IHJ=EDF=\alpha'$; hence $P=\frac{1}{2}wh^2 \tan \alpha \tan \alpha'$.

This expression is a maximum when $\alpha=\alpha'$; this may be shown by calculus. Hence the maximum value of $P=\frac{1}{2}wh^2 \tan^2 \alpha$.

Since $\alpha=\alpha'=\frac{1}{2}(90-\phi)$, and $90=\frac{\pi}{2}$, this may be also written

$$P=\frac{1}{2}wh^2 \tan^2\left(\frac{\pi}{4}-\frac{\phi}{2}\right), \quad . \quad . \quad . \quad . \quad . \quad (488)$$

which is the form in which it was originally deduced.

By the same hypotheses we may deduce a general formula for the pressure of a surcharged embankment upon a wall with an inclined back; the formula is, however, very complicated.

Poncelet's Formula.—Poncelet takes into consideration the friction on the surface BD, and hence the resultant reaction of the surface BD is R_5 (Fig. 77), which makes with the normal an angle ϕ', equal to the angle of friction of earth on masonry. For all practical purposes $\phi'=\phi$. Poncelet's formula for the pressure upon a retaining-wall with a vertical back with the surface of the earth level with the top of the wall, in terms of ϕ, is

$$R_5=P_1=\frac{wh^2 \cos \phi}{2(1+\sqrt{2}\sin \phi)^2}. \quad . \quad . \quad . \quad . \quad (489)$$

Rankine's Formula.—This formula is based upon the theory of internal stresses in a granular mass. The conclusions he arrives at are that the pressure at any point of *BD* varies with the distance from *B*, and that the action line of the resultant of the pressure is parallel to *BM* whenever *BM* makes an angle with the horizontal varying between zero and ϕ. If *BM* is horizontal, Rankine's formula is

$$P_2=\frac{wh^2}{2}\left(\frac{1-\sin\phi}{1+\sin\phi}\right).$$

If *BM* is parallel to *DF*, this formula becomes

$$P=\frac{wh^2\cos\phi}{2}.$$

More general forms of this formula are given by Rankine. Other theoretical formulas based on the hypotheses above given have also been deduced.

The earth pressure on a retaining-wall may also be determined by the graphic method. While no simpler than the analytic method, it may be utilized to check results obtained by calculation. The methods are given in works on graphic statics.

Having determined the earth pressure upon a retaining-wall, the form of cross-section is designed upon the same principles as that of a masonry dam.

I. The horizontal width of the profile at every point should be sufficient to prevent the wall from shearing off in any horizontal plane or sliding on its foundation. Walls laid without mortar are liable to rupture by shearing or *bulging*.

II. The resultant of the weight and the earth pressure should pierce each horizontal section far enough within the outer edge to make the wall safe against overturning. This is accomplished by making the moment of the wall about this edge twice as great as the moment of the earth pressure, or, preferably, by limiting the curve of pressure to the middle third of the wall.

III. The maximum unit pressure at the toe and heel should not exceed the bearing value of the foundation, nor should the maximum unit pressure on any joint exceed the allowable compressive stress of masonry.

As a retaining-wall is usually constructed to support an embankment of considerable height, in designing the profile it is only necessary to consider the pressure of the earth when the embankment is in its final condition.

The weight being the same, the most stable wall to resist overturning is therefore the one in which the center of gravity is at the greatest distance from the toe.

Poncelet deduced the following formula for converting a rectangular wall into a trapezoidal wall with a battered face, vertical back, and an equal moment about the toe. Its accuracy increases with the inclination of the face.

$$b' = b + \tfrac{1}{10} b'', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (492)$$

in which b' = base of trapezoidal wall;
b = base of rectangular wall;
b'' = assumed base of battered face $= \dfrac{h}{\tan \beta}$;
h = height of wall;
β = angle of inclination of face to the horizontal.

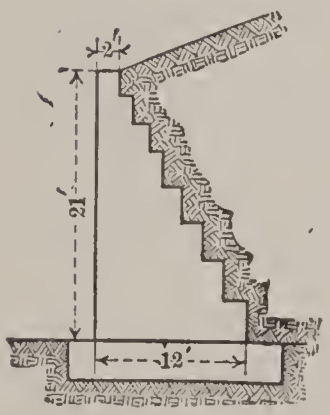


FIG. 78.

A profile in the form of a parallelogram leaning towards the embankment offers much greater resistance to overturning than a rectangular wall having the same base and height. Such a wall would, however, be apt to slide upon its base.

The most common form of retaining-wall, Fig. 78, is one which has a vertical or slightly battered face and a back made in rough steps. The top width of a retaining-wall is usually about 2 feet.

Empirical Formulas.—Although the theoretical formulas for earth pressure will give safe values for the thickness of retaining walls, their forms are very complex for surcharged walls and for walls with inclined backs. For these reasons and because they are based on hypotheses only approximately true, empirical formulas have largely replaced them.

Let h = height in feet of wall of rectangular cross-section;
b = mean thickness in feet of wall of rectangular cross-section.

Trautwine recommends the formulas for walls without surcharge

$$b=0.35h \text{ for first-class masonry} \quad . \quad . \quad . \quad . \quad . \quad (493)$$

$$b=0.40h \text{ " rubble masonry laid in mortar} \quad . \quad (494)$$

$$b=0.50h \text{ " well laid dry rubble} \quad . \quad . \quad . \quad . \quad . \quad (495)$$

French engineers use the formula

$$b=0.30h \text{ for first-class masonry.} \quad . \quad . \quad . \quad . \quad . \quad (496)$$

A wall with a rectangular cross-section can be transformed into one of equal stability with a battered face and vertical back by means of formula (492).

If the face is vertical and the back is stepped the formulas above given may be employed, the *mean thickness* of the vertical profile of the wall will be about 0.85 of the value of b, given above.

If the face or back of a wall is not vertical, the top should be at least two feet thick.

Surcharged Walls.—A surcharged wall must be made thicker than one without a surcharge.

If H = height of surcharge in feet, or vertical height of M (Fig. 77) above AB, the French engineers' formulas for surcharged walls of rectangular cross-section are

$$b=\tfrac{1}{3}h+\tfrac{1}{3}H \quad \text{if } H<\frac{h}{2},$$

$$b=\tfrac{1}{3}h+\tfrac{1}{15}H \quad \text{if } H>2h. \quad . \quad . \quad . \quad . \quad (497)$$

For intermediate values of H in terms of h the fractional coefficients of H in the formula are obtained by interpolation between $\frac{1}{3}$ and $\frac{1}{15}$.

Foundations.—The empirical rules apply only to walls on firm foundations. The failure of masonry retaining-walls laid in mortar is usually due to defects in the foundations which cause the walls to slide on their foundation or to settle unequally. This is especially true of retaining-walls along waterways where the soil is of a soft, compressible character.

Drainage.—If the earth in rear of a retaining-wall becomes saturated with water, it will greatly increase the pressure upon the back of the wall. To avoid such increased pressure a vertical layer of broken stone is placed against the back of the wall, and drainage-tubes, or *weepers*, are run through or under the wall, to allow the escape of the water.

Counterforts and Buttresses.—A *counterfort* is a projection upon the back of the wall designed to strengthen the wall to resist the pressure of the earth; a *buttress* is a similar projection on the face of the wall. Counterforts of simple masonry are of doubtful efficiency in strengthening a wall either against shearing or overturning because of their liability to separate from the wall itself. A better wall is usually secured by placing all the masonry in the wall itself. They are sometimes employed in long walls to divide it into panels, and have been largely employed in the masonry walls of fortifications to limit the field of destruction of an exploding shell or mine and to form the side walls of casemates. In walls of concrete constructed about a framework of steel, counterforts may be constructed which are an integral part of the wall and cannot be separated from it.

Buttresses are more efficient than counterforts in strengthening a wall against rupture by shearing or overturning. They are not often employed for this purpose except in Gothic architecture, as it is usually desirable to have the face of the wall a plane surface.

PROBLEMS.

57. Determine by the use of Coulomb's Formula the thickness of a rectangular retaining-wall 20 feet high to safely resist the pressure of an embankment of equal height. Specific gravity of earth 1.5. Specific gravity of masonry 2.3. Angle of repose of earth $=34°$. Make the moment of the weight of the wall twice that of the earth pressure. *Ans.* 7.0 feet.

58. Solve problem given above by Poncelet's Formula. *Ans.* 6.1 feet.

59. Solve the problem given above by Rankine's Formula. Compare the thickness with that obtained by using empirical formula $b = 0.4h$. *Ans.* 7.0 feet.

CHAPTER XVI.

MASONRY ARCHES.

Definitions.—A masonry arch (Fig. 79) is a structure designed to support pressure and transmit its effects to lateral points of

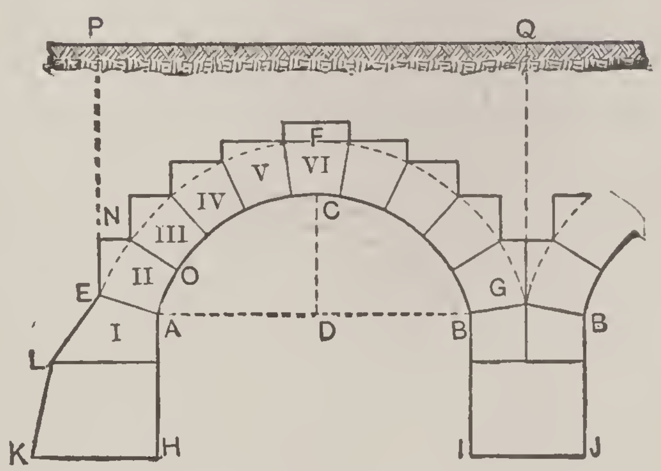


FIG. 79.

support. The arch is ordinarily composed of wedge-shaped blocks called *voussoirs*, which support each other by lateral pressure.

Soffit.—The inner cylindrical surface of the arch.

Back.—The outer surface of the arch.

Intrados.—The line of greatest curvature or the curve of right section of the soffit (ACB).

Extrados.—The line of greatest curvature or the curve of right section of the cylinder containing the outer extremities of the joints (EFG).

Abutment.—The mass of masonry designed to resist the thrust of the arch ($AELKH$). It is often a retaining-wall.

Pier.—A column of masonry designed to resist only the vertical component of the thrust of two adjacent arches ($BGBJI$).

Skewback.—The joint, usually inclined, between the extreme voussoir and the abutment or pier (EA).

Cushion-stone.—The stone whose upper surface is the skewback (*AEL*).

Springing-line.—The intersection of the skewback and the soffit (*A*).

Span.—The perpendicular distance between the springing-lines (*AB*).

Crown.—The highest rectilinear element of the soffit (*C*).

Rise.—The vertical distance between the crown and the plane of the springing-lines (*CD*).

Axis.—The intersection of the plane of the springing-lines by a vertical plane through the crown (*D*).

Keystone.—The highest stone of an arch (*VI*).

Springer.—The lowest stone of the arch (*II*).

Head.—The end surface of the arch.

Haunches.—The part of the arch about midway, vertically, between crown and springing-lines (*III*, *IV*).

String-course.—A row of voussoirs parallel to the axis.

Coursing-joint.—The joints between string-courses (*ON*).

Ring-course.—A ring of voussoirs parallel to the head of an arch.

Heading-joint.—The joint between the ring-courses.

Spandrel.—The volume above the arch limited by the back of the arch, vertical planes through the outside edges of the skewbacks, and a horizontal plane parallel to the plane of the springing-lines (*EPQGF*). A *spandrel wall* is one constructed in this space, parallel to the head of an arch; *spandrel filling* is the material deposited in the spandrel.

Form of Intrados.—*Full-center Arch.*—One whose intrados is a semicircle.

Segmental Arch.—One whose intrados is a circular arc less than 180°.

Elliptical Arch.—One whose intrados is a semi-ellipse.

Oval Arch.—One whose intrados is an oval or curve similar to an ellipse made up of three or more circular arcs tangent to each other.

Tudor Arch.—A pointed arch whose intrados is made up of two intersecting curves.

Direction of Axis.—*Right Arch.*—One in which the axis is perpendicular to the head.

Oblique or Skew Arch.—One in which the vertical plane through the axis makes an oblique angle with the plane of the head.

Rampant Arch.—One in which the axis is inclined to the horizontal plane.

Groined and Cloistered Arches.—The soffits of groined and cloistered arches are formed by the intersection of two cylindrical soffits having the same rise and intersecting axes. In a groined arch that part of each soffit which lies within the other is removed, thus preserving the crown of each arch throughout. The groins are the curves of intersection of the two soffits. The cloistered arch is a dome-shaped arch which rises from the springing-lines to a point. It is formed by retaining only that part of each soffit which lies within the other.

Annular Arch.—An annular arch is one generated by revolving a plane of right section about a line of this plane perpendicular to the span, but not intersecting the arch; it is a circular arched passageway.

Dome.—The soffit of a dome is generated by revolving a plane of right section about a vertical line intersecting its crown.

THEORY OF THE ARCH.

Modes of Rupture.—An arch, like a masonry dam, may rupture by rotation, by sliding, or by the crushing of the material. The mode of rupture depends upon the form of the arch and the depth of the voussoirs.

The ordinary mode of rupture of a *full center* arch, in which the ratio of the rise to the span is one-half, is shown in I, Fig. 80; the thrust of the arch at the haunches causes the

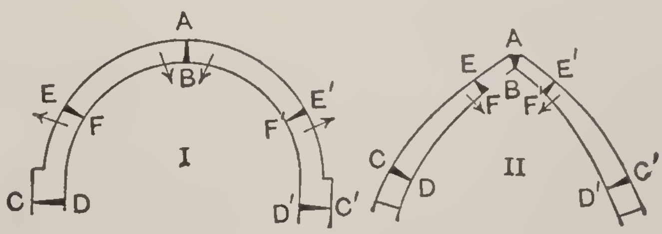


FIG. 80.

lower segments to rotate outwards about the outer edge of the skewback or of some joint of the abutment, and when the points F and F' are sufficiently separated, the weight above the crown causes the upper segments to rotate inwards. An *oval* or *elliptical* arch may also rupture as shown in II, Fig. 81. The thrust at the haunches causes the lower segments to slide on the cushion-stones and thus separate the points F and F'; when these are sufficiently separated the upper segments rotate inwards as before. In the arches above described, the points F and F' lie in a horizontal plane which passes midway between the crown and the horizontal diameter of the circle or the horizontal axis of the ellipse or oval. If this plane passes below the springing-lines of a segmental arch, the points F and F' are at the springing lines. A *segmental* arch usually ruptures as shown in I, Fig. 81. The thrust at the springing-lines causes the arch to slide

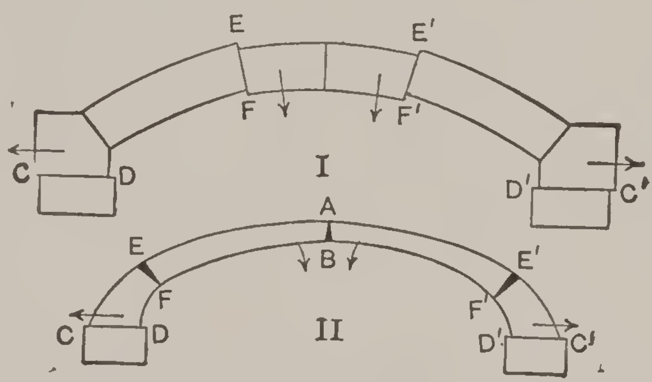


FIG. 81.

on its abutments and separates the points F and F' sufficiently to allow the voussoirs near the crown to slip through.

The ordinary mode of rupture of an arch in which the ratio of the rise to the span exceeds one-half is shown in II, Fig. 80. The pressure on the haunches causes the arch to open at some joint, as EF, near the crown. The lower segment then rotates inwards about some joints, as CD, near the springing-lines, thus bringing the points F and F' nearer together; as the points E and E' move, they cause the crown of the arch to rise and the upper segments to rotate about B.

The joints shown in Figs. 80 and 81, at which the arch opens, are called the *joints of rupture*.

If the arch does not rupture by rotation or by sliding, it can rupture only by the crushing of the material of which it is composed. If the arch ruptures in this manner, it will crush the material at the joints of rupture, since, as will be shown, the resultant pressure is nearer the intrados or extrados at these points than at any other points of the arch.

The Thrust at the Crown.—Let Fig. 82 represent an arch made up of two monolithic segments with a vertical joint at the crown. Assume the arch to be made of material of infinite strength and without elasticity. Each semi-arch will, under the

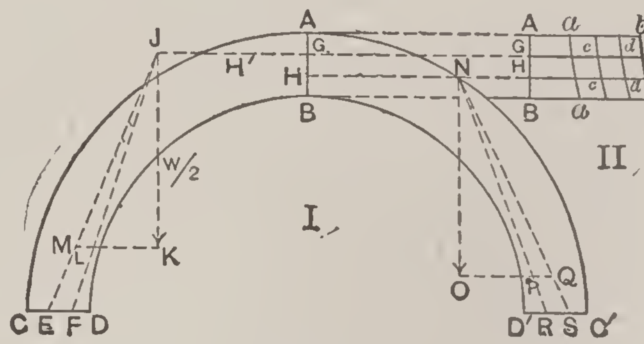


FIG. 82.

action of its own weight, attempt to rotate about its springing-line and thus produce a thrust upon the other half of the arch. As in a three-hinged truss, each semi-arch, when at rest, is acted upon by a system of three forces in equilibrium, consisting of its weight $\frac{W}{2}$, which acts through its center of gravity, the thrust at the crown H, or pressure of the other half of the arch, and the reaction at the skewback $R_{,}$. A system of three forces in equilibrium must either be a parallel or a concurrent system. $\frac{W}{2}$ and H cannot be parallel since one is a vertical force and the other is the reaction of two vertical surfaces and cannot be vertical. The system is therefore a concurrent one. Such a system admits of accurate solution if one of the forces is fully given and the action lines of the other two are known.

Of the three forces, the intensity and action line of $\frac{W}{2}$ can be determined when the dimensions of the arch and the specific gravity of the material are known. If the two semi-arches are symmetrical and are symmetrically loaded, the action line of H will be horizontal. The point of application of H may, however, be anywhere between A and B.

Of the force $R_{,}$ we know only that its point of application must lie between C and D. An accurate solution of the system is therefore impossible. We may, however, determine the *limits* within which the intensities of H and $R_{,}$ must lie.

We may assume that H acts at A or B, and that $R_{,}$ acts at C or D.

These hypotheses will give the limiting values of H and $R_{,}$.

Let H' = intensity of H when the point of application is at A and the center of moments is at D;

H'' = the intensity of H when the point of application is at A and the center of moments is at C;

H''' = intensity of H when point of application is at B and center of moments is at D;

H^{IV} = intensity of H when point of application is at B and center of moments is at C;

y'' = lever-arm of H with respect to D or C when point of application is at A;

y' = lever-arm of H with respect to D or C when point of application is at B;

x' = lever-arm of $\frac{W}{2}$ with respect to D;

x'' = lever-arm of $\frac{W}{2}$ with respect to C.

Then we shall have:

Point of application A:

$$\text{Minimum value } H' = \frac{Wx'}{2y''},$$

$$\text{Maximum value } H'' = \frac{Wx''}{2y''}.$$

Point of application B:

$$\text{Minimum value } H''' = \frac{Wx'}{2y'},$$

$$\text{Maximum value } H^{\text{IV}} = \frac{Wx''}{2y'}$$

To interpret these values make AB in II, Fig. 82, equal to AB in I, Fig. 82.

At the extremity of AB in II, Fig. 82, lay off $Aa = H'$, $Ab = H''$, $Ba = H'''$, and $Bb = H^{\text{IV}}$. Then is aa the curve whose ordinates give values of H, which, if applied at the proper points of the joint AB, will make the resultant of $\frac{W}{2}$ and H act through D, and bb is the curve whose ordinates will give the values of H, which, if applied at the joint AB, will make the resultant of $\frac{W}{2}$ and H act through C. The curves aa and bb are hyperbolas since Hy is constant.

The area $aabb$ may be taken as a measure of the stability of the arch, since the ordinate of any point of the surface will give a value for H which, if applied at the proper point of AB, will prevent the semi-arch from rotating about D or C. Any value of H less than the ordinate of aa, or greater than the ordinate of bb, will cause the semi-arch to rotate about D or C. Thus the intensities Ga, Gb, or any intermediate values, will, if applied at G of the crown joint, prevent the arch from rotating about D or C.

In the actual design of an arch some latitude must be allowed for errors in the value of H, due to the fact that $\frac{W}{2}$ cannot be accurately determined. Furthermore, if the material is of finite strength, the thrust cannot act at A or B without crushing the material, since the unit pressure, as was shown under eccentric loading, would then be infinite. For these reasons, the hypothesis is made that H and $R_{\prime}$ shall act within the middle third of their respective joints; the limiting points of application of H in I, Fig. 82, are therefore G and H; and of $R_{\prime}$ E and F, in which $GH = \frac{1}{3}AB$, and $EF = \frac{1}{3}CD$.

The limiting values of H consistent with satisfactory stability are shown in Fig. 82, in which

$$JK = NO = \frac{W}{2}.$$

KM and KL = the maximum and minimum values of H when the point of application is at G.

OQ and OP = the maximum and minimum values of H when the point of application is at H.

If in II, Fig. 82, we lay off $AG = GH = HB$, $Gc = KL$, $Hc = OP$, $Gd = KM$, $Hd = OQ$, the area $ccdd$ may be taken as the measure of the satisfactory stability of the semi-arch. The ordinate of any point of this area will give a value of H which, acting at the corresponding point within the middle third of AB, will cause the resultant of $\frac{W}{2}$ and H to intersect CD between E and F.

If the semi-arch is designed by employing such a value of H, the arch cannot yield by rotation under the crown thrust; it may, however, slide on the skewback if the action line of the resultant of $\frac{W}{2}$ and H is sufficiently oblique to the joint at the springing-line. In order that sliding shall not take place, the force of friction at this joint must be greater than the component of the resultant of $\frac{W}{2}$ and H parallel to the joint, or, in other words, the resultant must make a less angle with the normal to the joint than the angle of friction.

Assuming the angle of friction of masonry on masonry to be about 37 degrees, the angle between the actual resultant and the normal must be less than this angle. As the action line of the actual resultant cannot be accurately fixed, the angle between its most oblique action line possible, NS, Fig. 82, and the normal is, for safety, made less than 37 degrees.

If the two conditions as above given are fulfilled, the arch cannot rupture unless the masonry itself crushes. To prevent this, the unit pressure at A, B, C, and D should not exceed the allowable unit compressive stress of the masonry of which the arch is composed.

General Condition of Stability.—If we now assume the semi-arch to be divided into voussoirs, it is evident that the same conditions which we have imposed upon the semi-arch must be imposed upon each segment included between the crown and any joint; hence we have the following conditions of satisfactory stability for an arch.

1. *The resultant of the thrust at the crown and the pressure upon the arch between the crown and any joint should make a less angle with the normal to the joint than the angle of friction.* This will prevent sliding at the joints even if no reliance is placed upon the strength of the mortar.

2. *The resultant of the thrust at the crown, and the pressure*

upon the arch between the crown and any joint, should pierce the joint within the middle third.

3. *The unit pressure at the extrados or intrados of any joint should not exceed the allowable unit stress in compression of the material of which the arch is made.*

Méry's Curve of Pressure.—If we know the intensity and point of application of the thrust at the crown, it is an easy matter to construct graphically the resultant of the thrust at the

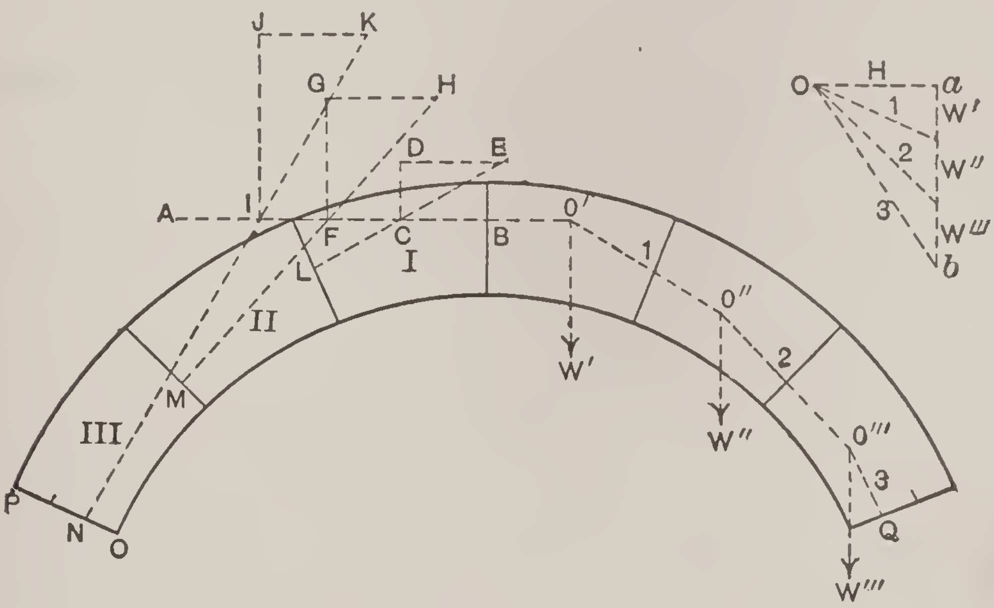


FIG. 83.

crown, and the weight of each segment, and find the corresponding center of pressure. Thus in Fig. 83 let

$DC = W'$ = action line and intensity of weight of voussoir I;

$GF = W' + W''$ = action line and intensity of weight of voussoirs I and II;

$JI = W' + W'' + W''' = \frac{W}{2}$ = action line and intensity of weight of voussoirs I, II, and III;

AB = action line of thrust at crown;

$JK = GH = DE$ = intensity of the thrust at the crown.

Combining W' and H, the resultant is EC, which pierces the joint between I and II at L.

Combining $W' + W''$ and H, their resultant is HF, which pierces the joint between II and III at M.

Combining $W' + W'' + W'''$ with H, their resultant pierces the skewback at N.

If we now construct a curve passing through $BLMN$ tangent to the several resultants, it will be Méry's *curve of pressure* or the *curve of resistance.*

If the curve of pressure of an arch is given, its stability can be readily tested. If at any joint the angle between the normal to the joint and the tangent to the curve of pressure is less than the angle of friction, the first condition of satisfactory stability is fulfilled. If the curve of pressure lies everywhere within the middle third, the second condition of satisfactory stability is fulfilled.

The curve of pressure may also be constructed by means of an equilibrium polygon.

In Fig. 83 lay off Oa equal to JK or H, and ab equal to $\frac{W}{2}$ or $W'+W''+W'''$. Divide ab into three parts, equal to the weights W', W'', W''', and draw lines 1, 2, and 3. From the force polygon Oab construct the equilibrium polygon $BO'O''O'''Q$. Then will the points where the sides of the polygon intersect the joints be the centers of pressure, and a curve tangent to the equilibrium polygon at these points will be the curve of pressure.

The equilibrium polygon is called the *line of pressure* since its lines are the action lines of the pressures at the joints.

It is evident, from the discussion given above, that the curve of pressure of a symmetrically loaded arch may always be constructed when the intensity and the point of application of the crown thrust are known.

In the segmental arch shown in Fig. 83 the theory of minimum crown thrust has been adopted. According to this theory the maximum pressure at any joint must not exceed twice its mean pressure, and the intensity of the crown thrust is the minimum thrust which will fulfil this condition.

In Fig. 83 the crown thrust, according to this theory, must be $ED=HG=KJ$, and its point of application must be B. If a thrust of less intensity than ED acts at B, the center of pressure at the joint PO will lie between N and O, or be outside of the middle third, and the pressure at O will be greater than twice the mean pressure on PO. If the point of application of ED is above B, the pressure at the top of the crown joint will be greater than twice the mean pressure on the crown joint. If the point of application of ED is below B, the center of pressure at the joint PO will lie between N and O, or outside the middle third, and the pressure at O will be greater than twice the mean pressure on PO.

The minimum crown thrust of a symmetrically loaded arch may be readily determined if the joints of rupture are known. In a segmental arch, like that shown in Fig. 83, we know that the skewback PO is a joint of rupture. If, therefore, we connect I, the point of intersection of the action lines of the crown thrust and the weight of the semi-arch, with N, the extremity of the middle third of PO, it will be the resultant line of pressure of the weight of the semi-arch and the *minimum* crown thrust. If we lay off IJ equal to the weight, KJ must be the intensity of the minimum crown thrust.

Load on an Arch.—In determining the value of $\frac{W}{2}$ for the semi-arch and the value of W', W'', etc., for each voussoir, we must consider not only the weight of the voussoirs themselves, but also the pressure to which they are subjected from the spandrel filling. In determining these values it is usual to reduce the area of the spandrel filling to an equivalent area having the same specific gravity as the masonry of the arch. In the arch shown in Fig. 84 the ratio of the specific gravity of the spandrel filling

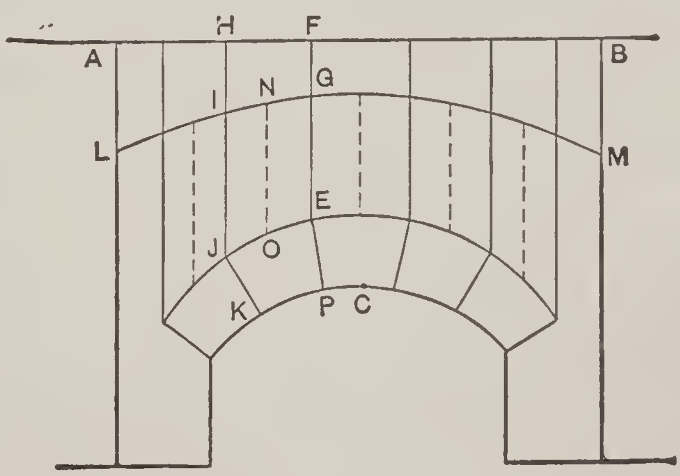


Fig. 84.

to that of the masonry is assumed to be $\frac{2}{3}$. We may therefore reduce the spandrel volume to one of the same specific gravity as the masonry by laying off from the back of the arch on each ordinate of the spandrel, as EF, a distance $EG = \frac{2}{3}EF$. The line LM connecting these points will be the cross-section of the upper surface of the spandrel filling having the same specific gravity as the masonry of the arch.

To find the weight resting on each voussoir approximately, the spandrel filling is divided by vertical planes, as shown in Fig. 84. The voussoir is assumed to form a single mass with the load upon it, hence its weight is assumed to act through the center of gravity of the mass.

To find the action line of the weight of any of these masses, as *PEGIJK*, find the area of surface *EGIJ* assumed as a rectangle whose width is *HF* and depth *NO*, and its moment with respect to any point, as *C*, the crown of the arch; find also the area of the voussoir *EPKJ*, assumed to be a trapezoid, and its moment with respect to the same point. The sum of the moments divided by the sum of the areas will give the approximate distance of the center of gravity of the mass from *C*, and a vertical line at this distance from *C* will be the action line of its weight.

In the above discussion we have neglected the friction along the planes *HJ* and *FE*, and have assumed that the block *KJIGEP* is a single mass. It would probably be more correct to assume the weight of the block *JIGE* as acting vertically through *O*, and at that point resolve it into normal and tangential components.

Depth of Keystone.—If the curve of pressure lies within the middle third of the joint, the unit pressure upon the extremity of the keystone joint cannot exceed $\frac{2H}{d}$, in which H equals the horizontal thrust in pounds of an arch one inch in length, and d equals the depth of the keystone in inches. H, however, cannot be known until the form of the right section of the arch is known.

In designing an arch the depth of the keystone must be assumed. Various formulas have been proposed by engineers for this purpose, usually based upon a study of arches already constructed and assumed to be satisfactory. As H is dependent upon the span and rise, these quantities must enter a general formula. In the following formulas for circular arches the dimensions are in feet; d = depth of the keystone, h = rise of arch, s = span of arch.

Perronet:

$$d = 1\tfrac{1}{12} \text{ feet} + \tfrac{1}{23}s \text{ full center arch.} \quad . \quad . \quad . \quad (498)$$

This formula makes d too great when $s > 50$ feet.

Dupuit:

$$d = 0.13s \text{ for full-centered arches, } \quad (499)$$

$$d = 0.073s \text{ for segmental arches when } \frac{h}{s} = \frac{1}{4}. \quad (500)$$

Dupuit's formulas are recommended by the Manual of the "Ponts et Chaussées," France.

Dejardin:

$$d = 1 \text{ foot} + 0.05s \text{ when } \frac{h}{s} = \frac{1}{2}, \quad (501)$$

$$d = 1 \text{ foot} + 0.026s \text{ when } \frac{h}{s} = \frac{1}{10}. \quad (502)$$

The coefficient of s is interpolated when $\frac{h}{s}$ varies between $\frac{1}{2}$ and $\frac{1}{10}$.

Depth at the Springing-line.—In order that the mean pressure shall be the same throughout, the arch must increase in depth from the crown to the springing-lines. In arches of large span the ratio of the depth at the keystone to the depth at the springing-lines varies from $\frac{2}{3}$ to $\frac{1}{2}$. In arches of small span the depth is usually constant.

Allowable Masonry Pressures.—The following are considered safe limits for the allowable unit pressures in arches:

Concrete, 60 to 70 pounds per square inch.

Brick in lime mortar, 85 pounds per square inch.

Brick in hydraulic cement mortar, 110 to 114 pounds per square inch.

Soft stone masonry in hydraulic cement mortar, 85 to 200 pounds per square inch.

Hard stone masonry in hydraulic cement mortar, 300 to 400 pounds per square inch.

Testing the Design of an Arch.—The curves of the intrados and extrados, the depth at the crown and springing-lines, the specific gravity of the masonry, the specific gravity and form of the spandrel filling must all be assumed. From the data thus given the load upon every voussoir is determined.

If the coursing-joints are made normal to the intrados, there will be little danger of the voussoirs sliding upon each other, since the resultant of the external forces acting on either side of the joint will make with the normal a less angle than the angle of friction. Rupture, as shown in I and II, Fig. 81, is prevented by strengthening the abutment.

In this manner the first condition of stability is fulfilled.

The second and third conditions of stability are fulfilled by making the distance between the intrados and the extrados sufficient at every point to cause the center of pressure to lie within the middle third of every joint.

If the arch is designed by the theory of minimum crown thrust, we may always determine the intensity of this thrust in a symmetrically loaded arch if we know the joints of rupture. If the arch is a full-centered one, as Fig. 80, we know its joints of rupture are EF and CD. The minimum crown thrust for the joint EF may be determined, as explained for the segmental arch, by assuming that at the joint EF the center of pressure is one-third of EF from F; if this thrust will cause the center of pressure at CD to lie within the middle third, the arch will have satisfactory stability. If the minimum crown thrust as above determined causes the center of pressure at CD to lie outside the middle third, the thickness of the arch at CD must be increased until the center of pressure lies within the middle third. It is usually done by moving C farther from D and making the extrados a straight line from the haunch to the springing-line.

If the joints of rupture are not known, we may construct for each joint the limits Gc and Gd, Fig. 82, within which the minimum crown thrust must lie for that joint; values common to all joints may be assumed as satisfactory values of the crown thrust to be employed in designing the arch.

If the arch shown in Fig. 82 is designed with tight joints and with a minimum crown thrust Gc, it is evident that unless there is some yielding in the material the actual minimum thrust may be less than Gc and act between G and A. If the crown thrust acts at any point between G and A, the maximum pressure will exceed twice the mean pressure. We can therefore insure compliance with the condition that the maximum shall not exceed twice the mean pressure only by making the joint open between

G and *A*. In some segmental arches recently constructed, lead plates were inserted at the crown and springing-lines, covering only the middle third of the arch, and the remainder of the joint was filled with a non-resisting material. In such an arch the maximum pressure at a joint cannot exceed twice the mean pressure.

Elastic Arch.—The arch, especially if a monolithic structure of concrete, may also be designed on the theory that it is a curved beam whose fibers have tensile and compressive resistance.*

Abutments. — The abutment should fulfil conditions similar to those of the arch itself. If the abutment resists by its weight alone, the resultant of the thrust of the arch at the skewback, and the weight of the abutment above any horizontal section, should pierce the section within its middle third. The abutment should also have sufficient strength in every horizontal plane to resist the horizontal component of the thrust.

In computing the dimensions of the abutments the line of pressure should be assumed in its most unfavorable position for the stability of the abutment. This is usually through the crown itself and the outer edge of the skewback.

Unsymmetrical Loading.—If the arch is not symmetrically loaded, the crown thrust will not be horizontal. Its direction may be determined either by assuming points of the curve of pressure at the skewbacks and constructing an equilibrium polygon passing through them, or by assuming points of the curve of pressure both at the skewbacks and crown, and constructing an equilibrium polygon passing through the three points. In either case it is necessary to experiment until a polygon is constructed which intersects every joint within its middle third, or it is clear that such a polygon cannot be constructed. In the latter case the dimensions of the arch must be changed. In determining the effect of a live load it is customary to assume a uniformly distributed live load extending from the crown to one of the abutments.

The line of the equilibrium polygon intersecting the crown joint will be the action line of the crown thrust for unsymmetrical as well as for symmetrical loading.

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* "A Treatise on Arches," by Malverd A. Howe.

CHAPTER XVII.

THE PRESSURE AND FLOW OF WATER.

The design of the structures employed in systems of water-supply and sewerage is based not only on the laws governing stress and resistance in solids, but also on the laws governing the pressure and flow of water.

Physical Properties.—Water is one of the fluids classed as incompressible because its change of volume under pressure is so small that it is neglected in all practical problems. Its density, however, increases slightly under pressure, and decreases slightly as its temperature rises from near the freezing- to the boiling-point.

The weight of a cubic foot of fresh water under normal conditions is 62.425 pounds; in practical problems it is usually taken as 62.5 pounds. This is usually an error on the safe side.

The volume of water is expressed either in units of volume or in units of capacity. The unit of volume is the *cubic foot* in all countries where the English system is employed; where the metric system is used the unit is the *cubic meter*. The unit of capacity in this country is the *standard gallon*, which contains 231 *cubic inches;* in England it is the *imperial gallon*, which contains 277.27 *cubic inches;* where the metric system is used it is the *liter*, containing 0.264 *standard gallons*, or the *dekaliter*, containing 2.64 gallons.

1 cubic foot = 7.4805 standard gallons = 62.5 pounds.

1 standard gallon = 0.1337 cubic feet = 8.356 pounds.

Static Principles.—1. *Every molecule of still water is subjected to equal pressure from all directions.*

This results from the hypothesis that in a perfect liquid there is no cohesion between its molecules, and that the molecules move without the development of friction. This is not strictly true since there is slight cohesion between its molecules, but in all ordinary problems it may be neglected.

2. *The upper or free surface of still water is horizontal or normal to the action line of the force of gravity.*

The surface must be normal to the resultant water pressures on the surface molecules; otherwise these molecules would move along the surface. Since each surface molecule is at rest, the resultant water pressure on it must be directly opposed to the action line of its weight, which is the only other force acting on it. As the action line of the weight is vertical, that of the resultant pressure must be vertical and the free surface must therefore be horizontal or normal to the action line of the force of gravity.

From this principle it follows that, if the free surface of water is not horizontal, the water is not still, but is in motion; if still water is subjected to any force whose action line is not vertical, the surface will not be horizontal. A flowing river is a body of water with an inclined surface, and a pond rippled by the wind is a body of water acted upon by force in addition to that of gravity.

3. *The unit pressure on each molecule of a body of still water varies directly with its depth below the free surface.*

Since there is neither cohesion nor friction, the water pressure on the upper surface of any molecule is due to the weight of the molecules above it; as these are of equal weight, the pressure on the molecule must be proportional to its depth. According to the first principle, the pressure from every other direction must be equal to this pressure. Thus if

p = pressure in pounds per square foot,
p' = pressure per square inch,
h = depth in feet,
h' = depth in inches,

$$(1)\ p=62.5h, \quad (2)\ p'=\frac{62.5}{144}h, \quad (3)\ p'=\frac{62.5}{1728}h'. \qquad (503)$$

If the weight or pressure of the atmosphere which rests on the water be added, the total unit pressure will be $p+p_0$ or $p'+p_0'$, in which p_0 and p_0' are the pressures of the atmosphere in pounds per square foot and inch. At the sea-level p_0' is about 15 pounds.

The factor h is called *the hydrostatic or static head*, or simply *the head;* since it varies directly with the pressure, the head is often employed in the sense of pressure.

4. *The pressure of still water is, at every point, normal to the surface pressed.*

The pressures on the elementary areas of a plane surface will therefore form a system of parallel forces whose resultant is a single force. The pressures on the elementary areas of a curved surface will in general form a system of non-parallel forces which cannot be represented by a single resultant. If, however, the curved surface is one of revolution, the total pressure on it may sometimes be represented by a single resultant. Thus the total pressure on a semi-cylinder whose axis is vertical or whose extreme elements lie in a horizontal plane may be represented by a single force.

If the pressure on the semi-cylinder is assumed to be uniform, or the same on each unit of area, the total pressure may be represented by a single force whatever be the position of the cylinder.

5. *The total pressure on any plane surface subjected to the pressure of still water is equal to the weight of a prism of water, whose base is the area under pressure, and whose height is the depth of the center of gravity of the area below the free surface of the water.*

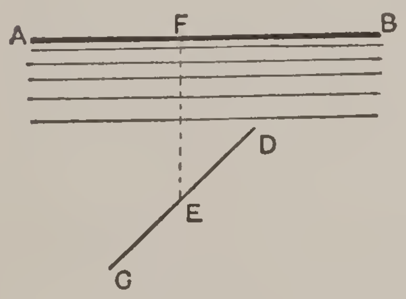


FIG. 85.

In Fig. 85 let CD be a plane whose center of gravity lies in the horizontal line E.

From the third principle we have for the pressure of the elementary area δa of the plane, at a depth h below the surface,

$$\delta p = 62.5 h \delta a.$$

For the total pressure P we have

$$P = 62.5 \Sigma h \delta a.$$

Since $\Sigma h \delta a$ is the sum of the products of each elementary area by its distance from the free surface, it will, from the principle of the center of mass, be equal to AH, in which A equals the area of the plane in feet, and H is the distance in feet of its center of gravity below the free surface. Substituting we have

$$P = 62.5 AH,$$

in which the second member is the weight of a prism whose base is A and whose height is H. In the figure H is equal to EF.

Since H is the head at the center of gravity, we may also state that the pressure on a submerged plane is the product of its area, the head at its center of gravity, and the weight of a unit volume of water.

6. *If pressure be applied to a unit area of the surface of water completely filling a closed vessel, an equal pressure will at once be felt at every other unit area both within and at the surface of the water, and hence on every unit of area of the vessel in contact with the water.*

This results from the incompressibility of water and the movement of its particles without developing friction. It is the principle employed in the hydraulic or hydrostatic press.

7. *The resultant pressure of still water on a plane area acts at the center of percussion of he area with respect to the axis formed by the line of intersection of the plane and free surface.*

As the elementary pressures form a system of parallel forces, they must have a single resultant; the point in which this resultant pierces the area is called the *center of pressure.*

As the pressure is zero at the surface and increases uniformly from that surface, were the area free to rotate under the action of the pressure it would rotate about its intersection with the surface. This line may therefore be considered its *spontaneous axis*, and the point where the resultant pierces the area, or the *center of pressure*, then becomes the *center of percussion.*

The distance of the center of percussion from its spontaneous axis may always be found from the following equation derived from Mechanics

$$D=\frac{I}{Ad'}=\frac{r^2+d'^2}{d'}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (504)$$

in which D = distance of center of percussion from the spontaneous axis, or in this case the distance of the center of pressure from the intersection of the plane and the water surface;

d' = distance of center of gravity of area from the same line;

I = moment of inertia of area about the same line;
Ad' = moment of area about the same line;
r = radius of gyration of the plane area about an axis through its center of gravity parallel to its spontaneous axis.

As the value of d' for any plane area can easily be determined, and as the values of r for all simple areas are given in engineering manuals, it is easy to determine the value of D for any simple area.

In Fig. 86 let CD be a rectangular plane area whose edges projected in C and D are parallel to the surface AB.

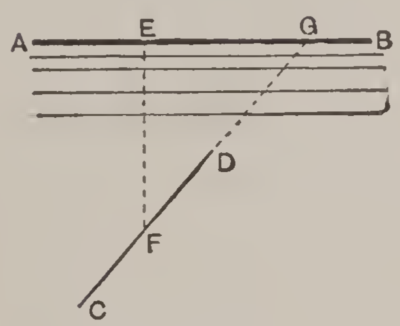


FIG. 86.

Let d = length of edge CD,
b = length of edge C. Then

$$A = bd,$$

$$r^2 = \frac{d^2}{12},$$

$$d' = FG,$$

$$D = \frac{\frac{d^2}{12} + FG^2}{FG}.$$

If the upper edge of the plane is in the surface AB, FG becomes equal to $\frac{d}{2}$, and the value of D reduces to

$$D = \tfrac{2}{3}CG = \tfrac{2}{3}d.$$

Hence the center of pressure of a rectangular surface whose upper edge lies in the free water surface is two-thirds the depth of its lower edge below the surface.

The center of pressure of a curved surface can be accurately determined only when the elementary pressures can be replaced by a single resultant.

8. *Archimedes' Principle. Every solid, floating or immersed in water, is subjected to a vertical force acting upwards, whose intensity is equal to the weight of the water displaced by it.* This is called the buoyant effort of the water.

When at rest a solid lighter than water will therefore displace its own weight of water. A solid heavier than water will lose in weight an amount equal to the weight of an equal volume of water.

PROBLEMS.

60. What is the static pressure per square inch in sea-water at a depth of 50 feet? Weight 64 pounds per cubic foot.

Ans. 22.2 pounds.

61. What is the total pressure upon the surface of an inclined isosceles triangle whose vertex is at the surface and whose base is parallel to and 9 feet below the surface? The base of the triangle is 10, and its altitude 12 feet. *Ans.* 22,500 pounds.

62. The base of a conical vessel filled with water is a circle whose radius is 6 inches; the free surface of the water is a circle whose radius is 1 inch. If a pressure of 10 pounds is applied to the free surface, what will be the resulting pressure on the base?

Ans. 360 pounds.

63. The moment of inertia of a circle about an axis through its center is $\frac{\pi d^4}{64}$. Where is the center of pressure of an inclined submerged circular plate whose radius is 10 feet, whose center is 4 feet below the free surface, and whose circumference is tangent to the free surface? *Ans.* 12.5 feet.

Discharge through Orifices.—An *orifice* is an opening in the side of a vessel containing water; it is entirely below the free surface of the water. The stream which issues from an orifice is called a *jet.*

Torricelli's Law. The theoretical velocity of the jet is the same as that developed by a body falling in vacuo through a height equal to the depth of the center of the orifice below the free surface of the water. Or

$$V = \sqrt{2gH},$$

in which V = velocity of discharge in feet per second;
H = depth in feet of center of orifice below the surface;
g = acceleration due to force of gravity = 32.2 feet per second.

The theoretical discharge per second is the product of this velocity and the area of the orifice; or

$$D = aV,$$

in which D = discharge in cubic feet per second;
a = area of orifice in square feet;
V = theoretical velocity in feet per second.

If the vessel is made of thin plates and the orifice is not near the bottom or a side, the actual form of the jet is shown in A, Fig. 87. The different molecules reach the orifice by converging

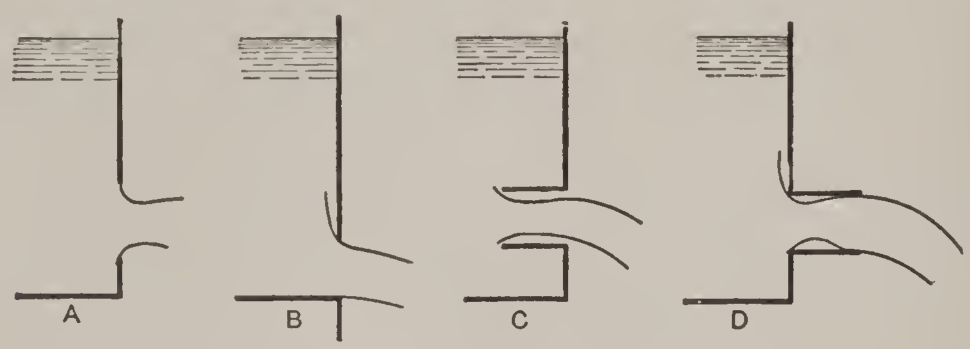


FIG. 87.

paths and make the cross-section of the jet at a short distance from the orifice sma er than the orifice itself.

By comparing the actual path of the jet with its theoretical path, it is found that the velocity at the contracted section necessary to produce this path is only 0.97 of the theoretical velocity. By careful measurement the area of this contracted section is found to be only 0.64 of the area of the orifice itself. The actual discharge when measured is found to agree with the actual velocity and actual area of the contracted section, and is only 0.62 of the theoretical discharge.

If, therefore, V' = actual velocity at contracted section,
a' = actual area at contracted section,
D' = actual discharge,
V = theoretical velocity at the orifice,
a = area of orifice,
D = theoretical discharge, then

$$V' = c_v V = 0.97 V, \qquad (505)$$

$$a' = c_a a = 0.64 a, \qquad (506)$$

$$D' = cD = 0.62 D, \qquad (507)$$

$$c = c_v c_a, \qquad (508)$$

in which c_v is the coefficient of velocity;
c_a is the coefficient of contraction;
c is the coefficient of discharge.

The value of the actual discharge may be expressed as follows:

$$D' = cD = caV = ca\sqrt{2gH}, \quad \dots \quad (509)$$

from which the discharge of any orifice may be computed when the coefficient of discharge, the area of the orifice, and the depth of the center of the orifice below the water surface are known.

If the plate in which the orifice is made is thin and clean, the edges of the orifice are sharp, and the orifice is not too near the bottom or a side of the vessel, the coefficient of discharge will be 0.62.

This value of the coefficient cannot be employed, however, if the thickness of the plate exceeds the diameter of the orifice, or if the edges of the orifice in a thick plate are not normal to the surface of the plate in contact with the water.

The formula for the discharge of an orifice is usually put in the form

$$D = 0.62a\sqrt{2gH} = 0.62aV, \quad \dots \quad (510)$$

in which D = actual discharge in cubic feet per second;
a = area of orifice in square feet;
H = head in feet over center of orifice;
g = 32.2 feet per second;
V = theoretical velocity due to head.

From this formula the value of any one of the quantities D, a, H, or V may be found when the others are given.

If the orifice is near either the bottom or one of the sides of the vessel, as shown in B, Fig. 87, the contraction is only *partial*, and the discharge will be greater than that given by formula (510).

Short Tubes.—Short tubes are often inserted into orifices. These are of two varieties: the *re-entrant tube*, which projects into the vessel and has its outer end flush with the outer face of the vessel, and the *projecting tube*, whose inner surface is flush with the inner face of the vessel.

The *Borda tube* (C, Fig. 87) is a cylindrical re-entrant tube so short that the jet passes through the tube without coming into contact with its sides. Its coefficient of discharge is only 0.5, which is less than that of any other form. This small value is due to the crowding of the molecules seeking entrance. The formula of discharge for a Borda tube is

$$D=0.5a\sqrt{2gH}=0.5aV. \quad . \quad . \quad . \quad . \quad (511)$$

The *standard tube* is a short projecting cylindrical tube (D, Fig. 87). As the jet enters the tube it is contracted as in leaving an orifice, but before it leaves the tube it expands so as to completely fill the cross-section at the outlet. The actual contraction is therefore nothing, and the coefficient of discharge must be equal to the coefficient of velocity. By actual measurement the common coefficient has been found to be 0.82. The increase in the discharge over a simple orifice is explained by the formation of a vacuum in the tube at the point where the jet is contracted; this reduces the back pressure. The vacuum is caused by the rushing water carrying with it the confined air. The formula for the standard tube is therefore

$$D=0.82a\sqrt{2gH}=0.82aV. \quad . \quad . \quad . \quad . \quad (512)$$

By changing the form of the projecting tube its discharge coefficient may be modified. If made bell-shaped, like the vertical section of a jet when issuing from an orifice (A, Fig. 87), the coefficient will be about 0.95 if the discharge is measured in terms of the area of its smallest cross-section.

Coefficients have been determined, by careful measurement, for many forms of projecting tubes, convergent, divergent, and combined. These are found in engineering manuals. An orifice in a plate whose thickness is greater than the diameter of the orifice is simply a projecting tube and has the same coefficients.

If the tube is near the top, bottom, or sides of the vessel, the molecules will enter the tube with less interference, the coefficient will be increased, and the jet will be only partially contracted.

PROBLEMS.

64. What will be the discharge in gallons per second from an orifice 2 inches square, whose center is 4 feet below the surface?

Ans. 2.06 gallons.

65. What head will be required to make the discharge through a 2-inch Borda tube one gallon per second?

Ans. 2.29 feet.

66. What is the diameter of a standard tube which under a head of 9 feet discharges 2 gallons per second?

Ans. 1.57 inches.

Miner's Inch.—The miner's inch is the discharge in a unit of time through an orifice, one square inch in area, under a head fixed by law or custom; it is employed in measuring water sold for mining and irrigating purposes. A head of six inches is often prescribed, which makes the discharge per second

$$D=\frac{0.62}{144}\sqrt{\frac{64.4}{2}}=0.0244+\text{ cubic feet.} \quad . \quad . \quad . \quad (513)$$

This formula is deduced under the hypothesis that the coefficient of discharge is that of an orifice in a thin plate. In practice the shutter through which the water flows is not a thin plate, but a board at least an inch thick. If the coefficient is assumed as 0.72, a mean between the coefficients for an orifice in a thin plate and that for a standard tube, it will correspond more nearly to the conditions of practice.

The practical method of measuring the discharge of a stream in miner's inches is shown in Fig. 88. The water is discharged through an orifice, shown by the shaded area, one inch deep whose center is 6 inches below the top of the plate, B. A sliding shutter, A, in the plate permits of the widening of the orifice until the entire discharge takes place through it when the surface of the water is on a level with the top of the plate, B. The number of miner's inches is at once read off on the scale.

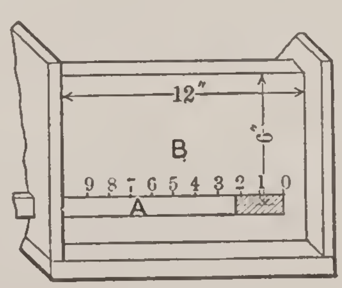


FIG. 88.

Weirs.—A weir is a notch cut in the side of a vessel, through which the water flows. It is usually rectangular in shape with vertical sides; the bottom is called the *sill*. Triangular notches are also employed.

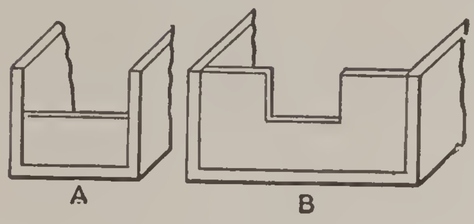


FIG. 89.

A weir of the form shown in A, Fig. 89, is called *a weir without end contractions;* one constructed as shown in B, Fig. 89, is *a weir with end contractions*.

Francis' formula, commonly used for a weir with end contractions, is

$$D = 3.33[b - \tfrac{1}{5}H]H^{\frac{3}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (514)$$

The end contractions are assumed to have the effect of shortening the length of the weir. For weirs without end contractions this becomes

$$D = 3.33bH^{\frac{3}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (515)$$

In this formula H is the head of the water in feet, with respect to the level of the sill, taken just above the point where the fall of surface, due to the weir, is perceptible (Fig. 90); b is the width of the weir in feet. H must be measured with great care when accuracy is desired. For this purpose a *hook gauge* is employed. This is a graduated rod, provided with a vernier reading to thousandths of a foot, whose index is a hook with point upwards. The hook is raised from underneath the surface until the point makes a slight pimple on the surface; the zero of the scale is set at the level of the sill.

Weirs are employed to measure the discharge of a stream not easily measured by the discharge of an orifice; tables of coefficients for different lengths are given in the engineering manuals. For the accurate measurement of small streams the weir is cut in a thin plate of metal, and H is carefully measured with a hook gauge. For a rough determination H may be measured as shown in Fig. 90, in which $h = H$.

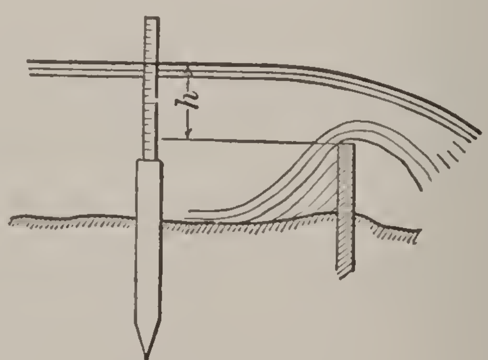


FIG. 90.

The formulas above given are for weirs in the dams of reservoirs in which the water is still, and the velocity over the weir is that due to its head above the level of the weir alone. If the weir is placed in a running stream, the velocity at the weir for the same head over the sill will be greater than in the case of still water, and the discharge will be greater than that given by formulas (514) and (515). If the velocity of the stream is measured, and the height H_1 due to that velocity is computed, the discharge over the weir is ascertained by substituting $(H+H_1)$ for H in formulas (514) and (515).

PROBLEMS.

67. What is the discharge in cubic feet per second of a weir 3 feet long under a head of 6 inches if there is no end contraction? If it is contracted at the ends?

Ans. 3.489 and 3.414 feet per second.

Flow of Water through Pipes.—In Fig. 91 let the water level in the reservoir and the diameter of the pipe both be constant

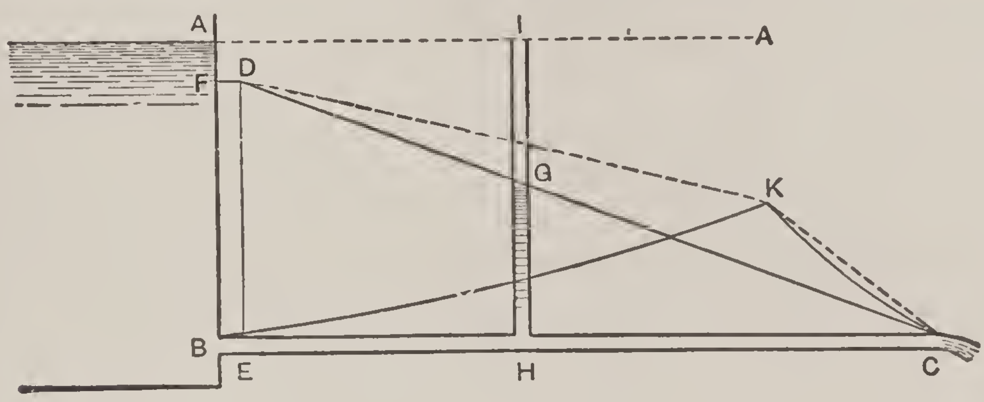


FIG. 91.

and the pipe be running full; assume that there is no resistance to the flow of water into the orifice or entrance of the pipe, and that there is no resistance in the pipe itself. The velocity of the current in the pipe will then be constant throughout the pipe, and uniform in each cross-section; its value will be

$$V^2=2gH, \quad \text{and the head which produces it } H=\frac{V^2}{2g}, \qquad (516)$$

in which H equals the head in feet, V equals the velocity in feet per second due to the head H, and g equals 32.2.

Loss of Head at Entrance.—In the above hypothesis the error was made of assuming that there was no resistance offered, and therefore no pressure required, or head expended, in forcing the water into the pipe.

The entire pipe BC may be considered as made up of a standard tube BE to the end of which is fastened a pipe of equal diameter into which the standard tube discharges. In the discussion of the standard tube it was shown that the resistance in the tube was sufficient to reduce the discharge to 0.82 of the theoretical discharge. As the tube runs full at its end E, there is no final contraction, and the coefficient of contraction must be equal to unity. Since $c=c_v c_c$, the coefficient of velocity must therefore be equal to that of discharge, and the velocity of the water as it leaves the tube BE to enter the pipe EC will be only 0.82 of the theoretical velocity. As the pipe is of uniform cross-section, this velocity will remain constant, and will be the velocity of the stream when it leaves the pipe at C.

Let v = velocity in the pipe or the velocity of discharge;
V = theoretical velocity;
h = velocity head;
h' = entrance head or head lost at entrance;

$$H=h+h'=\frac{V^2}{2g}.$$

Then for a standard tube

$$v=0.82V,$$

$$v^2=0.67V^2=\tfrac{2}{3}V^2,$$

$$V^2=\tfrac{3}{2}v^2,$$

$$h=\frac{v^2}{2g},$$

$$H=\frac{V^2}{2g}=\frac{3}{2}\frac{v^2}{2g},$$

$$h'=H-h=\frac{3}{2}\frac{v^2}{2g}-\frac{v^2}{2g}=\frac{1}{2}\frac{v^2}{2g},$$

$$\text{or}\quad h'=\tfrac{1}{2}h.$$

Hence we see that the head expended in forcing the water into and through the standard tube, which is called *the head lost at entrance or head due to influx*, is about 0.5 of the head required to produce the velocity of efflux or discharge. It follows from this that if the velocity of discharge is small, the loss of head at entrance will also be small, and conversely.

If the pipe has a bell-shaped or other inlet whose coefficient of discharge and velocity is not 0.82 but c, since $v=cV$, or $\frac{v}{c}=V$, and $H=\frac{v^2}{c^2 2g}$,

$$h'=H-h=\left(\frac{1}{c^2}-1\right)\frac{v^2}{2g}=\left(\frac{1}{c^2}-1\right)h, \quad . \quad . \quad . \quad (517)$$

for the loss of head at entrance. The expression $\left(\frac{1}{c^2}-1\right)$ may be represented by m. Equation (517) then reduces to

$$h'=H-h=\frac{mv^2}{2g}=mh. \quad . \quad . \quad . \quad . \quad . \quad . \quad (518)$$

In the above discussion, the resistance in the pipe EC has been neglected. It is clear, however, that this resistance cannot affect the value of m or the ratio of h', the head lost at entrance, to h, the head which is required to produce the velocity of discharge. This ratio is dependent solely on the coefficient of discharge of the inlet-tube.

Loss of Head Due to Friction.—In the original hypothesis a second error was made in assuming that the flow of the water met with no resistance in the pipe. As a matter of fact the molecules at and near the surface meet with considerable resistance. As the resistance occurs at the surfaces of contact, it is usually called *frictional resistance.* It is found by experiment to be governed by the following laws:

1. *The resistance varies directly with the area of the surface of contact.* It is therefore a function of the product of the length and perimeter of a pipe running full.

2. *The resistance is independent of the normal pressure on the surface of contact.* In this respect it differs from friction between solids. This peculiarity is accounted for by the incompressibility of water and lack of friction between the molecules.

3. *The resistance varies with the roughness of the surface of contact.* This roughness causes eddies and retards the flow of the molecules near the surface. The retardations affect the flow of the other layers of molecules inversely as their distance from the surface. The velocity is therefore variable in any area of cross-section.

4. *The resistance varies directly with some function of the mean velocity.* It is usually assumed that the resistance varies with the function $\frac{v^2}{2g}$, as experiments indicate that for the velocities of practice the resistance varies with the second power of v. In pipes of very small cross-section it varies with the first power of v.

Since the velocity of the current is the same at every cross-section, the force expended in overcoming the frictional resistance in the pipe must be equal to this resistance.

If R = frictional resistance, in pounds, of the pipe beyond the standard tube,
πd = inner perimeter of cross-section in feet,
l = length in feet of pipe whose resistance is R,
f' = constant depending on roughness of interior surface,
v = mean velocity in feet per second of flow at any cross-section,
$g = 32.2$ = acceleration in feet per second due to force of gravity,

we may write

$$R = \frac{\pi d l f' v^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (519)$$

If p'' = mean unit pressure, in pounds per square foot on the cross-section, required to overcome the frictional resistance of the pipe in a length l,
a = area of cross-section of pipe in square feet,
h'' = head in feet required to overcome frictional resistance in length l,
$G = 62.5$ pounds = weight of a cubic foot of water,

the force required to overcome the frictional resistance R will be

$$p''a = h''Ga.$$

Equating the expressions for the force and resistance we have

$$h''Ga = \pi d l f' \frac{v^2}{2g},$$

$$h'' = \frac{\pi d}{a} \cdot \frac{f'l}{G} \cdot \frac{v^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (520)$$

Substituting f for the constant $\frac{f'}{G}$ we have

$$h'' = \frac{\pi d}{a} \cdot \frac{f l v^2}{2g} = \frac{4}{d} \cdot fl \cdot \frac{v^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad (521)$$

and

$$f = \frac{a}{\pi d} \cdot \frac{h''}{l} \cdot \frac{2g}{v^2} = \frac{d}{4} \cdot \frac{h''}{l} \cdot \frac{2g}{v^2}. \quad . \quad . \quad . \quad (522)$$

In these expressions

f = the coefficient of friction and is a constant whose value must be determined by experiment.

$\frac{a}{\pi d} = \frac{d}{4}$ = ratio of the area of cross-section of interior of pipe to its perimeter; this is called the *hydraulic mean radius*, or the *mean radius*. It is a constant for any given pipe.

$\frac{h''}{l}$ = ratio of head required to overcome the frictional resistance in a pipe to the length of the pipe. It is called the *hydraulic mean gradient* or the *hydraulic gradient.*

Many experiments have been made to determine accurately the value of the coefficient f. It is found that the theoretical expression for it is not sufficiently accurate to admit of the use of a single value for all cases. It varies with the material of the pipe, its condition of cleanliness, its diameter, and the velocity of flow. In each case it is therefore necessary to select from the tables found in engineering manuals the empirical value which corresponds most nearly to the given conditions. Care must be exercised in taking out the proper value, as authors employ different expressions for it.

From equation (522) we may deduce the value of f:

$$f=\frac{2g}{4v^2}\cdot\frac{dh''}{l},$$

$$f_1=\frac{2g}{v^2}\cdot\frac{dh''}{l},$$

$$f_2=\frac{dh''}{v^2l}.$$

Each of these is employed by some author as the coefficient of friction. For their relative values we have

$$f=\tfrac{1}{4}f_1=16.1\,f_2 \quad \text{and} \quad f_1=64.4\,f_2.$$

For approximate results the following values may be employed:

	f	f_1	f_2
Smooth pipes......	0.006	0.024	0.0004
Rough pipes.......	0.012	0.048	0.0008

These values are somewhat greater than those given in the tables.

Where greater accuracy is desired the following formula has been recommended:

For smooth pipes $f=.005\left(1+\frac{1}{12d}\right)$, d=diameter in feet.

For rough pipes $f=.01\left(1+\frac{1}{12d}\right)$, d=diameter in feet.

Velocity Head.—In the above discussion it has been assumed that the water flowed through the pipe at a uniform velocity of v feet per second. A fraction of the total head must be expended in producing this velocity.

If v=the velocity and h=the head, since gravity is the only force acting on the water, we have

$$h=\frac{v^2}{2g}.$$

This head h is called the *velocity head.*

Fanning gives the following table showing the fractions of the total head expended in overcoming the various resistances in a pipe one foot in diameter when the inlet is a standard tube:

Length of pipe in feet.	5	50	100	1000	10,000
Velocities, feet per second. .	63.46	51.11	43.11	17.38	5.39
Head expended at entrance.	31.58	20.49	14.57	2.37	0.23
Head expended in overcoming friction.	5.88	38.94	56.57	92.94	99.33
Velocity head.	62.54	40.57	28.86	4.69	0.44
Total head, H.	100.00	100.00	100.00	100.00	100.00

Formulas.—Since the total head, H, must be equal to the entrance head, plus the velocity head, plus the friction head, we may write the equation

$$H=h'+h+h''=\left(\frac{1}{c^2}-1\right)\frac{v^2}{2g}+\frac{v^2}{2g}+\frac{4flv^2}{2dg}=\left((m+1)+\frac{4fl}{d}\right)\frac{v^2}{2g}, \quad (523)$$

or

$$v=\sqrt{\frac{2gH}{m+1+\frac{4fl}{d}}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (524)$$

The coefficient m is equal to 0.5 if the entrance is in the form of a standard tube, and 0.08 if in the form of a bell mouth which offers little resistance.

As the standard tube is the common form, it will be substituted in the formulas (523) and (524). These formulas then become

$$H=\frac{\left(1.5+\frac{4fl}{d}\right)v^2}{2g}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (525)$$

$$v=\sqrt{\frac{2gH}{1.5+\frac{4fl}{d}}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (526)$$

Since the discharge is equal to the velocity multiplied by the area of cross-section, we may also write

$$D = av = 0.7854d^2v = 0.7854d^2\sqrt{\frac{2gH}{1.5+\frac{4fl}{d}}} = 6.303\sqrt{\frac{d^5H}{1.5d+4fl}}. \quad (527)$$

Solving with respect to d^5,

$$d^5 = \frac{1}{(6.303)^2}\frac{(1.5d+4fl)D^2}{H}, \quad . \quad . \quad . \quad . \quad (528)$$

$$d = 0.4788\sqrt[5]{(1.5d+4fl)\frac{D^2}{H}}. \quad . \quad . \quad . \quad . \quad (529)$$

To solve this equation a value of d must be assumed for the term under the radical sign. The true value of d is obtained when its value determined by solving the equation is equal to the assumed value.

Long Pipes.—Fanning's table, given above, shows that as the length of the pipe is increased, more and more of the total head H is expended in overcoming the frictional resistance, and less and less is expended at the entrance and in giving velocity. As the velocity in a city water main is usually only a few feet a second, it is customary to consider the head H as entirely expended in overcoming the frictional resistance. Hence we may assume $H = h''$ and $L = l$, in pipes having a velocity not exceeding 10 feet a second, or a length exceeding one thousand diameters.

Under this hypothesis the four equations become

$$H = \frac{4flv^2}{2gd} = 0.0621\frac{fLv^2}{d} \quad \text{or} \quad \frac{H}{L} = 0.0621\frac{fv^2}{d}, \quad . \quad . \quad (530)$$

$$v = \sqrt{\frac{2gHd}{4fL}} = 4.012\sqrt{\frac{Hd}{fL}} = \left(4.012\sqrt{\frac{d}{f}}\right)\sqrt{\frac{H}{L}}, \quad . \quad (531)$$

$$\left.\begin{aligned} D &= 0.7854d^2v = 0.7854d^2\sqrt{\frac{2gHd}{4fL}} = 3.152\sqrt{\frac{Hd^5}{fL}} \\ \text{or} \quad D^2 &= \left(9.925\frac{d^5}{f}\right)\frac{H}{L}, \end{aligned}\right\} . \quad (532)$$

$$d = 0.4788\sqrt[5]{\frac{4fLD^2}{H}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (533)$$

The practical problems of flow usually appear under one of the following forms, in which H, d, and L are in feet, v in feet per second, and D in cubic feet per second:

1.	Given d and $\frac{H}{L}$.	Required v and D.
2.	" d and v.	" $\frac{H}{L}$ and D.
3.	" d and D.	" $\frac{H}{L}$ and v.
4.	" $\frac{H}{L}$ and v.	" d and D.
5.	" $\frac{H}{L}$ and D.	" d and v.

In forms 1, 4, and 5 both H and L may be given, and in forms 2 and 3 one of the two may be given and the other required.

To solve	1	employ	formulas	(531)	and	(532).
"	" 2	"	"	(530)	"	(532).
"	" 3	"	"	(530)	"	(532).
"	" 4	"	"	(530)	"	(532).
"	" 5	"	"	(533)	"	(531).

PROBLEMS.

67. A pipe 1 foot in diameter has a hydraulic gradient of $\frac{1}{100}$; what is the velocity of flow and discharge per second if $f = 0.012$?
Ans. 3.66 feet; 2.87 cubic feet.

68. The velocity of flow in a pipe 2 feet in diameter is 3 feet a second; what is its hydraulic gradient and discharge per second if $f = 0.010$?
Ans. .0028; 9.425 cubic feet.

69. A pipe 6 inches in diameter delivers 1 cubic foot of water per second; what is the velocity of flow and the hydraulic gradient if $f = 0.012$?
Ans. 5.09 feet; 0.0386.

70. The hydraulic gradient of a pipe is $\frac{1}{100}$ and the velocity of flow 4 feet a second; what is its diameter and its discharge per second if $f = 0.010$?
Ans. 0.994 feet; 3.104 cubic feet.

71. The hydraulic gradient of a pipe is $\frac{1}{100}$ and its discharge is 3 cubic feet per second; what is the velocity of flow and its diameter if $f = 0.012$?
Ans. 1.017 feet; 3.693 cubic feet.

Hydraulic Grade Line.—In Fig. 92 let BC be a straight horizontal pipe of uniform diameter of which BE is the entrance or standard tube.

Let $AB=H$;

$$AF=h'+h=(m+1)\frac{v^2}{2g};$$

$$DE=h''=\frac{4flv^2}{2gd};$$

$BC=L$;

$EC=l$;

v = mean velocity in any cross-section;

f = coefficient of friction of pipe.

Transposing equation (521) we have

$$\frac{h''}{l}=\frac{4fv^2}{2gd}, \quad \ldots \ldots \quad (534)$$

in which $\frac{h''}{l}$ is the mean hydraulic gradient, or the tangent of the angle made by the *hydraulic grade line* or *virtual slope* with horizontal line. In Fig. 92, therefore, the line DC is the hydraulic grade line.

Since the head AF is expended in forcing the water into the pipe and in giving it a velocity v at E, the remaining head

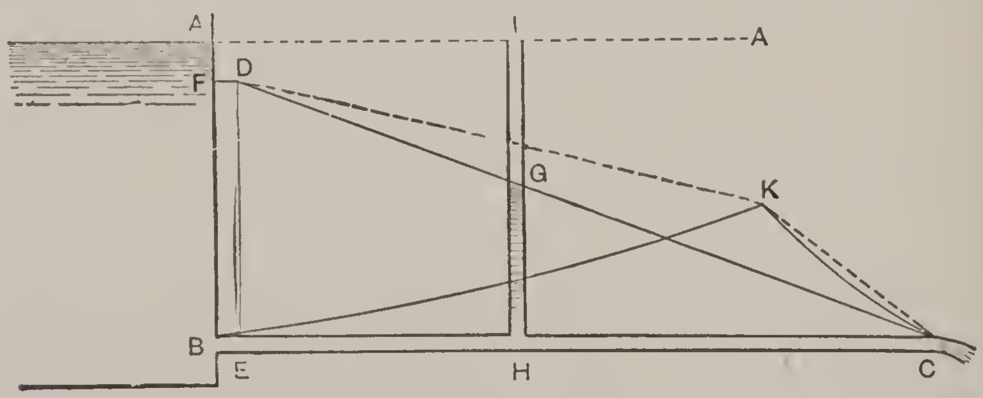


FIG. 92.

at E is DE. Hence if a tube, open at the top, be inserted in the top of the pipe at E, the water in that tube will stand at the level D. This height or head DE which measures the pressure at E in a pipe in which the water is in motion is called the *hydraulic or pressure head.*

From equation (534) we see that if f, v, and d are constant, the value of h'' will vary directly with l; that is, to overcome the resistance in any fraction of l will require the expenditure of the same fraction of h''. Hence we may determine the hydraulic or pressure head h_1'' at any point, as H, from the proportion

$$h_1'' : h'' \text{ or } DE :: HC : CE \text{ or } l.$$

Hence $h_1''=GH$. The loss of head between E and H is therefore $h''-h_1''$ or $DE-GH$.

If a tube, open at the top, be inserted in the top of the pipe at H, the level of the water will therefore be at G, the intersection of DC and GH.

As the same is true of every other point of the pipe EC, it follows that no water can be delivered at an elevation above DC while the water is flowing through BC with a velocity of v.

Tubes inserted, as above explained, to measure the hydraulic or pressure head are called *piezometric tubes* or *piezometers*. The *hydraulic grade line* or *virtual slope* may then be defined as *the line connecting the water level of a series of piezometric tubes inserted in a pipe through which water is flowing.*

If the outlet at C is closed, the water will at once rise in each piezometric tube until it reaches the level A; the water will cease to flow in any portion of the pipe BC. The head $HI=H$ at any point, as H, is called the *hydrostatic* or *static head.*

If DE is very large as compared with AF, as is the case in long pipes, AB and DE may be assumed to coincide and AC may be taken as the hydraulic grade line. This is the usual custom in practice.

In equation (534) l is the actual length of the pipe in feet; if, therefore, the pipe is either curved, or inclined to the horizontal plane, we must make BC equal to the actual length of the pipe. In practice, however, the length of the horizontal projection of the pipe between its inlet and outlet is so nearly equal to its true length that the ratio $\frac{AB}{BC}$ is not appreciably affected by substituting its horizontal projection between inlet and outlet for the true length.

If equation (534 is solved with respect to v, we have

$$v^2=\frac{2gd}{4f}\cdot\frac{h''}{l}. \quad \dots\dots\dots \quad (535)$$

Since the discharge per second is equal to the velocity multiplied by the area of cross-section, we see that all pipes of the same diameter and the same coefficient of friction will deliver the same volume of water provided they have the same hydraulic grade line. If AC be considered the grade line, and the length of the pipe does not differ materially from BC, then the velocity and the discharge will remain constant whatever be the position of the pipe, so long as the outlet is at C, the inlet below A, and the whole pipe lies below AC. The pressure in the pipe at any point will, however, vary with its depth below the hydraulic grade line.

If the pipe is laid on the line BKC, Fig. 92, it must be treated as if made of two pipes united at K. The grade line of the part BK is DK or AK, which is higher than that of the pipe BC; hence it will deliver water at a higher elevation, but the velocity and the discharge will be less. The hydraulic grade line of the part KC is higher and steeper than that of BC, and it can deliver water at a higher elevation and in greater quantity per second than BC, provided it runs full. However, as the pipe BK delivers less per second than BC, the part KC will not run full, and hence the pressure or hydraulic head in that part of the pipe will be zero and the water would not rise in the piezometric tubes. If, however, the diameter of the pipe BK be sufficiently increased, while that of KC remains the same, the pipe KC will eventually run full and the pressure in the pipe will be represented by the line KC, and the velocity and discharge will both be greater than the velocity and discharge of BC.

If the pipe is laid along the line AC, the discharge is the same as the discharge from BC, but there is no pressure at any point of the pipe, and an open conduit could be substituted for the pipe.

The actual pressure in a pipe laid along the hydraulic grade line is p_0, the pressure of the atmosphere; hence if a closed pipe at any point passes above this line, the pressure at that point will be less than the atmospheric pressure, air will be disengaged from the water and collect at the high point and interrupt the continuity of flow.

Other Losses of Head.—In the discussions above given it has been assumed that the level of water in the reservoir is constant. This is not usually the case. The total head H is therefore variable, and in all practical problems it is usually its *minimum* value which is used. This is the difference of level between the inlet and the outlet of the pipe.

The only loss of head that we have considered in the pipe itself is the loss due to friction, and this is the only one that need be considered if the pipe is straight and of uniform diameter.

If the pipe has a sharp bend, however, a certain fraction of the head is expended in forcing the water around the bend. This loss is placed under the usual form

$$h''' = n\frac{v^2}{2g}, \quad \dots \dots \dots \quad (536)$$

in which n is a coefficient whose value is determined by experiment.

If the area of cross-section of the pipe is changed, the relation between the pressure heads and velocities before and after the change is given by the equation

$$h = \frac{v_1^2 - v_2^2}{2g} - (h_2 - h_1), \quad \dots \dots \quad (537)$$

in which h = loss of total head;

$\frac{v_1^2 - v_2^2}{2g}$ = loss of velocity head;

$h_2 - h_1$ = gain of pressure head.

From this expression h can be determined, when we can measure v_1, v_2, h_1, and h_2.

For the loss of head due to the eddies formed at a section of sudden enlargement the following formula has been deduced from (537):

$$h_{\text{IV}} = \left(\frac{a_2}{a_1} - 1\right)^2 \frac{v_2^2}{2g} = m''\frac{v^2}{2g}, \quad \dots \dots \quad (538)$$

in which h_{IV} = loss of head;

a_1 = area of cross-section before enlargement;

a_2 = " " " after " ;

v_2 = velocity after enlargement;

m'' = a coefficient.

For the loss of head due to a sudden contraction a similar formula is employed:

$$h_{v}=\left(\frac{a_2}{a'}-1\right)^2\frac{v_2{}^2}{2g}=\left(\frac{1}{c_a}-1\right)^2\frac{v^2}{2g}=m'''\frac{v^2}{2g}, \quad . \quad . \quad (539)$$

in which h_{v} = loss of head;
a' = area of contracted vein after passing from the larger into the smaller pipe;
a_2 = area of cross-section of smaller pipe;
c_a = coefficient of contraction;
m''' = a coefficient.

The value of c_a depends on the ratio of the area of cross-section of the pipe before and after contraction.

A fraction of the head is also lost whenever the cross-section of the pipe is diminished by the partial closing of a valve. The loss due to this cause is also put in the general form and the value of the coefficient determined by experiment:

$$h_{v1}=m^{IV}\frac{v^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (540)$$

Pipes of Varying Diameters.—In the pipe heretofore considered the diameter has been constant throughout. In practice, however, this is not usually the case; the diameter of a water-main decreases as the draft on it decreases, and it is therefore made of a series of lengths each having its own diameter. The lengths are connected by conical pipes called *reducers*, which make the change in the cross-section of the stream at each reduction a gradual one, so that discharge in a unit of time will be the same at every cross-section.

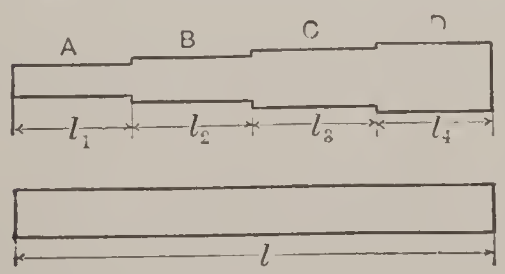


FIG. 93.

To find a pipe of uniform diameter having, under the same head, the same discharge, in Fig. 93, let

$l_1,\ l_2,\ l_3,\ l_4$ = lengths of sections A, B, C, and D;
$d_1,\ d_2,\ d_3,\ d_4$ = diameters of sections A, B, C, and D;
$v_1,\ v_2,\ v_3,\ v_4$ = velocities of sections A, B, C, and D;
$h_1,\ h_2,\ h_3,\ h_4$ = heads required to overcome friction in A, B, C, and D;
$l,\ d,\ v,\ h$ = corresponding dimensions of the pipe of uniform diameter.

Under the assumption that the entire head is expended in overcoming friction, we have

$$h = h_1 + h_2 + h_3 + h_4,$$

$$\therefore\ \frac{4fl}{d}\frac{v^2}{2g} = \frac{4f_1l_1}{d_1}\frac{v_1^2}{2g} + \frac{4f_2l_2}{d_2}\frac{v_2^2}{2g} + \frac{4f_3l_3}{d_3}\frac{v_3^2}{2g} + \frac{4f_4l_4}{d_4}\frac{v_4^2}{2g}. \quad (541)$$

Since the discharge of each section is the same and is equal to the discharge of the uniform pipe, we have

$$\frac{\pi}{4}d^2v = \frac{\pi}{4}d_1^2v_1 = \frac{\pi}{4}d_2^2v_2 = \frac{\pi}{4}d_3^2v_3 = \frac{\pi}{4}d_4^2v_4. \quad . \quad . \quad (542)$$

Hence

$$v_1 = v\frac{d^2}{d_1^2}, \quad v_2 = v\frac{d^2}{d_2^2}, \quad v_3 = v\frac{d^2}{d_3^2}, \quad v_4 = v\frac{d^2}{d_4^2}. \quad . \quad (543)$$

If in equation (541) we assume that $f = f_1 = f_2 = f_3 = f_4$, which may ordinarily be done, we have, by dividing each term by $\frac{4f}{2g}$ and substituting for v_1, v_2, etc., the values above deduced,

$$\frac{lv^2}{d} = \frac{l_1v^2}{d_1}\left(\frac{d^2}{d_1^2}\right)^2 + \frac{l_2v^2}{d_2}\left(\frac{d^2}{d_2^2}\right)^2 + \frac{l_3v^2}{d_3}\left(\frac{d^2}{d_3^2}\right)^2 + \frac{l_4v^2}{d_4}\left(\frac{d^2}{d_4^2}\right)^2, \quad (544)$$

whence

$$l = \frac{d^5}{d_1^5}l_1 + \frac{d^5}{d_2^5}l_2 + \frac{d^5}{d_3^5}l_3 + \frac{d^5}{d_4^5}l_4, \quad . \quad . \quad . \quad . \quad (545)$$

$$\frac{l}{d^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5} + \frac{l_4}{d_4^5}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (546)$$

from which either l or d can be deduced if all the other quantities in the equation are known.

PROBLEMS.

72. A pipe 2000 feet long is made of two equal lengths; one has a diameter of 8, and the other of 6 inches. What is the discharge per second under a head of 20 feet? $f=0.01$.

Ans. 0.71 cubic feet.

73. Construct the hydraulic grade line in the above problem.

Ans. $h_1=3.854$, $h_2=16.145$.

Flow of Water in Open Channels.

Assume that water is flowing in the channel shown in Fig. 94 at a uniform velocity and with a constant cross-section; or in other terms that its *régime* is permanent. If we consider a small

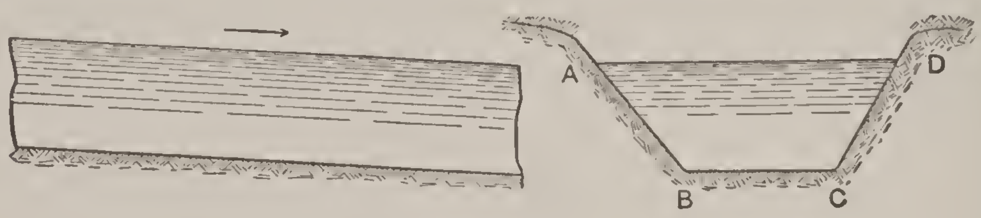


Fig. 94.

volume of water included between two parallel planes of cross-section, it is evident, since the terminal sections are similar in all respects, that the pressures will be equal, and hence the only force tending to move the prism is the component of its weight which acts parallel to the surface.

The intensity of this component will be

$$\frac{62.5alh}{l},$$

in which a = area of cross-section of stream;

l = length of prism;

$\frac{h}{l}$ = sine of angle of slope of surface.

As the velocity is uniform by hypothesis, this force must be equal to the resistance which opposes the flow. This resistance, as in a pipe, is the frictional resistance, so called, at the bed and side slopes of the channel.

This resistance varies with the area of the surface, its roughness, and with some function of the velocity of the flow. In determining the resistance, the free surface of the water is omitted, as it is found by experiment that the frictional resistance along this surface may be neglected. The area of frictional contact is therefore equal to the length of the channel between the cross-sections considered, multiplied by the length of the perimeter $ABCD$; this perimeter is called the *wetted perimeter*. If the same function of the velocity be assumed as in the case of pipes, the resistance becomes

$$\frac{plf'v^2}{2g},$$

in which p=length of wetted perimeter in feet;

l=the length of prism in feet;

f'=frictional constant;

v=mean velocity of flow, or $\dfrac{\text{discharge in cu. ft.}}{\text{area of cross-section in sq. ft.}}$

g=32.2 feet per second.

Equating the moving force and the resistance we have

$$62.5al\frac{h}{l}=\frac{plf'v^2}{2g}. \qquad (547)$$

Making $f=\dfrac{f'}{62.5}$ we have

$$\frac{fv^2}{2g}=\frac{ah}{pl}, \qquad (548)$$

in which f=coefficient of friction;

$\dfrac{a}{p}=R$=hydraulic mean radius;

$\dfrac{h}{l}=S$=sine of the slope of the surface.

Hence

$$v=\sqrt{\frac{2g}{f}}\sqrt{RS}, \quad \text{or} \quad v=c\sqrt{RS}. \quad . \quad . \quad . \quad (549)$$

This last equation is usually called the Chézy formula and is the basis of those since devised. As originally employed a constant value was given to c.

In an elaborate series of experiments undertaken by the French engineers Darcy and Bazin about the middle of the nineteenth century, it was soon developed that c was not constant, but its value was greatly affected by the character of the walls of the canal. Also that in a channel in which $\sqrt{S}$ was constant, c did not vary exactly with the ratio $\frac{v}{\sqrt{R}}$.

The formula deduced by Bazin from these experiments was

$$v=\sqrt{\frac{1}{\alpha+\frac{\beta}{R}}}\sqrt{RS}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (550)$$

in which α and β are constants whose value depends on the character of the walls of the canal. For English units these values are:

	α	β
1. Cement and carefully planed wood	.000046	.0000045
2. Smooth ashlar, brick, unplaned wood	.000058	.0000133
3. Rubble masonry	.000073	.0000600
4. Earth	.000085	.0003500
5. Channels carrying detritus and coarse gravel, mountain torrents	.000122	.0007000

These experiments were made principally in an artificial canal 2 meters wide, 1 meter deep, and about 600 meters long.

About the same time was made the elaborate study of the Mississippi River by Captain Andrew A. Humphreys and Lieutenant Henry L. Abbot of the Corps of Engineers, U. S. Army.

The formula deduced by these engineers is, in its simplest form,

$$v=\frac{K}{\sqrt[4]{S}}\sqrt{RS}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (551)$$

When the results of the American investigations were compared with the French, it was seen at once that Bazin's formulas could not be applied to a channel of the dimensions of the Mississippi, nor the formulas deduced by Humphreys and Abbot to the small channels experimented on by Darcy and Bazin.

The theory of the flow of water in open chanels was next made the subject of careful study by two Swiss engineers, Ganguillet and Kutter, who sought an empirical formula which would conform to the data obtained by the American and French engineers, as well as that furnished by other experimenters.

They found that the value of c varied with the following general laws:

1. It increases with R, and most rapidly when R is small.

2. It increases with the decrease in the roughness of the bottom and the walls of the channel. This effect is greatest when the velocity is small.

3. It increases with S if R is less than one meter and if the bottom and sides of the channel are smooth.

4. It decreases as S increases if R exceeds one meter, and also in small channels if the surface of the bottom and sides is very rough.

The formula deduced by them and known as Kutter's formula is the one now generally employed. It is based on Bazin's formula, but incorporates the data compiled by Humphreys and Abbot.

$$v=\left[\frac{a+\frac{l}{n}+\frac{m}{S}}{1+\left(a+\frac{m}{S}\right)\frac{n}{\sqrt{R}}}\right]\sqrt{RS}=\left(\frac{y}{1+\frac{x}{\sqrt{R}}}\right)\sqrt{RS}=c\sqrt{RS}, \quad (552)$$

in which v=mean velocity in feet per second;

R=hydraulic mean radius or hydraulic mean depth in feet;

S=sine of slope;

n = coefficient of roughness of bottom and sides;
a = constant = 41.66 in English measure and 23 in metric system;
l = constant = $\sqrt{\text{one meter}}$ = 1.81132 in English measure;
m = constant = .0028075 in English measure and .00155 in metric system.

Inserting the numerical values the formula becomes

$$v = \left[\frac{41.66 + \frac{1.81132}{n} + \frac{.0028075}{S}}{1 + \left(41.66 + \frac{.0028075}{S}\right)\frac{n}{\sqrt{R}}} \right] \sqrt{RS} = c\sqrt{RS}. \quad . \quad (553)$$

The value of the coefficient n, as determined from the results of experiments, varies with the character of the bottom and sides of the channel as follows:

Well-planed timber	0.009
Cement plaster	0.010
Same with one-third sand	0.011
Unplaned wood	0.012
Ashlar and brick	0.013
Canvas	0.015
Rubble	0.017
Canals in firm gravel	0.020
Rivers and canals in perfect order free from stones and weeds	0.025
Rivers and canals in fair condition	0.030
Rivers and canals in bad condition	0.035
Mountain torrents	0.050

As the formula is a complicated one, numerous tables and graphic diagrams have been computed and inserted in engineering manuals giving the value of c corresponding to channels of different materials, different slopes, and different mean hydraulic depths. In all ordinary computations these tables are used. (See Trautwine's "Engineers' Pocket Book.")

Having determined the velocity, the discharge is ascertained from the formula

$$D = av. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (554)$$

in which a = area of cross-section in square feet;
v = velocity in feet per second.

Kutter's formula is applicable to the flow in pipes as well as to the flow in open channels.

PROBLEMS.

74. What is the velocity of flow in a semi-circular brick conduit flowing full when the slope is $\frac{1}{100}$ and the radius of the conduit is 5 feet? By Bazin's formula? By Kutter's formula?

Ans. 19.87 feet; 21.31 feet.

75. What is the discharge per second by each formula?

Ans. 780.3 cubic feet; 836.85 cubic feet.

Direct Measurement of Velocity.—The velocity at any point in the cross-section of a stream may be determined by direct measurement. The three methods ordinarily employed are by means of *floats*, *current-meters*, or *Pitot's tubes*.

The stretch selected for making the measurement is one in which the slope of the surface is uniform; the form of cross-section is constant, and symmetrical with respect to a vertical line through the axis of the stream; and the axis of the stream is a right line.

To measure the velocity by means of floats, two parallel lines of cross-section, from one to two hundred feet apart, are marked by stretching a cord across the stream, or by placing two poles in line, on either bank, at each cross-section. The time which is required for each float to pass from the upper to the lower cross-section is observed and recorded.

If the surface velocity is desired, some material is employed whose specific gravity is only little less than water. It will therefore be almost wholly submerged and offer little surface for the action of the wind or air.

If the velocity at any point below the surface is desired, a double float is employed. This consists of a surface float which offers little resistance to movement through the water, and a submerged float which has a large surface exposed to the action of the current. The lower float is in the form of a cylinder or globe, and is connected to the surface float by a thin wire.

If the mean velocity in any vertical plane is desired, the float is in the form of a weighted pole which extends from the surface almost to the bottom.

The current-meter has a wheel of some form which is revolved by the current; the number of revolutions are automatically recorded. The wheel may be attached to a pole and thus lowered to any point in the plane of cross-section. The current-meter is rated by moving it at uniform rates of speed through still water.

Pitot's tubes as modified by Darcy consist of two vertical glass tubes with horizontal arms at the lower extremities; these horizontal arms are fixed at right angles to each other. Each horizontal arm is so tapered as to leave only a small inlet for the water. In each tube near the bottom is a valve which may be closed when it has been lowered to the desired level. In using the instrument it is lowered into the water with one of the horizontal arms pointing up-stream. The valves are closed and the instrument is raised so that the difference of level of the water in the two tubes may be read on a vertical scale. The velocity at the desired depth is deduced from the formula

$$v=0.84\sqrt{2gh}, \quad \ldots \ldots \quad (555)$$

in which h = difference of level of the water in the two arms.

Velocity Contours.—A velocity contour in any plane of cross-section of a stream is a line formed of points whose velocity is the same. In a circular pipe flowing under pressure the maximum velocity will be at the axis and the velocity contours will be concentric circles. In an open channel the point of maximum velocity is about $\frac{3}{10}$ the depth below the surface; some of the velocity contours will enclose this point.

Discharge.—If a sufficient number of observations are made in any plane of cross-section to plot its velocity contours, the discharge may be obtained by the formula

$$D=Av+A'v'+A''v''+\text{etc.}, \quad \ldots \ldots \quad (556)$$

in which A, A', A'', etc. = the areas between the successive velocity contours;

v, v', v'', etc. = the mean velocities of those areas, or approximately the velocities midway between the contours.

In gauging a river it is customary to divide the cross-section by vertical lines into a number of sections of equal width. Practically this may be done by attaching tags to the cord which is stretched across the stream, by driving poles into the bottom, by anchoring floats, or by establishing points in the plane of cross-section by intersecting range lines fixed by poles on shore. The mean velocity of each section is ascertained by floating a pole midway between the points established, by measuring the velocity in the same vertical plane midway between the surface and bottom, or by measuring the surface velocity in the same vertical plane and assuming that the mean velocity is 0.9 the surface velocity.

The area of cross-section of a stream is ascertained by making soundings at regular intervals with a pole or line. The depth and position of each sounding are plotted on cross-section paper to a convenient scale. This facilitates the computation of the areas of the sections into which it may be divided.

The discharge is then obtained by substituting the areas and mean velocities in formula (556).

When only approximate results are desired it is assumed that the mean velocity of the entire cross-section of a stream is about three-fourths of the maximum surface velocity.

CHAPTER XVIII.

TIMBER.

TIMBER is wood of a quality and size suitable for engineering construction. The abundance and cheapness of timber in the United States, its strength, lightness, and durability, and the ease with which it may be procured and worked, make it one of the most valuable of engineering materials.

The value of any variety of timber depends upon *strength*, *durability*, *cost*, and, for interior work, *appearance*. All of these qualities are more or less dependent upon the physical structure of the tree itself.

Structure.—The timber trees of common use in building construction are *exogenous*, that is, they increase in size by the addition of rings of annual growth which are formed immediately under the bark. The palms and bamboo are the principal exception to this rule, and they are used only where other woods are too expensive. They lack the strength and stiffness essential in large structures. In carpentry the exogenous trees are classified as *soft* and *hard* woods, the former designation being applied to *coniferous* trees, and the latter to *broad-leaved* trees.

Coniferous Trees.—The typical coniferous trees are the *pines*, which are again divided into soft and hard pines. The term soft is applied to white pine, and the term hard pine to all other varieties. *White* or *soft pines* of large size are found along the northern boundaries and along the Atlantic coast of the United States, and in Canada. It was formerly the most important timber in the United States, but is now so scarce that it is used only for finishing. It is white, soft, straight-grained, light in weight, and easily worked; it does not warp in seasoning.

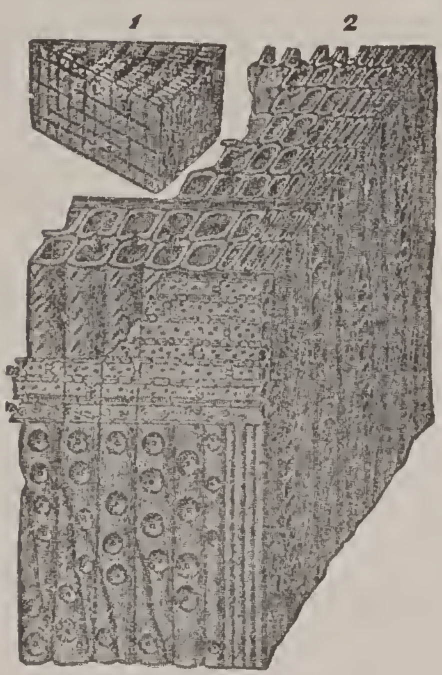
The *white* and *red cedar*, the *fir*, and the *spruce* are medium to large-sized trees whose woods have qualities similar to those of soft pine.

The *yellow pine*, which is a large tree and the strongest and hardest of the hard pines, is found along the Atlantic and Gulf coasts. It is dark in color, fine-grained, resinous, and inferior only to the best broad-leaved trees in strength and stiffness. It may be employed wherever large timbers are required, and is also used for interior finishings and floors. The *bull-pine* of the Far West and the *loblolly-pine* of the South are inferior pines of the same class. *Larch* or *tamarack* and *bastard spruce* are medium to large-sized trees whose woods have qualities similar to those of the inferior hard pines. *Cypress* and *hemlock* are also coniferous trees used to a limited extent in building construction. *Douglass fir* or *Oregon pine* is one of the principal building materials on the Pacific coast and is being introduced in the East. Its qualities are similar to those of yellow pine.

If a sector of a full-grown pine-tree is carefully examined, at the center will be found a small pith; then a great number of concentric rings, varying in width and spacing; and finally an envelope of bark. The rings are alternately light and dark one light and one dark ring representing a year's growth; the light wood is *spring* growth and is comparatively soft and weak; the dark ring is *summer* growth and is dense and strong. The strength of the wood may therefore be measured by the ratio of summer to spring wood in a unit of volume. In a cross-section, the rings of summer wood are narrow near the pith and near the bark; in longitudinal section the rings decrease in thickness from the ground to the top; hence the strongest timber will come from the lower part of the tree, midway between the pith and the bark. The average amount of summer wood is 24 to 40 per cent of the entire tree.

The *sap-wood* is a zone of light, weak wood, thirty or more rings wide, next to the bark; the outer portion of it is the growing part of the tree. The *heart-wood* is the inner and darker portion of the sector and has no part in the growth of the tree; it is much stronger and denser than the sap-wood. Heart-wood results from the gradual change of sap-wood due to the infiltration of chemical substances from the sap. The proportion of heart-wood depends upon the age of the tree, forming about, 60 per cent of an old, long-leaf pine.

If the sections of a coniferous tree (Fig. 95) are examined under a microscope, the wood will be found to consist of a number of parallel vertical tubes, or cells, of wood fiber, called *tracheids*, arranged in radial lines. The cells are from $\frac{1}{20}$ to $\frac{1}{50}$ of an inch in length, and from $\frac{1}{50}$ to $\frac{1}{100}$ of these dimensions in diameter. They are closed at the ends, the end walls being thin in comparison with the side walls. In the summer wood the cells are flattened and have much thicker walls than in the spring wood; the increased thickness and flattening account for the greater strength of the summer wood. The summer wood is, however, somewhat weakened by *resin-ducts* which appear on the cross-section in grayish spots. At right angles to the tracheids are found groups of smaller horizontal tubes, called *pith*, or *medullary, rays;* among the pith-rays are also found horizontal resin-ducts which are smaller than the vertical ones.



Wood of Spruce.
FIG. 95.*

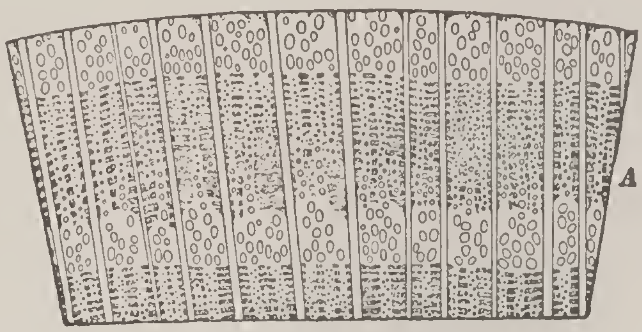
As the coniferous trees are composed almost entirely of the vertical cells or fibers with few pith-rays, timber from them is uniform in structure and is easily worked and split.

Broad-leaved Trees.—*Oak* is the best of this class of trees. Although porous and of coarse texture, it is heavy, hard, strong, and tough; the sap-wood is whitish, the heart-wood brown to reddish brown. It shrinks and checks badly, giving trouble in seasoning, but is durable and only slightly attacked by insects. Although there are perhaps twenty different varieties of oak, the market product is generally classified as *live*, *white*, and *red* oak. The live-oak possesses the qualities above given to the highest degree, but is not widely distributed, nor can timbers of great length be obtained from this variety. The white oaks are widely distributed, grow to a large size, and are therefore the most important of the oaks. Red oak is inferior to white oak, in being of a coarser texture, more porous, more brittle, less durable, and more troublesome in seasoning. The trees are also widely distributed and grow to a large size.

* Figures from Bulletin No. 10, on Timber, U. S. Dept. Agriculture.

The other important broad-leaved trees are the *ash*, *basswood*, *beech*, *birch*, *butternut*, *catalpa*, *cherry*, *chestnut*, *elm*, *gum*, *hickory*, *locust*, *maple*, *poplar*, *sycamore*, and *walnut*. As a rule these woods are of local importance, or their employment is confined to a limited field in engineering structures.

If a sector of an oak-tree is examined, it will be found to have a small pith, rings of annual growth composed of spring and



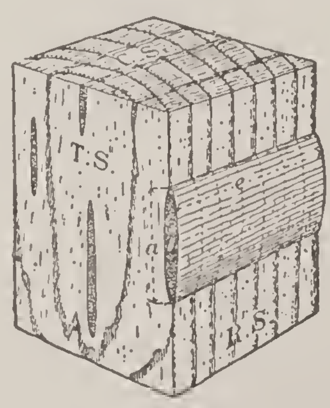
Cross-section of Oak.

FIG. 96.

summer wood, sap-wood and heart-wood. The spring wood, however, contains large pores separated by a porous tissue which with smaller pores extend into and sometimes through the adjoining summer wood (Fig. 96). The strength of a piece of oak may be measured by the ratio of its solid to its porous area in any section. To the pores are largely due the *patterns* which are found in oak; these patterns are more conspicuous in the

Board of Oak. Top cross-section; right face, radial section: front face, tangential section.

FIG. 97.



Block of Oak, showing medullary or pith ray.

FIG. 98.

spring than in the summer wood (Fig. 97). In white oak the pores are numerous and small; in red oak fewer in number, but larger. The summer wood, excepting the tissue which runs into it from the spring wood, is dark and strong. The dark portion, divided into strands by the tissue and by the concentric lines of short, thin-walled cells, consists of thick-walled cells, or fibers, and is the chief element of strength in oak.

The pith-rays are prominent, appearing on the cross-section as grayish lines with tapering ends; and on the radial section as bands or mirrors (Figs. 97 and 98).

Physical Properties.

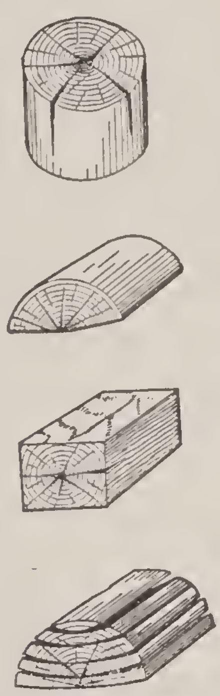
Specific Gravity.—The specific gravity of the wood-fiber itself is 1.6, but the specific gravity of a unit of volume of wood ranges from 0.3 to 1.3. This range is due to the cellular structure of wood and the amount of moisture in the specimen. Woods composed of thin-walled cells have the smallest specific gravity. The term *water-logged* is applied to timber whose cells are filled with sufficient moisture to destroy its flotation.

Grain.—Timber is classified as *fine-grained* when its rings of annual growth are close or it is susceptible of polish; and as *coarse-grained* when its annual rings are far apart or it is difficult to polish. In most timber the grain is parallel to the axis of the tree, but *spiral and wavy grains* are common; the last being well illustrated in *curly maple*. The surface under the bark is more or less irregular, depending upon the tendency of the tree to preserve any particular form. This tendency is marked in the maple, in which the surface depressions are small and numerous, giving the tangent boards or veneering sawed from it the beautiful figures which are found in *bird's-eye maple*.

Color.—Variations in color in wood may be due to the ratio of the summer to the spring wood, to the density of the wood-fiber, to incipient decay, or to pigments absorbed in its growth. When due to the first or second cause, the darker the color the stronger is the wood; when due to the third cause, a dark color indicates weakness; when due to the last cause, color is no indication of strength or weakness, but only of the species to which the tree belongs.

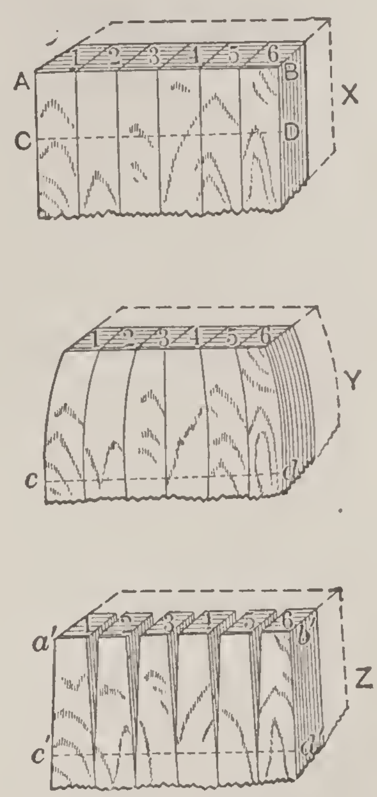
Moisture.—Water in wood occurs in three conditions: it forms over 90 per cent of the contents of the living cells; it saturates the walls of all cells; it entirely or partially fills the cavities of dead cells, fibers, and pores. The total amount of contained water remains nearly constant throughout the year, so that the time of felling in itself has but little effect upon the durability of timber. The more rapid decay of summer-felled timber is due to the fact that in summer the conditions of moisture and heat are more favorable to decay, and the wood rots before it has had time to season. In winter the cold prevents decay, and the timber seasons slowly.

Shrinkage.—Shrinkage is due to the change in the thickness of the walls of the cylindrical cells, or fibers, resulting from a loss of moisture. The amount of shrinkage increases with the thickness of the cell-walls. As the side walls of each cell are thicker than the end walls, the lateral shrinkage is greater than the longi-



Effects of Shrinkage.

FIG. 99.



Formation of Checks.

FIG. 100.

tudinal; and as the walls of the cells of the summer wood are thicker than those of the spring wood, summer wood shrinks more than spring wood. Lateral shrinkage is somewhat checked and longitudinal shrinkage increased by the pith-rays, whose direction is perpendicular to that of the main cells of the timber. A piece of wood in drying is therefore subjected to severe stresses in perpendicular planes which cause cracks or *checks*. The radial shrinkage will be less and more uniform than the tangential, since the summer and spring woods alternate in the former direction and are continuous in the latter direction. The latter causes the radial cracks or *permanent checks* and the warping of tangential boards (Fig. 99). The more rapid drying out and shrinking of the ends and sides of timbers cause *temporary checks*, which gradually disappear as the timber becomes thoroughly seasoned. The former are called *end checks* and are very common. (Fig. 100.)

Mechanical Properties.

The value of any variety of wood in building construction depends largely upon the following mechanical properties.

Strength.—From the fibrous structure of wood itself and from its growth in annual rings, it is apparent that it will not offer the same resistance to the various kinds of straining forces. It offers the greatest resistance to tensile, bending, and compressive forces which tend to elongate or crush the fibers. Less resistance is offered to a shearing force which acts in a plane perpendicular to the fibers or grain, and least of all to a shearing force which acts parallel to the fibers or grain. From the complex nature of the structure of wood, a considerable variation in strength must be expected in different pieces cut from the same tree, even if subjected to the same straining forces. Uniformity of strength can be secured only by careful selection.

The table on page 333 gives the breaking and safe unit working stresses of oak, white pine or spruce, and yellow pine.

From experiments on *bamboo* Professor Johnson draws the conclusion that bamboo in its natural form under a bending force is twice as strong as oak, weight for weight, when the oak is taken in specimens of solid square cross-section. The same holds true for crushing strength parallel to the fibers.

Stiffness under Bending Forces.—A certain degree of stiffness under bending forces is essential in timbers exposed to considerable stress, to prevent the undue deformation of the structure. The stiffness of wood ordinarily varies directly with its weight and strength to resist flexure, and inversely with the amount of moisture in it.

			Breaking Unit Stress.	Safe Unit Stress.	Factor of Safety.
Tension	With grain	Oak	10,000	1,000	10
		White pine	7,000	700	10
		Yellow pine	12,000	1,200	10
	Across grain	Oak	2,000	200	10
		White pine	500	50	10
		Yellow pine	600	60	10
Compression	With grain	Oak	4,500	900	5
		White pine	3,500	700	5
		Yellow pine	5,000	1,000	5
	Across grain	Oak	2,000	500	4
		White pine	800	200	4
		Yellow pine	1,400	350	4
Shearing	With grain	Oak	800	200	4
		White pine	400	100	4
		Yellow pine	600	150	4
	Across grain	Oak	4,000	1,000	4
		White pine	2,000	500	4
		Yellow pine	5,000	1,250	4
Bending		Oak	6,000	1,000	6
		White pine	4,200	700	6
		Yellow pine	7,200	1,200	6
Torsion		Oak	1,800	450	4
		White pine	....	.....	4
		Yellow pine	1,200	300	4
Modulus of longitudinal elasticity		Oak	900,000		
		White pine	800,000		
		Yellow pine	1,200,000		

Hardness.—Hardness measures the ability of the wood to resist abrasion and indentation, and is an important quality in floors, tenons, mortises, etc. The hardness of wood also varies directly with its weight, and inversely with the amount of moisture in it. The maximum hardness of any specimen is developed by placing the fibers in the direction of the applied force.

Flexibility.—Flexibility is the property of bending without rupture. This property of wood is developed by moistening, by steaming, and by reducing the material to a form in which one or both of the dimensions of the cross-section are small as compared with the length.

Toughness.—Toughness measures the capacity of the wood to resist shock; it is a function of both strength and flexibility.

Defects.

Defects in timber are due to peculiarities of growth and treatment, to the action of moisture, and to the action of insects.

Defects due to Growth and Treatment.—These are *wind-shakes*, circular cracks separating the annual rings which are caused by the twisting action of the wind; *belted timber*, a term applied to timber which has been killed by girdling before felling; *knots*, defects caused by the growth of the wood at limbs which impair the ease of working and strength; *twists*, a term applied to timber having a spiral grain; *heart-shakes*, splits in the center of the tree; *star-shakes*, splits radiating from the center; *checks*, cracks due to improper seasoning; *rind-gall*, a swelling caused by the growth of layers over the place where a branch has been removed; *waney timber*, a term applied to boards or beams which are not of uniform cross-section. This defect is found in boards cut from the surface of the tree.

Defects due to Moisture.—The defects due to moisture are *dry* and *wet rot*. *Dry rot* is fermentation of the sap and decay of the surrounding timber caused by bacteria. Dry rot attacks only felled timber which has not been thoroughly seasoned, and is encouraged by dampness, poor ventilation, or premature painting. *Wet rot* is the slow oxidation of the wood-fiber under the action of air and moisture. It may attack either live or dead timber, and is common in felled timber exposed to alternate wetness and dryness. Decay is promoted by moderate heat and checked by cold.

Defects due to Insects.—Timber immersed in the waters of the ocean in the torrid or temperate zones is soon destroyed by marine insects. These insects generally belong to one of two classes—the *teredo navalis* or the *limnoria terebrans*. They are found in both warm and cold climates, being more plentiful in the former; they will not work in foul or muddy water, or in a temperature below 40° F. They confine their operations to a zone extending from the bottom to a point somewhat above the low-water line. The teredo is first deposited upon the timber in the shape of an egg from which emerges a small worm. This worm enters the timber through a pin-hole, but as it proceeds into the timber it grows, and the hole is enlarged. Consequently timber attacked by the teredo shows no external evidences of its inroads unless examined closely. The teredo lines its hole with calcium carbonate, and therefore prefers a calcareous shore. The limnoria resembles a wood-louse; it prefers a silicious shore and soft wood; it does not bore, but destroys the wood by eating gradually from the surface inwards. Of land insects *wood-worms*, *borer-bees*, and *ants* are the most destructive; the latter are very common in tropical and subtropical countries.

Durability and Preservation of Timber.

If protected from destroying insects, timber may be kept unimpaired for centuries by placing it in a dry, well-ventilated place, or by immersing it completely in fresh or salt water. Timber placed in a damp or poorly ventilated place, or exposed to the weather without protection, will decay rapidly. Since the decay is caused primarily by the moisture and albuminous sap in the timber, the simplest method of preserving the timber is to remove this moisture and then cover the wood with an impervious coating of paint or other substance. This coating will also protect the wood from the attack of insects. If the coating is applied before the wood is seasoned, it will confine the moisture and dry rot will result.

Seasoning.—Seasoning timber consists in expelling the moisture by natural or artificial means. The rapidity with which the moisture may be safely removed depends upon the size, shape, and structure of the timber; the ends of sticks always dry much more rapidly than the interior. It is impossible to remove all water, and even if it were possible, the most thoroughly seasoned timber will at once begin to take up water as soon as it is exposed to moisture.

Natural seasoning consists in piling the timber under cover, on skids, with strips between successive layers, so that the air may circulate freely and dry out the moisture. The time required depends upon the kind of wood and its dimensions, varying from one to two years for soft woods, and from one and one-half to four years for hard woods. *Water-seasoning* is the removal of sap by the immersion of the timber in water for about two weeks. After removal from the water the wood must be thoroughly dried before using. *Artificial seasoning* consists in exposing the timber to a current of hot air in a drying-kiln. The temperature should not be too high; one-inch pine boards require four days at 180° to 200° F. Oak should first be air-seasoned for three to six months, and then exposed to a temperature not exceeding 150° F.; one-inch boards require ten days. Timber may also have its sap removed by *steaming;* this method prevents checking and cracking, but requires subsequent drying. The beneficial effect of seasoning as affecting strength is twofold: by shrinkage, a greater number of fibers per square inch is obtained in the plane of the cross-section, and the wood-substance itself becomes firmer and stronger. The last is the more important consideration.

Preservative Processes.—Seasoning is a protection against decay only when the seasoned timber is kept in a dry, well-ventilated place, or its surface is protected by an unbroken, impervious coating of *paint*, *varnish*, or other preservative.

A paint consists of a *base*, a *vehicle*, and a *solvent*. The *bases* commonly employed are red and brown oxide of iron, red and white lead, carbon-black, and graphite. The vehicle is usually *linseed-oil*, raw or boiled, made of flaxseed; as substitutes, *cotton-seed-* and *fish-oils* are employed. The *solvent* is usually the *spirits of turpentine*, obtained from the long-leaved pines; as substitutes *benzine* and *naphtha* are used.

Varnish is made by dissolving gum or resin in oil and turpentine or alcohol; it differs from paint in being transparent.

In engineering practice it is often necessary to employ timber under conditions in which it is impossible to protect it by a simple coating of paint. Special preservative processes have therefore been invented which render the exterior coating of paint unnecessary. They depend on one or more of the following principles: 1st. Expelling the sap and replacing it by substances of a durable nature; 2d. Expelling the sap and filling up the pores of the wood with a substance which will prevent the entrance of moisture; 3d. Saturating the timber with salts of a metallic base which will combine with the albuminous matter of the sap and make it inert; 4th. Introducing an antiseptic liquid or salt which will prevent the growth of bacteria.

The following are the most important methods in use:

Bichloride of Mercury.—This process, also called *Kyanizing*, consists in steeping the timber for several days under pressure in a solution of *bichloride of mercury* (*corrosive sublimate*); this is one of the strongest antiseptics known, and coagulates the fermentable substances. The salt is, however, a virulent poison, which is an objection to this process. A plant has been in operation, however, near Lowell, Mass., for over forty years, and no bad effects have been recorded under proper handling. Timber treated by this process lasts two or three times as long as unprotected timber. The process possesses the defect, common to nearly all, of having the sublimate gradually washed out by the action of external moisture.

Zinc chloride.—This process, also called *Burnettizing*, consists in subjecting the timber to a weak solution of *zinc chloride* under pressure, after first subjecting it to steam under partial vacuum. The sap is expelled by vaporization, and its place is taken by the chloride solution, which has a strong affinity for wood fiber and is antiseptic. The objections to the process are that the salt will wash out in service, and that the timber is rendered brittle if too large a quantity of chloride is deposited in one place. The process has been modified in the United States by the *Wellhouse*, or *zinc-tannin*, process, which consists in mixing a small quantity of glue with the chloride, and afterwards injecting a solution of tannin with a view to closing the pores of the wood with a kind of artificial leather. This modification is of so comparatively recent origin that its efficacy is as yet a matter of speculation.

These two zinc-chloride processes have been extensively used in this and foreign countries, and are without doubt successful in greatly increasing the life of timber. It is estimated that 1,500,000 railroad-ties are yearly treated by the zinc-chloride method in the United States alone.

Sulphate of Copper.—This process, also called *Boucherie's*, consists in forcing a solution of copper sulphate (blue vitriol) into the timber by hydraulic pressure. This solution drives out and replaces the sap. This process has been extensively employed in France and Germany. Its disadvantages are that the salt dissolves out, and also that an iron spike or nail is liable to decompose the sulphate, forming sulphuric acid, which attacks the wood. The durability of wood is, however, about doubled by this process.

A modification of this process known as the *Thilmany process* consists in treating the timber to a second bath, of barium chloride, forming barium sulphate, an insoluble salt, in the ends of the pores.

Creosote.—This process, in one of its forms (*Bethel's*, *Seeley's*, *Hayford's*, or *Creo-resinate*), is in many respects the best process for preserving timber, and consists in impregnating the timber with heavy oil of tar, known as *creosote*, or *dead-oil*. In *Bethel's process*, the earliest used, seasoned timber was first subjected to a partial vacuum, and the oil was then introduced at a temperature of 120°, and under pressure. *Seeley's process*, designed to permit the use of green timber, was a modification in which the timber was boiled in dead-oil for a sufficient time to expel any moisture; the hot oil was then replaced quickly by cold oil, forming a partial vacuum in the cells. The oil was forced into the wood by the difference between the external and internal pressure, by capillary action, and by the application of extraneous pressure to the liquid oil.

The *Hayford process* is a further modification designed to prevent timber from checking, which it is liable to do if subjected while green to a heat of 212° F. The timber is heated in dry air under pressure, and then at a lower temperature subjected to a partial vacuum which causes the sap vapors to be withdrawn. The dead-oil is then introduced under pressure.

The creosoting methods have been adopted with a view to preventing the loss of the preservative by exposure to moisture; the oil, being insoluble, better resists leaching out than the soluble salts. So long as the timber retains the oil it will not decay, but the creosote will wash out in time, though more slowly than the metallic salts. While giving the most satisfactory results, the cost of the most approved method (Hayford's) renders a more economical method desirable.

To render the creosote in the timber better able to resist the action of the water, experiments are now being carried on at various places with a combination of zinc chloride and creosote. A further modification of Hayford's process, known as the *creo-resinate process*, is also being employed, in which, instead of creosote, a mixture of creosote (30%) and pulverized resin (70%) is used; this, it is claimed, seals the pores with an insoluble water-proof compound.

Protection against Limnoria and Teredo.—For subaqueous work the creosote method possesses the great advantage of protecting the timber against the attack of these insects, so long as it remains thoroughly impregnated. The creosoting should, however, be thorough; no less than from 15 to 19 pounds of oil per cubic foot of wood should be injected for complete protection. The form of protection, commonly employed for piles, is a coating of thin sheets of metal, such as yellow metal, copper zinc, etc.; to prevent galvanic action these are secured by nails of the same metal. These sheets extend from about the line of extreme high tide to 4 feet below the mud line, and are usually put on over a layer of tarred felt, which increases the efficacy of the protection, and will protect the wood for a time after the metal covering is broken. The best results require that the sap-wood of the pile be removed. Protection may also be obtained by covering with terra-cotta pipe filled with concrete; by covering with layers of canvas saturated with some preservative; or by studding thickly with broad-headed nails. Palmetto is the only wood of the United States which in its natural state is immune from the attack of the teredo and limnoria. Some Philippine woods are said to possess this property.

Market Forms and Designations.

Sawing.—Two general methods of sawing are employed in the preparation of timber for the market—*bastard*, and *quarter* or *rift sawing*. In the former method the cuts are all parallel to each other; in the latter the log is first cut into quarters and then each quarter divided by cuts parallel to its bisecting radius, or parallel to its faces alternately. The sections made by bastard sawing, which is the more common and cheaper method, are *tangential* sections; those made by quarter sawing are approximately *radial* sections. Quarter-sawed lumber wears better, warps, splinters, and shrinks less, and in most hard woods produces a handsomer surface.

Products.—*Ceiling;* standard sizes are $\frac{3}{8}$ to $\frac{3}{4}$ inch thick and 4 to 6 inches wide, including tongue. It is matched, and dressed on both sides. *Flooring;* standard sizes are from 1 inch to $1\frac{1}{4}$ inches thick and 4 to 6 inches wide, including the tongue. *Stepping* embraces all sizes from 1 inch to 2 inches thick and from 7 inches up in width; *plank*, all sizes from $1\frac{1}{2}$ to 5 inches in thickness and from 7 inches up in width; *dimension timber*, all sizes from 6 inches up in thickness and 7 inches up in width. Timber is generally measured by *board measure* (B. M.), that is, by the number of superficial feet the piece would contain if it were sawed into boards one inch thick. Thicknesses less than one inch are either counted as one inch, or sold by the square foot.

Specifications.—Specifications vary with the locality, the lumber-manufacturers' associations usually having certain local standards. As a rule, each kind of lumber is divided into two or three classes according to the allowable imperfections. The terms *clear*, *prime*, and *merchantable* are employed to designate the three classes in order of value. The symbols S 1 S, 2E, or S 2 S, etc., are employed to designate that boards are to be surfaced one side and two edges, or surfaced two sides, etc. Surfacing means planing by hand or by machinery. In making specifications for lumber it is usually best to make them conform to the classification established by the lumber association which furnishes the lumber for the locality. This prevents unnecessary disputes.

Tests.—The general character of the material is determined by careful inspection. The tests for seasoning consist in carefully drying the wood in an oven; the difference in weight before and after drying gives the amount of moisture driven off. In thoroughly seasoned lumber this should not exceed 10 per cent. Tests for crushing, bending, shearing, and tensile stresses when necessary are made with testing-machines.

Joints and Pieces in Timber Construction.

Joints.—Rankine gives the following rules for the construction of the joints of a wooden frame or truss when its parts are subjected to the action of external forces:

I. Cut the joint and arrange the fastenings so as to weaken as little as possible the pieces they connect.

II. Place the abutting surfaces of a compression-joint as nearly as possible perpendicular to the action line of the pressure which it must transmit.

III. Proportion the area of the joint to the pressure which it must resist, and form and fit the surfaces accurately so that the stress is distributed uniformly.

IV. Proportion the fastenings of a tensile joint so that they may be as strong as the pieces they connect.

V. Arrange the fastenings so that the joint shall not give way by the fastenings shearing or crushing the pieces.

To this may be added Tredgold's caution:

VI. Joints should be so formed that the expansion or contraction of timber will not injure the pieces connected.

Lap-joint.—A *lap-joint* (Fig. 101) is made by overlapping the pieces and fastening them together by bolts or straps. It is a simple, strong joint for rough work. In military bridges a

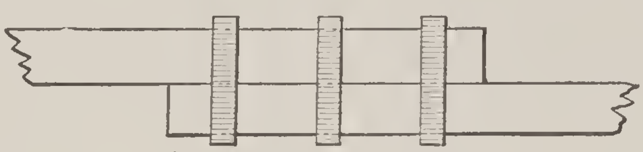


Fig. 101.

lap-joint is often made with rope-lashings instead of bolts and straps.

Butt-joint.—A *butt-joint* (Fig. 102) is one in which the ends of the pieces are cut square and abut against each other. The

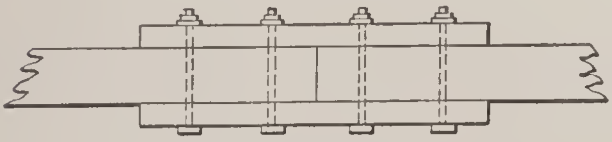


Fig. 102.

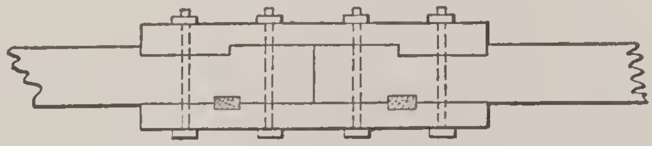


Fig. 103.

pieces are held together by *fish-plates* which overlap the joint and are bolted together through the pieces. The fish-plates may also be notched into the pieces, or have keys notched into the fish-plates and pieces, as in Fig. 103.

In order that a tensile joint like Fig. 102 shall be equally strong in all its parts, the tensile strength of the pieces, the tensile strength of the two fish-plates, the shearing strength of the bolts, and the bearing value of the bolts must all be equal. If keys are employed as in Fig. 103, the stress on the bolts, and the strength of the fish-plates and pieces, are all reduced.

Scarf-joint.—A *scarf-joint* (Fig. 104) is a lap-joint in which the overlapping parts are so reduced that their combined area at the joint is no greater than the area of cross-section of either piece. The scarf-joint may be strengthened by bolts, straps, or fish-plates, and the pieces may be notched into each other. A

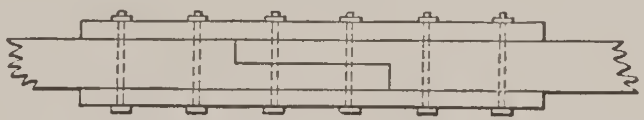


FIG. 104.

simple scarf-joint is one in which the plane of contact of the two pieces is a single inclined plane. The opening of this joint, due to the shrinkage of the pieces, is almost imperceptible.

Compression-joint.—A butt-joint, and a scarf-joint like Fig. 104, are suitable joints for resisting a compressive force if strengthened to resist bending by placing fish-plates on all sides. A lap-joint may be employed when the compressive force is not great.

Tensile Joint.—A lap-joint and a butt- or scarf-joint properly strengthened by fish-plates are suitable joints to resist a tensile

FIG. 105.

force. If the tensile force is small, a joint like Fig. 105 without fish-plates may be employed. The joint is kept closed by the use of *folding wedges*.

Flexure-joint.—A joint to resist flexure must have plane abutting surfaces, normal to the fibers, in the part of the beam subjected to compression, and a fish-plate to transmit the tensile stress in the fibers on the opposite side of the neutral axis. A butt-joint with a fish-plate bolted to the surface whose fibers are subjected to the greatest tensile stress, and a scarf-joint in which the joint above the neutral axis is a simple butt-joint and the lower part is fished as above described, are suitable joints to resist flexure. A scarf-joint of the form shown in Fig. 104 has

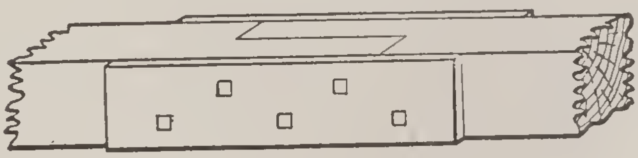


FIG. 106.

been found to be stronger when the fish-plates are parallel to the bending force, as in Fig. 106, than it is when the fish-plates are perpendicular to the force, as in Fig. 104. In temporary structures, a lap-joint may be employed to resist flexure.

Common Mortise-and-tenon Joint.—A *mortise* is a hole made in one piece to receive the *tongue*, or *tenon*, which terminates the other (Fig. 107). The end surfaces of the piece on either side

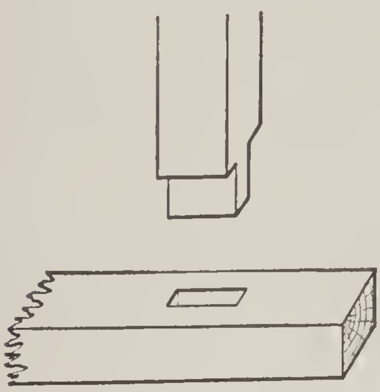


FIG. 107.

FIG. 108.

of the tenon are the *shoulders;* the surfaces adjacent to the mortise upon which the shoulders rest are the *cheeks*. A pin may be employed to hold the tenon in place. The width of tenon is usually about one-third of the width of the piece of which it forms part; its length is slightly less than the depth of the mortise, unless the mortise passes entirely through the beam.

If the pieces connected by a mortise-and-tenon joint are beams subjected to forces of flexure, a modified form of the simple joint is employed (Fig. 108). This is called a *tusk-tenon;* the mortise may extend wholly through the piece as shown, or it may extend only part way through. The area of cross-section of the tenon at the shoulders is increased so as to strengthen the beam, of which it forms part, to resist the shear at its extremity; the axis of the mortise lies in the neutral surface of the beam from which it is cut so as to reduce as little as possible the strength of this beam to resist flexure. A joint of this description is employed in connecting heavy floor-beams.

Oblique Mortise-and-tenon Joint.—The oblique mortise-and-tenon joint is shown in Fig. 109. According to Tredgold the depth of the mortise should be equal to one-half the depth of the piece in which the mortise is cut; in practice it is usually less than this, since a deep mortise greatly reduces the strength of the beam. The effect of the thrust in the strut *A* is to shear off the beam *B* in a horizontal plane and in the vertical planes

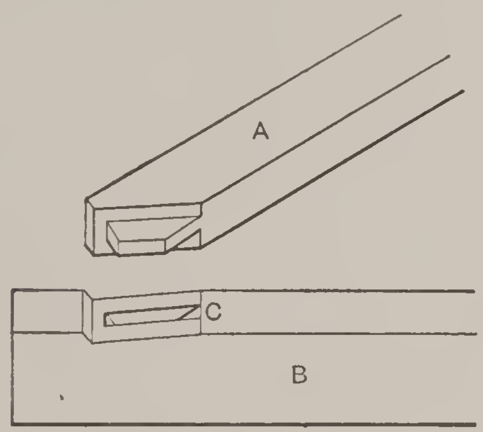


FIG. 109.

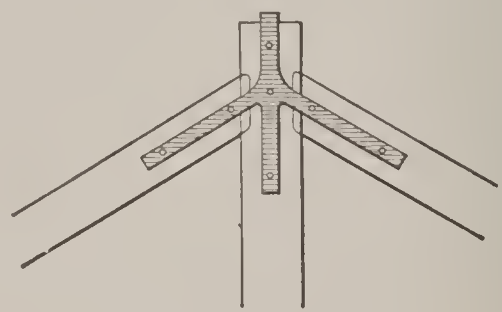


FIG. 110.

of the sides of the tenon; hence the shearing resistance of these surfaces must be greater than the horizontal component of the compression in the strut *A*. If this condition cannot be fulfilled, the joint must be strengthened by means of a strap around, or bolt through, both pieces. The tenon is then omitted. When several pieces intersect at a common point, as at the head of a king-post, the strap may be designed to unite all the pieces, as shown in Fig. 110.

Bridle-pieces.—A tie may be made of two pieces which are bolted together and inclose the pieces connected by them; the ties are then called *bridle-pieces*. Blocks are placed between the bridle-pieces at the points where the bolts are inserted (Fig. 111).

Halving and Notching.—These are the means employed to reduce the joint depth of two beams at the points of intersection. This reduces the strength of the pieces cut. The beams forming the sills of a building are joined by halving (Fig. 112). The ends of roof-rafters are often notched (Fig. 113).

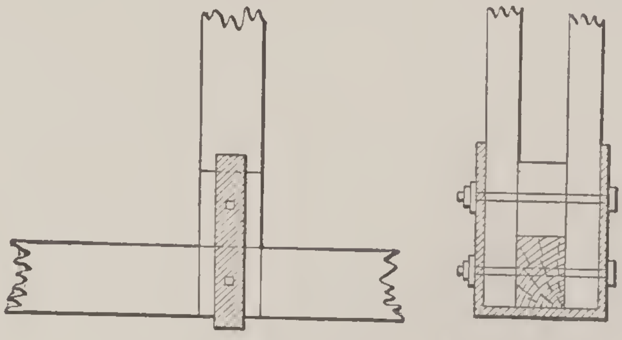


FIG. 111.

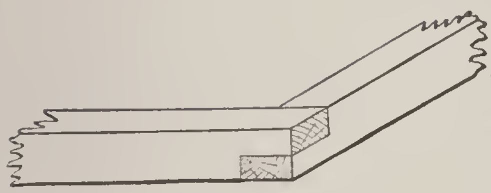


FIG. 112.

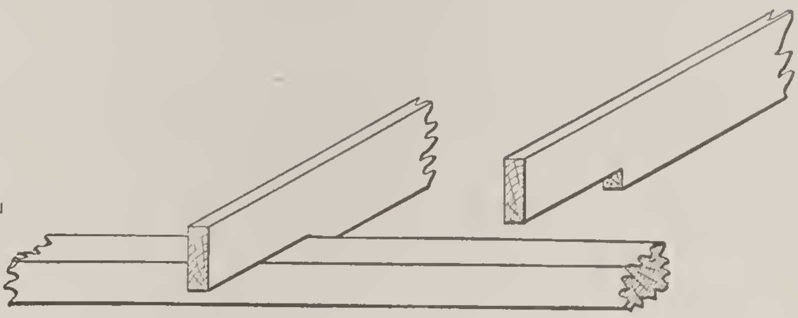


FIG. 113.

STRENGTHENING BEAMS.

If the span is so great that the cross-section of a single beam is insufficient to support its load, the beam may be strength-

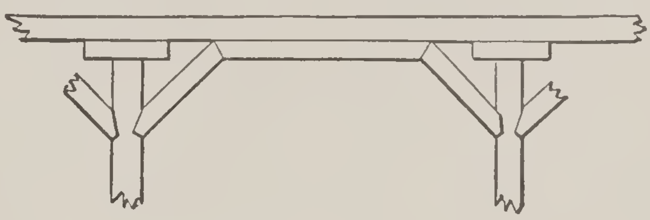


FIG. 114.

ened by the use of *shores*, *corbels*, or *straining-beams* (Fig. 114).

Shores.—A *shore* is a prop or strut which supports some point of a beam between its extreme supports. By this means the maximum bending moment in the beam may be greatly reduced. Shores are employed when a suitable surface can be found underneath the beam upon which the shores may rest.

Corbels.—A *corbel* is a short beam placed between the beam and the top of the pillar or other support. The effect of a corbel is to increase the width of the supported portion of the beam and thus decrease the maximum bending moment in the entire beam between the pillars. When the corbels are long, their ends are in turn supported by shores.

Straining-beams.—A *straining-beam* is one which supports the middle part of the beam itself and is in turn supported by shores. Not only is the bending moment at the middle point of the beam thus diminished, but the resistance of the straining-beam is added to that of the main beam thus strengthened.

Built-up Beams.—*Built-up beams* are made by superposing simple beams and so uniting them that they cannot slide on their surfaces of contact. The beams may be united by bolts, or the beams may be held together by bands, and the sliding prevented, by notching the beams or by introducing keys or folding wedges along the plane of contact (Fig. 115).

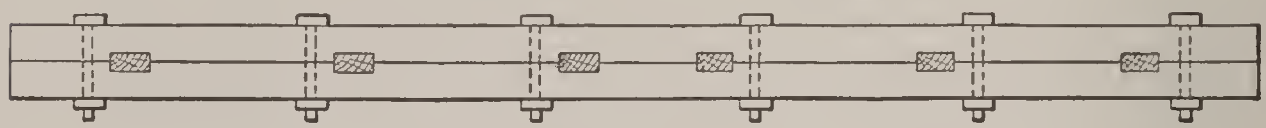


FIG 115.

If each layer is made of two or more pieces, those in the upper layers are united by simple butt-joints and those in the lower layers by butt- or scarf-joints strengthened by fish-plates.

Curved Beams.—A curved beam may be made of several layers of boards, thin enough to be bent over a form, which are spiked or bolted to each other; or it may be made of beams whose axes are chords of the arc, and which are united by inclined butt- or *miter-joints*. Curved beams are employed in the construction of military bridges; their ends are joined by suitable tie-rods.

Straps, Bolts, and Shoes.—Joints in timber-work are often strengthened by the use of iron *straps*, *bolts*, and *shoes*. Straps are made of wrought iron and are in the form of the letter U with square angles. The ends are provided with screw-threads and are connected by an iron plate held in place by nuts. Bolts have been heretofore described. Rectangular plates or washers are employed to prevent straps, bolt-heads, and nuts from cutting into the wood. Cast-iron shoes are also employed instead of straps at the panel points of wooden trusses. The struts fit in sockets formed in the shoes, and the ties are attached to bolts which pass through the shoes.

Strengthened Beams.—A beam may be strengthened by *trussing* it as shown in Fig. 116. The stress in the beam is then wholly compression and is uniform throughout. In long beams,

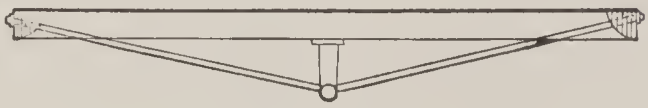


FIG. 116.

two *stanchions* or struts, counterbraced, are employed instead of one. If the tie is a single rod, it lies in the plane of the axis of the beam; if the tie is composed of two rods, they are on opposite sides of the beams.

For further information consult Tredgold's "Carpentry"; Kidder's "Building Construction and Superintendence, Part II"; Reports of Forestry Division, Department of Agriculture, U. S.; Snow's "Principal Species of Wood"; Johnson's "Materials of Construction"; Thurston's "Materials of Engineering, Part I."

CHAPTER XIX.

METALS.

BECAUSE of their strength and durability the useful metals and their alloys form a very important class of engineering materials. They may be safely exposed to conditions in which timber would fail. The principal metallic substances employed in engineering are *iron*, *steel*, *copper*, *lead*, *tin*, and *zinc*.

IRON AND STEEL.

The simplest classification of the three varieties of iron derived from the ores, based on the percentage of carbon and on their distinguishing properties, is given in the following table:

	Percentage of Carbon.	Properties.
Cast iron	5 to 2	Fusible, not malleable.
Steel	2.0 to 0.10	Fusible to malleable.
Wrought iron	0.2 to 0.05	Malleable, not fusible.

The lines of demarcation are not sharp; cast iron and hard steel on the one hand, and wrought iron and soft steel on the other, are somewhat similar in composition and in qualities. Besides iron and carbon, the different members of the group contain silica, sulphur, phosphorus, and manganese in various proportions, which greatly modify the range of strength and the qualities of the metal of each group. Taken as a whole, the group forms the most important structural material in engineering practice, because of its strength, its durability, its comparative cheapness, and the ease with which it may be fashioned into the forms best suited to resist all classes of straining forces.

CAST IRON.

Cast iron is made by remelting, in a *cupola furnace*, certain grades of pig iron, called *foundry pig*, which are the product of the blast-furnace. From the cupola furnace the molten metal is poured into cavities formed in molding sand by the use of wooden *patterns*. The products of the molds are called *castings*. If the process of remelting is repeated two or three times, the product is improved at each remelting. Scrap-iron is therefore usually mixed with the foundry pig. On account of the ease of manufacture, cast iron is largely used in engineering practice, in the manufacture of hollow columns, pipes, and ornamental forms not easily made of non-fusible metal and not requiring the strength or toughness of steel or wrought iron.

Commercially, cast iron is classed as *gray* and *white;* the gray is the class ordinarily employed in engineering practice. It is soft, tough, and slightly malleable when cold, and may be turned, drilled, and planed. It has a gray, granular fracture, with a metallic luster. It is usually subdivided into several minor classes differing in hardness. White iron is hard, brittle, and sonorous; it cannot be worked and has a white crystalline fracture with a vitreous luster. It is used only where a very hard metal is required. *Chilled cast iron* is gray iron with a white iron face.

PROPERTIES.

Specific Gravity.—The specific gravity varies according to the composition and method of casting from 6.9 to 7.5; in ordinary calculations a cubic foot is assumed to weigh 450 pounds.

Expansion and Contraction.—Under the influence of changes of temperature a cast-iron bar will elongate or contract about 0.0000062 of its length for each degree of Fahrenheit. Above 120° F. it begins to lose strength; at a red heat this loss is 33 per cent; below the freezing-point its strength is unreliable. The elongation or contraction of a bar of one inch cross-section is $\frac{1}{5000}$ of its length for each ton of applied force.

Strength.—The ultimate unit strength of any casting depends on the composition of the metal, the size and shape of the casting, and the care exercised in its manufacture.

The following values are considered safe values to use in engineering practice:

	Breaking Unit Stress, Pounds per Square Inch.	Allowable Unit Stress, Pounds per Square Inch.
Tension	18,000	3,000
Compression	80,000	10,000
Shearing	20,000	3,000
Bending	36,000	6,000
Torsion	25,000	4,000
Elastic limit in tension	6,000	
Modulus of longitudinal elasticity	15,000,000	

DEFECTS.

The defects in castings arise from improper composition of the metal and careless manipulation in the process of casting. The former causes weakness and brittleness, and the latter *blow-* or *air-holes*, *honeycomb* or *cavities*, *cracks*, *flaws*, and *cold-shut*. Air-holes, honeycomb or cavities result from air or impurities being retained in the molten metal while cooling; cracks and flaws result from unequal cooling. Cold-shut is a surface of weakness caused by filling the mold with partially cooled metal, through two or more apertures; the metal does not cohere perfectly at the surfaces of contact.

DURABILITY AND PRESERVATION.

Cast iron is durable for an indefinite time if kept in a dry place or immersed in fresh water; if exposed to the alternate action of the air and moisture, or to the ordinary gases arising from the combustion of coal, it *rusts;* if the gray variety of cast iron is exposed to the action of sea-water, it gradually softens and becomes porous. It may be protected against the action of moisture, gases, and sea-water by coating it with an impervious layer of preservative. The first coat should be applied soon after the casting is made and before it has time to rust; in subsequent applications all rust must be removed before the preservative is applied. Any of the ordinary paints may be employed for this purpose; water-pipes are usually heated, and plunged into a bath of hot coal-tar pitch varnish. White iron resists the action of sea-water better than the gray variety. Although durable, cast iron will in time lose part of its strength under the action of excessive loads, or when subject to shocks.

SPECIFICATIONS AND TESTS.

In Cooper's "Highway Bridges" the specifications for cast iron are as follows:

"All castings must be tough, gray iron, free from cold-shuts or injurious blow-holes, true to form and thickness, and of a workmanlike finish. Sample pieces one inch square, cast from the same heat of metal in sand molds, shall be capable of sustaining, on a clear span of 12 inches, a central load of 2400 pounds when tested in the rough bar. A blow from a hammer shall produce an indentation on a rectangular edge of the casting without flaking the metal."

This transverse strength is based on a maximum fiber stress in the test-bar of 43,200 pounds. Johnson gives the following specifications as employed by the St. Louis Water Department:

"All of the iron castings shall be made from a superior quality of iron, remelted in the cupola or air furnace, tough, and of an even grain, and shall possess a tensile strength of not less than 18,000 pounds per square inch.

"Test-bars of the metal 3 inches by $\frac{1}{2}$ inch when broken transversely, 18 inches between supports, and loaded in the center, shall have a breaking load of not less than 1000 pounds, and shall have a total deflection of not less than $\frac{3}{10}$ of an inch before breaking.* Said bars to be cast as nearly as possible to the above dimensions without finishing; but correction will be given by the Water Department for variations in thickness and width, and the corrected result must conform to the above requirements."

The transverse strength is based on a maximum fiber stress of 36,000 pounds.

Imperfections in the metal are discovered by very careful inspection and by the tests mentioned. In testing water-pipes,

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* The tensile strength may be raised to 20,000 or even 25,000 pounds per square inch, while the deflection may be made $\frac{3}{8}$ of an inch for ordinary good cast iron, and $\frac{1}{2}$ inch for a better quality. For a superior quality it may be made $\frac{5}{8}$ inch, with a breaking load of 1250 pounds.

each section is submitted to a hydraulic pressure of 200 to 300 pounds per square inch, and while under pressure is sharply struck at different points with a hammer.

Malleable Cast Iron—Malleable cast iron is made by a process of annealing or partially decarbonizing white iron. By this process its toughness and tenacity are greatly increased.

Steel.

Structural steel is made by one of two processes, the *open-hearth* and the *Bessemer.*

In the open-hearth process certain grades of pig iron are melted in a reverberatory furnace, either basic- or acid-lined, and either scraps of wrought iron or steel, or pure oxides of iron (ores) are added to secure the proper composition. The molten metal is then molded into *ingots*, which are *rolled* or *forged* into the required shapes. In the Bessemer process certain grades of pig iron, melted in the cupola furnace, or direct from the blast-furnace, are poured into a *converter*, which has a basic or acid lining, and there subjected to a blast of air and the addition of molten pig or spiegeleisen to secure the proper composition; the molten metal is then molded into ingots. The ingots are reduced to structural forms by reheating and rolling. The open-hearth process is under more perfect control and admits of more frequent analyses; its product is therefore more reliable, uniform, and homogeneous. For these reasons it is preferred by engineers for structural work. Bessemer steel is largely used for steel railway rails.

Steel wire is made by drawing thin rods through slightly conical holes in steel plates; the diameters of the rods are gradually reduced by successive drawings. The tensile strength of the material is greatly increased by the process.

Structural Steel Divisions.—Structural steel is usually divided into *rivet*, *low* or *soft*, and *medium* steel, depending upon its tensile strength and ductility.

Kind.	Tensile Strength.	Elongation.
Rivet or very soft	48,000 to 59,000 lbs.	26 per cent
Soft, low, or mild	54,000 to 62,000 "	25 " "
Medium	60,000 to 70,000 "	22 " "

These limits are not strictly adhered to either by engineers or manufacturers. Steel having a greater tensile strength and less elongation is called *high* or *hard* steel.

PROPERTIES.

Specific Gravity.—The specific gravity of steel is about 7.8; in ordinary calculations a cubic foot is assumed to weigh 490 pounds.

Expansion and Contraction.—The expansion or contraction of a bar per unit of length for each degree Fahrenheit of change of temperature is 0.0000067; the expansion and contraction of a bar of one square inch cross-section is about $\frac{1}{13000}$ for each ton of applied weight.

Ductility.—Ductility is an important property of structural steel, since, combined with strength, it gives the toughness which is essential in all structures subject to shocks and moving loads. The percentage of elongation under a tensile stress which will be required of the steel is always stated in the specifications. This percentage increases as the steel approaches the composition of wrought iron, or as the amount of carbon is decreased.

Hardness and Fusibility.—The hardness and fusibility both increase with the percentage of carbon, or as the metal approaches the composition of cast iron.

Hardening, Tempering, and Annealing.—Steel is *hardened* by suddenly cooling it from a temperature of about 1300° F., called a *low yellow heat.* For this purpose it is plunged into a bath of mercury, water, or oil. It may then be *annealed* or have its original properties restored by reheating it to a low yellow heat and cooling it slowly; for the latter purpose, when removed from the furnace it is covered with quicklime or charcoal. The hardened steel may also be *tempered* by reheating to a temperature below 700° F., called a *dull red heat,* and cooling it slowly. The amount of softening depends on the range through which it is cooled.

Weldability.—Soft steel has the property of weldability, that is, two pieces being heated and superposed may be united by repeated blows of a hammer. Medium steel can be welded only imperfectly by ordinary processes, and hard steel not at all.

Electric welding, which consists in melting the surfaces by an electric current and uniting them by pressure, can be applied to medium and hard steel. A welded joint can be relied on to carry one-half the load of a solid bar.

Strength.—The strength of steel depends upon its chemical composition, and upon the mechanical and heat treatment of the metal during the process of manufacture. The range of strength is very great, the tensile strength alone varying from 40,000 pounds in a very low steel to 240,000 pounds in steel wire employed in cable manufacture.

The following are considered safe values in engineering practice:

Stress.	Breaking Unit Stress, Pounds per Square Inch.	Allowable Unit Stress, Pounds per Square Inch.	
Tension	60,000 to 70,000	8000 (in floor-beams subjected to sudden loads) to 16,000 (in buildings)	
Compression in columns	50,000	10,000 (in bridges) to 16,000 (in buildings)	
Shearing	48,000 to 56,000	12,000	
Bending	....	12,500 (in floor-beams of bridges) to 16,000 (in floor-beams of buildings)	Increase 25 per cent for pins.
Torsion	60,000	10,000	
Bearing	....	20,000	

Unit stresses in wind bracing, about 25 per cent greater.
Modulus of longitudinal elasticity, 29,000,000 pounds.
Limit of longitudinal elasticity, one-half of breaking unit stress.
Average per cent of elongation in 8 inches: medium steel, 22 per cent; low steel, 25 per cent.
Average per cent of reduction of area: medium steel, 43 per cent; low steel, 47 per cent.

DEFECTS.

The principal defects in steel are due to excess of sulphur and phosphorus. The former makes the metal brittle when hot, or *red-short*, and the latter brittle when cold, or *cold-short*. These defects can be avoided only by specifying the maximum allowable percentage of each of these elements in the composition of the steel.

DURABILITY AND PRESERVATION.

Steel rusts under the action of alternate exposure to dryness and moisture, and the action of gases resulting from the combustion of coal; it is protected by covering it with some impervious coat of oil or paint which is renewed at intervals. The surface to which the paint is applied should be dry and chemically clean. At the mill, the metal is cleaned in an acid followed by an alkali bath. Old paint is removed from steel structures by a sand-blast or by wire brushes.

SPECIFICATIONS.

The following specifications are those required by Mr. L. L. Buck, Chief Engineer of the Williamsburg Suspension Bridge over the East River, New York:

Structural Steel.—"All steel shall be made in an open-hearth furnace lined with silica.

"The finished steel shall not contain more than the following proportions of the elements named:

Phosphorus	.07 of one per cent
Sulphur	.04 " " " "
Manganese	.50 " " " "
Silicon	.10 " " " "
Copper	.02 " " " "

"Specimens cut from the finished material shall have the following physical properties:

Description of Material.	Strength in Pounds per Square Inch.	Elongation Per Cent in 8 Inches.	Reduction Per Cent in Area.
Shapes and universal mill-plates	60,000 to 68,000	22	44
Shear-plates	60,000 to 68,000	20	44
Rivet-rods	54,000 to 60,000	25	50

"All specimens cut from plates and shapes shall bend cold 180° around once the thickness of the specimen; when at or above red heat, 180° flat; and when quenched in water at a temperature of 80° F., 180° around three times the thickness of the specimen. Specimens cut from rivet-rods shall bend

180° flat when cold, when at or above red heat, or when quenched from a light yellow heat in water at a temperature of 60° F.

"The elastic limit of the steel shall not be less than one-half the ultimate strength.

"All bending tests shall show no signs of fracture on the outside of the bent portion.

"The fracture of all tension tests shall have a cup or angular shape and shall have a fine silky texture, of a bluish gray or dove color, free from black or brilliant specks.

"All rolled or forged material shall be entirely free from piping, checks, cracks, and other imperfections, and shall have smooth-finished surfaces and edges.

"Rivets cut out of the work, when required by the engineer or his representative, shall be tough and show a silky texture without a crystalline appearance.

"Rigid tests will be made to guard against all red-shortness."

Steel for Castings.—"Steel for castings shall be made in an open-hearth furnace lined with silica.

"The finished steel shall not contain to exceed the following limits of the elements named:

Phosphorus..........	.06 of one per cent
Sulphur.............	.04 " " " "
Manganese..........	.08 " " " "
Silicon..............	.35 " " " "

"All steel castings shall be carefully and thoroughly annealed.

"All castings shall be sound and as free from blow-holes as the latest and best practice can produce.

"Test-pieces taken from coupons on the annealed castings shall show an ultimate strength of not less than 60,000 pounds per square inch, an elongation of not less than 20 per cent in 2 inches, and shall bend 90° around three times their thickness without rupture.

"All steel castings must be true to the drawings, with smooth surfaces, and all reentrant angles must be neatly filleted. They must be planed smooth and true where the drawings require, and all holes for bolts must be drilled accurately."

Steel for Wire.—"All steel for wire shall be made in an open-hearth furnace lined with silica.

"The finished steel shall not contain to exceed the following limits of the elements named:

Phosphorus	.04	of	one	per	cent
Sulphur	.03	"	"	"	"
Manganese	.50	"	"	"	"
Silicon	.10	"	"	"	"
Copper	.02	"	"	"	"

Wire.—"The wire for the cables and for the suspenders and ties must have an ultimate strength of 200,000 pounds or more to the square inch, and must have an elongation under test of at least 2½ per cent in 5 feet of observed length, and of at least 5 per cent in 8 inches of observed length.

"It must be capable of being coiled around a rod of its own diameter without cracking."

In specifications for less important structures the allowable percentages of phosphorus are alone stated; very frequently reliance is placed on the mechanical tests alone.

Wrought Iron.

Wrought iron is made by *puddling* in a *reverberatory furnace* certain grades of *pig iron*, known as furnace pig, which are the product of the blast-furnace. From the reverberatory furnace the pasty metal is dumped into *squeezers* where the cinder is expelled, and then passed through *rolls* which reduce it to the market forms. High grades are obtained by cutting up the first or *muck* bars and repuddling once or twice. Before it was replaced by steel in this country the ordinary market forms were similar to those of structural steel.

PROPERTIES.

Specific Gravity.—The specific gravity of wrought iron is from 7.4 to 7.9; in ordinary computations its weight is assumed as 480 pounds per cubic foot.

Expansion and Contraction.—Under the influence of temperature a bar expands and contracts 0.0000067 of its length for each degree Fahrenheit; under the action of an applied force a bar one inch in cross-section is extended or compressed $\frac{1}{12000}$ of its length for every ton of applied weight.

Ductility and Weldability.—Wrought iron is less ductile than structural steel, but can be welded more easily. A welded joint has also about one-half the strength of an unwelded piece of the same area of cross-section.

Strength.—In general the strength of wrought iron may be assumed to be about 84 per cent of that of soft steel. The following may be considered safe values:

	Breaking Unit Stress. Pounds per Square Inch.	Allowable Unit Stress, Pounds per Square Inch.
Tension	48,000	12,000
Compression	48,000	10,000
Shearing	50,000	9,000
Bending	48,000	12,000 (50% more on pins)
Torsion	50,000	10,000

Limit of longitudinal elasticity, 26,000 pounds.
Modulus of longitudinal elasticity, 27,000,000 pounds.
Percentage of elongation in 8 inches, 15 to 20 per cent.
Percentage of reduction in area, 12 to 30 per cent.

Defects.—The defects in wrought iron are also those arising from excess of sulphur and phosphorus, *red-* and *cold-short.*

Durability and Preservation.—Wrought iron is durable under the same conditions and is protected against rust in the same manner as steel.

Specifications and Tests.—The following specifications for wrought iron are given by Fowler in his "Roof-trusses":

"Wrought iron must be tough, fibrous, and of uniform quality. Finished bars must be thoroughly welded during the rolling, and be straight, smooth, and free from injurious seams, blisters, cracks, or imperfect edges.

"For tension tests the piece shall have as near one-half square inch of sectional area as possible, and a length of at least 8 inches with uniform section, for determining the elongation.

"The elastic limit shall not be less than 26,000 pounds per square inch for all classes of iron.

"Standard test-pieces from iron having a section of $4\frac{1}{2}$ square inches or less shall show an ultimate strength of not less than

50,000 pounds per square inch, and an elongation in 8 inches of not less than 18 per cent.

"Standard test-pieces from bars of more than $4\frac{1}{2}$ square inches section will be allowed a reduction of 500 pounds for each additional square inch of section, provided the ultimate strength does not fall below 48,000 pounds, or the elongation in 8 inches below 15 per cent.

"All iron for tension members must bend cold through 90° to a curve whose diameter is not over twice the thickness of the piece, without cracking.

"Not less than one sample out of three shall bend to this curve through 180° without cracking.

"When nicked on one side and bent by a blow from a sledge, the fracture must be wholly fibrous."

JOINTS AND PIECES IN IRON-WORK.

Joints in iron-work are either *lap-* and *butt-joints* with fish-plates, the pieces being held together by bolts or by rivets, or they are *eye-bar* and *pin joints.* If several plates are united to form a single plate, care is taken to break joints.

Ties.—Ties are usually made of steel or wrought iron eye-bars, the segments being united by pin-joints. If stiffness is required in a tie, it is made in the form of a laced or latticed column.

Struts.—Cast-iron struts are in the form of hollow circular or square columns. They are employed in structures not subjected to the shocks of suddenly applied heavy live loads. Horizontal beams rest on and are bolted to brackets cast on the columns. Steel and wrought-iron struts are in the form of built-up columns made of channels, Z bars, or other convenient structural shapes.

Beams.—Steel rolled I beams are manufactured in market sizes from a depth of 3 inches and a length of 21 feet to a depth of 24 inches and a length of 36 feet. They vary in weight from $5\frac{1}{2}$ to 100 pounds per linear foot. These I beams may be employed in pairs, threes, etc., placed side by side to secure greater strength.

When I beams fail to give the requisite strength, plate and box girders are constructed as heretofore described.

Other Metals.

Copper.—After iron and steel, copper is the most important metal in engineering practice. Its principal uses are in the forms of sheets for roofing, and of wire for electrical purposes.

Properties.—Metallic copper is both *malleable and ductile* and can therefore be reduced to sheets or wires. The properties above mentioned are found in the highest degree in pure copper. It is *harder* and more *tenacious* than any other metal excepting iron. Its tensile strength is 30,000 pounds, which may be doubled by wire-drawing. It is very durable under ordinary atmospheric conditions and needs no coating of paint or other preservative. Sheathing-copper is formed into sheets 14×48 inches, whose weight varies from 14 to 34 ounces per square foot; these are united by soldering or brazing.

Tin.—In engineering practice tin is used in the form of tin-plate, which is sheet iron tinned on both sides; good *tin-plate* is plated with pure tin; the cheaper or "*terne*" *plates* with a dark alloy. The coating of the former is bright, even, and thick, and devoid of dark spots or roughness due to imperfections in the coating or incomplete coating.

Zinc.—Zinc, being extremely durable when exposed to the action of the weather, is largely used as a coating for sheet iron and iron wire. Galvanized iron is made by dipping the sheet iron in melted zinc. Tin plate and galvanized iron exposed to the action of the atmosphere are usually protected by a coating of paint.

Lead.—Lead is used in the form of sheets and pipes.

Alloys.—Various alloys of these metals are also used to some extent in engineering practice. These are described in text-books in chemistry.

For further information consult Thurston's "Materials of Engineering, Parts II and III; Johnson's "Materials of Construction."

CHAPTER XX.

NATURAL AND ARTIFICIAL STONE.

NATURAL STONE.

NATURAL stone is a valuable material in engineering practice because of its durability and wide distribution; it has a more limited field of usefulness than timber because of its great weight, and because it is suitable only for resisting crushing and shearing forces. The value of any variety of stone depends upon the *strength*, *durability*, *appearance*, *ease of working*, and *its action when exposed to the weather*, or to the atmosphere of the position in which it is to be used. All of these qualities are more or less dependent upon its structure.

Structure.—According to their predominating constituents, stones are classified as *silicious*, *calcareous*, and *argillaceous;* the most important constituent of the first being *quartz*, of the second *lime*, and of the third *clay*. According to the manner in which they are formed they are *sedimentary* or *stratified*, *igneous* or *unstratified*, and *metamorphic.*

The ordinary building-stones are given in the following table:

Class.	Stratified.	Unstratified.	Metamorphic.
Silicious	Sandstone	Granite, Trap	Gneiss
Calcareous	Common limestone	....	Marble
Argillaceous	....	....	Slate

Stratified and Unstratified.—The stratified stones are found in layers varying in thickness from a few inches to many feet. As they are easily separated along the planes of stratification, they are more easily quarried than unstratified stones. A stratified stone should be laid with its planes of cleavage perpendicular to the maximum force to which it is subjected; in this position it offers the greatest resistance. It should never be laid with its planes of stratification parallel to its exposed surface, since in this position it flakes under the action of frost.

Some of the metamorphic stones, gneiss and slate, have surfaces, or planes of cleavage, similar to the planes of stratification. They should therefore be treated as stratified stones.

Unstratified stones may be taken out in blocks, limited only by their strength to resist the bending stresses involved in their movement; they are therefore largely used in structures where massive effects are desired. The metamorphic rocks without planes of cleavage, e.g. marbles, are also equally strong in all directions, and may be treated as unstratified stones.

Silicious.—The silicious stones are *sandstones*, *granite*, *syenite*, *trap*, and *gneiss*. In engineering practice the term granite is employed as a general term covering granite, syenite, and gneiss.

Sandstone is formed of grains of sand cemented together by silicious, ferruginous, calcareous, or clayey material. The quality of the stone depends largely upon the cementing material. The granites are of more or less crystalline structure, the quality of the stone depending largely upon the size of the crystals and the uniformity of the structure. Trap is not properly a building material, but is used in engineering practice in road construction.

Calcareous.—The calcareous stones are either *limestones* or *marbles*. The limestones are of two general classes, the *common* and the *magnesian*. The common limestones are of the *oolitic* and the *coquina* varieties, and are composed of almost pure carbonate of lime; the magnesian limestones, or *dolomites*, contain 20 to 40 per cent of magnesia.

The marbles are crystallized limestones containing impurities which impart characteristic colors.

Argillaceous.—The only important argillaceous stone is *roofing-slate*. *Bluestone* is an argillaceous sandstone employed for curbing and sidewalk slabs.

PHYSICAL QUALITIES.

Specific Gravity.—Stones suitable for building construction range in specific gravity from 2.2 to 2.8. The unstratified and metamorphic stones are heavier than the stratified ones. In ordinary computations the weight is assumed as 160 pounds per cubic foot.

Rift and Grain.—The rift of a stone is the direction parallel to its plane of stratification or cleavage. Its grain is perpendicular to this plane. It is usually possible to separate the stone in planes in both directions. Paving-blocks are thus made.

Color.—The chief coloring-matter in stone is iron; therefore a very light or nearly white color indicates an absence of iron. If the iron is in a form which cannot undergo further oxidation, the color is permanent, as in *brownstone*, in which the iron is in the form of *red oxide;* if, however, it is in the form of a *carbonate* or *sulphide*, oxidation usually takes place when the stone is exposed to the atmosphere and the color of the stone is gradually changed. In a building-stone permanency of color is often an important consideration.

Hardness and Toughness.—These qualities enable a stone to resist abrasion and concussion. For ornamental work, stones of the unstratified or metamorphic varieties, such as fine granite and marble, which take a polish, are the most valuable. For street pavements, a hard, tough stone of granular structure, such as coarse granite or Medina sandstone, is best. Trap, which is the hardest and toughest of stones, is very valuable for broken-stone roads, but is too smooth for paving-blocks.

Moisture.—The moisture found in stones in their natural state is called *quarry-water;* this is removed by evaporation when the stone is exposed to the air. The quarry-water renders the stone softer and is an important factor in working many varieties, which become exceedingly hard on exposure. The unstratified and metamorphic stones absorb very little water, not over 0.16 per cent; while the most porous stratified rocks absorb as much as 5.50 per cent. The latter therefore are much less durable in a climate where the stone is subjected to a temperature frequently rising above and falling below the freezing-point. On this account quarries of porous rock are usually closed during freezing weather.

MECHANICAL PROPERTIES.

Strength.—The strength of stones to resist a crushing force seems to be a function of their specific gravity, since the heavy, unstratified, and metamorphic stones are stronger than the lighter, stratified ones. From the complex character of their structure it is apparent that great differences may be expected in the same class. A minimum crushing strength of 5000 pounds per square inch is considered desirable in a building-stone; some granites have five times this strength. The stresses per square foot allowed in building regulations are 60 tons in granite, 40 in limestone and marble, and 30 in sandstone. When its resistance to crushing is an important feature in the design of a structure, as in high reservoir walls and bridge piers, the stone must be carefully selected. As stone is lacking in fibrous structure, its strength to resist tension and bending is very small compared with its strength to resist crushing and shearing.

DURABILITY AND PRESERVATION.

Durability.—In engineering practice stones are subjected to two classes of disintegrating agencies, *chemical* and *mechanical.* The principal *chemical* agents are the *acids* of the air, notably *sulphuric* and *carbonic* acids resulting from the combustion of gas, coal, etc., in cities and manufacturing districts. Every constituent of stone except quartz is subject to attack by acids. The silicious stones having the largest percentages of silica, or the *quartzites*, are therefore the most durable, and the limestones the least so. If the cementing substance of a stone is attacked, the stone will crumble; if the grains are attacked, a spongy, porous stone will result. The durability of a stone, so far as chemical agents of destruction are concerned, is chiefly dependent upon its mineral constituents. A porous texture will, however, expedite the deterioration. Laboratory tests and microscopic examinations furnish an indication of the durability of a stone under the attack of chemical agents, but the only thoroughly reliable test is the exposure of the stone for a long period of time to the conditions under which it is to be used. Examination of exposed ledges at the quarry shows how well the stone resists deterioration under the conditions obtaining there, but is not a sure indication of its resisting power under different conditions.

The *mechanical agents* to which stones are subjected in engineering practice are the *friction* and *concussion* of moving bodies, *driving winds and rains*, and *changes of temperature*, especially in the vicinity of the freezing-point, when they are saturated with moisture. The effects of friction and concussion are best seen in the paving-stones of a large city, where granite blocks must be replaced every eighteen years, and sandstone in eight. Driving winds and rains slowly wear away the surface of stones; the action is ordinarily appreciable only in ornamental work where sharp edges are exposed to their action. Silicious, unstratified stones, which are hard and tough, best resist the above agents. The most destructive agent is the range of the temperature in temperate climates, which subjects the elements of stone to unequal expansion and contraction, opens the joints of masonry structures and allows rain to enter, and finally expands this water, when the temperature falls below the freezing-point, and crumbles the stone. A non-porous stone of uniform structure will best resist changes of temperature. If the range of temperature is very great, as in a conflagration in which the heated stones are drenched with cold water, the silicious stones of uniform texture, as the sandstones, are much more durable than those of more complex structure, like the granites. Limestones are said to be more durable below the temperature at which the stone is calcined. In the Census Report of 1880 the order of durability is given as follows: *sandstone, granite, marble, ordinary limestone, porous sandstone.*

Preservation.—The preservation of stone may be effected in the same manner as wood by coating it with an impervious covering. The various substances used for this purpose are *paint, boiled linseed-oil, melted paraffin*, and a mixture of *alum* and *soft soap* dissolved in water. Other processes have been devised for filling the pores with an insoluble salt. Ransome's process consists in saturating the stone with a solution of *silicate of soda or potash* and then applying a solution of *chloride of calcium.* The resulting reaction produces *chloride of soda*, which is easily washed off, and an insoluble *silicate of lime*, which fills the pores of the stone. There are several other processes having the same general object.

Quarrying.—Were the stone in every quarry a homogeneous monolithic structure, it could be removed from its bed only by drilling a row of vertical holes parallel to the face of the quarry and a row of horizontal holes parallel to, and at the desired distance below, the top. These holes charged with explosive and fired simultaneously would loosen the stone from its bed when it could be broken into smaller pieces in a similar manner. This process is abridged in many quarries due to the planes of stratification and cleavage, and also due to the joints which divide the mass of stone into volumes of different dimensions. Stratified and foliated stones do not, as a rule, require the process of undercutting, or *gadding*, as the stones are easily separated along the planes of stratification and cleavage. They are also divided by joints perpendicular to these planes. Unstratified rocks are also usually traversed by two or more sets of intersecting joints which divide them into more or less regular rectangular or rhomboidal prisms. The planes of stratification and cleavage and the joints limit the size of the product which may be obtained from any quarry, but also decrease the cost of the quarried stone.

The drills used in quarrying are hand-, churn-, or steam-drills. *Hand-drills* are short blunt chisels which are forced into the stone by blows from a hand-hammer or sledge. *Churn-drills* are longer drills which are forced into the stone by dropping them a short distance. They are suitable for holes from 3 to 5 feet deep. In *steam* or *compressed-air drills* the man-power of churn-drills is replaced by one of those agents. The blows are delivered with great rapidity. In quarries of sandstone, limestone, and marble a *channeling-machine* is often employed instead of the steam or compressed-air drills. This machine operates in a similar manner, but cuts a continuous groove instead of a series of holes. A *hollow revolving drill* or *diamond drill* is employed when large holes are required; this drill removes a core and thus furnishes specimens for the study of the character of the stone. *Explosives* employed in quarrying dimension-stone are *slow-acting* and used in small quantities. The direction of the line of fracture is influenced by the form of the hole around the charge. Large, irregular fragments are obtained by the use of slow-acting explosives without any attempt to control the direction of the line of fracture. Small irregular fragments are obtained by the use of high explosives.

Dressing.—The quarried stone is reduced to proper dimensions for use by employing *hand-drills*, *plugs and feathers;* it is then *dressed* with the ordinary mason's tools. *Rock-face* finish presents a rough, undressed surface, which is frequently surrounded by a margin of *drove-work;* the latter is made with a wide chisel or drove. *Pointed-face* finish is rock-face finish trimmed down with a sharp-pointed tool called a *point. Pean-hammered* or *ax-hammered* finish is a comparatively smooth surface made by hammering the surface with a *pean-hammer;* this is a heavy axe with a blunt cutting-edge. *Patent-hammered* finish is similar to the pean-hammered finish and is made by hammering the surface of the stone with a hammer whose head is made of several plates of steel so fastened together as to form a grooved face. The finish is known as 4-cut, 6-cut, 8-cut, 10-cut, and 12-cut, depending upon the number of plates per lineal inch in the hammer. This is the ordinary finish of smooth-surface walls. *Bush-hammered* finish is similar to the last, the hammer-head being a single piece with a grooved face. *Tooth-chiseled* finish is made with a saw-toothed chisel; the softer stones are sometimes finished in this manner. *Sawed face* is, as its name implies, produced by sawing the stone. *Fine sand and pumice* finish are made by rubbing the surface with fine sand or pumice. A *polished surface* is then given by rubbing with *polishing-putty*, or a moist woolen cloth and oxalic acid.

Specifications.—Specifications for building-stones usually require the bidder to submit samples for appearance and samples for tests. The former are six-inch or one-foot cubes whose faces are dressed in the different styles required in the structure. Two-inch cubes or test-pieces are made with smooth faces.

Tests.—The tests to which specimens of building-stones may be subjected are chemical and mechanical. The chemical or laboratory tests are made for the purpose of determining the composition of the stone and the probable effect of its various elements when exposed to the conditions to which it will be subjected in the structure. The mechanical tests are for the purpose of determining the amount of absorption, its action when exposed to frost in a saturated condition, its action when exposed in a conflagration, and its crushing strength. The *absorption test* consists in drying the specimen in an oven and then immersing for a long time in water; after it is removed from the water it is exposed to the air until the surface-water is evaporated. The difference in weight before and after immersion will give the weight of the water absorbed.

The *freezing test* consists in exposing a specimen saturated with water to the alternative action of a temperature above and below the freezing-point. The test may be made in summer by exposing the specimen to the temperature of the air each day, and the temperature of a refrigerating apparatus each night for a long period of time. The total loss of weight will measure the action of the frost. The *fire test* consists in exposing it to high heat and dashing it with cold water. The *crushing strength* is determined by crushing the specimen in a testing-machine.

Brick.

Brick is an *artificial stone* which is cheaper than natural stone, almost equally strong, and is less affected by the ordinary chemical and mechanical agents of destruction. Its field of usefulness is, however, limited by the small size of the individual stones, which ordinarily have a volume of only one-twentieth of a cubic foot.

MANUFACTURE.

Clay.—The clay of which common bricks are made consists principally of *silicate of alumina*. *Sand*, *lime*, *iron*, and *magnesia* are usually present in varying proportions. Sand if not in excess is beneficial, as it preserves the form of the brick when exposed to the high temperature of burning; in excess it destroys the cohesion and renders the brick brittle and weak. Silicate of lime renders the brick fusible and liable to deformation in burning. Carbonate of lime in excess is calcined in the process of burning, slakes in contact with moisture, and causes the brick to crumble. Iron, which in burning forms the red oxide, gives the brick strength, hardness, and a red color. By selecting the clay and introducing coloring material bricks of many colors and shades of color are produced.

Mixing.—The clay is converted into a plastic mass for molding by mixing with water and kneading In small plants the mixing is effected by shoveling the clay into a circular trough, where it is moistened with water, and broken up by a heavy wheel which moves around the trough, or by mixing the clay and water in a *pug-mill*. The latter is a vertical cylinder or box in whose axis is a vertical revolving shaft. To this shaft are attached horizontal arms set in such a manner as to knead the clay and force it downward and out through an orifice in the bottom.

Molding. — The paste is molded by throwing it into plank molds, open at the top and bottom, which are placed on a molding-board. The mold is carefully filled and struck off with a straight-edge. To prevent the paste from adhering to the sides of the molds they are kept wet and sprinkled with sand. In large plants the operations of mixing and molding are done by a continuous process in a brick-making machine. In one form the plastic clay passes from the mixer into a hopper from which it is forced through a rectangular aperture whose cross-section is that of a brick; here it is cut by a wire to the proper length and is carried off on an endless chain to the drying-floor. In another form it drops from the hopper into molds attached to the circumference of a large wheel; from these molds the bricks are also deposited on an endless chain.

Classification.—Common brick are classified according to the way in which they are molded, into *hand-made*, *machine-made*, and *pressed brick*. Machine-made brick are usually more regular than hand-made brick, but some machines injure the structure of the brick itself. Pressed brick are very regular, compact bricks, produced by compressing a partially dried brick in a suitable mold under heavy pressure. In the old style of kiln, brick are classified as *overburned* or *arch brick*, *cherry* or *hard* brick, and *salmon*, *pale*, or *soft* brick. The first are hard and brittle, and do not form a good bond with mortar; the last are too soft for general use and are used only in interior walls supporting light weights.

Requisites of a Good Brick.—A good brick should have plane faces, parallel sides, and sharp edges and angles; it should be of fine, compact, uniform texture, should be quite hard, and give a clear, ringing sound when struck. It should not absorb more than one-tenth of its weight of water. Its specific gravity should be at least two. Its crushing strength should be at least 7000 pounds per square inch, and its modulus of flexure at least 1000 pounds per square inch.

Tests.—In purchasing bricks it is best to specify that the bidder shall submit samples representing the *worst* of the brick to be furnished. The qualities above enumerated are determined as follows: The shape and structure are determined by careful inspection of the entire brick, and its cross-section when broken. The amount of absorption is determined as in the test for stone. The conditions of practice are best represented by testing whole bricks, but, as some kinds have a more or less impervious skin, it is well to make a similar test on a broken brick to determine the condition of the interior. The specific gravity, the crushing and bending strengths are determined as in tests of other materials.

Size and Weight.—There is no legal standard of size in the United States. In the Eastern States the usual size is $8\frac{1}{4} \times 4 \times 2\frac{1}{4}$, but in the West the dimensions are somewhat smaller. On account of the shrinkage in burning, the size is not exactly uniform. Where uniformity of size is essential it is usual to specify that the brick shall be *culled*, i.e., assorted in sizes. Pressed brick weigh about 150 pounds per cubic foot, hard brick about 125 pounds, and inferior soft brick about 100 pounds per cubic foot. Common brick average about $4\frac{1}{2}$ pounds each.

SPECIAL FORMS.

Paving-bricks.—Paving-bricks are used for street pavements, stable floors, etc.; they are ordinarily made of clay in the form of shale, which has a higher percentage of flux than common brick-clay, and is burned at a higher temperature than common brick. The result is a compact semi-vitrified brick or paving-block. A good paving-brick should be hard, tough, and absorb little water even when its surface is worn off. The hardness may be tested by grinding the brick on a stone, and the toughness by placing the brick in a *rattler* used for cleaning castings, or by dropping it repeatedly on a hard floor.

Fire-bricks.—Fire-bricks are used where high temperature is to be resisted. They are made of nearly pure clay, or of a mixture of nearly pure clay and clean sand or crushed quartz. The presence of oxide of iron is injurious. Fire-bricks should contain less than 6 per cent of oxide of iron, and less than an aggregate of 3 per cent of combined lime, soda, potash, and magnesia. Good fire-brick should be uniform in size, regular in shape, easily cut, strong and infusible.

Terra-cotta.—The term terra-cotta is usually applied to brick which has been molded into ornamental forms. It is made of different varieties of selected clays mixed together with ingredients which make the mixture slightly fusible. The forms are given by plaster molds.

Tiles.—The term tile is applied to a variety of forms of brick employed in building construction. The *dense tiles* employed in fire-proof floor construction are made of fire-clay mixed with other clays, and are shaped in strong molds under heavy pressure. *Porous tiles* are made in a similar manner; with the clay is mixed some sawdust or straw which is reduced to ash in the burning. Roofing-tiles are dense tiles often made with a glazed surface.

Glazed and Enameled Bricks.—These are special varieties of brick having a glazed or enameled surface. The surface may be given any desired color.

Vitrified Sewer-pipe and Blocks.—Sewer-pipe is made of selected clay and has a glazed coating. The pipes are molded by machinery, dried, placed in a kiln, and gradually exposed to a high heat. At the proper temperature coarse salt is thrown on the fire; the salt vaporizing combines with the silica of the clay and forms a soda-salt, or glass. Sewer-blocks are similar in composition to sewer-pipes; they are used to replace the bottom courses of a brick sewer.

Lime, Cement, Mortar, and Concrete.

Lime and *cement* are the products of the burning or *calcination* of certain classes of calcareous stones. The receptacle in which the operation takes place is called a *kiln.* The character and properties of the product of calcination depend upon the chemical composition of the limestone and the temperature at which the stone is burned. These products are *common lime*, *hydraulic lime*, and *cement.* Any one of them if reduced to powder and mixed with water will form a paste which is the cementing material of a large class of artificial stones. If the paste is mixed with sand only, the resulting mixture is called *mortar.* Mortar is itself employed either as a paste to unite individual stones in masonry, or it is employed as an artificial stone. *Con-*

crete is an artificial stone usually made by mixing cement mortar in the plastic state with broken stone, gravel, etc.

Liquid asphalt and coal-tar are also employed in the place of lime or cement paste in making mortar and concrete.

LIME.

Common Lime.—*Rich*, *fat*, or *pure* lime is produced from limestone which is practically pure calcium carbonate; marble, white chalk, oolitic and coquina limestone, in which the impurities do not exceed 2 or 3 per cent, may be employed for this purpose. The calcination at moderate heat drives off the carbonic acid and water in the stone and leaves *quick* or *caustic lime.* This product is white, amorphous, highly caustic, and has great avidity for water. If lumps of quicklime are sprinkled with water, they will at once swell, burst into small fragments, and finally crumble into a powder known as *slaked lime* or calcium hydrate. The process of slaking is accompanied by the evolution of heat and steam, and by an increase in volume, varying between two and three and a half times the original volume. The same effects are produced without the evolution of heat and steam if the quicklime is exposed to the atmosphere; it absorbs moisture slowly and slakes gradually; this is known as *air-slaking.*

If a pat made of slaked lime and water is exposed to dry air, it will crack due to contraction, and the outside will harden, due to its absorption of carbonic acid and conversion into calcium carbonate. If the mass is large, the interior will become friable in dry air, and remain pasty in damp air. If a similar pat is placed under water, it will disintegrate, and if the quantity of water is sufficient to dissolve it, will wholly disappear.

Meager or *poor lime* is produced by the calcination of limestone composed of calcium carbonate mixed with sand. The process of slaking is retarded and rendered less complete by the sand; if the lime is very poor, it cannot be reduced to a powder by slaking, but must be thus reduced by grinding. Poor lime should not be used in engineering work.

Manufacture.—The kiln in which the limestone is calcined is a large cylindrical, conical, or egg-shaped chimney which is ordinarily lined with fire-brick. The process may be either *intermittent* or *continuous.* If intermittent, the bottom of the kiln forms the fuel-chamber, which is separated from the limestone chamber above it by a perforated arch of limestone or fire-brick; this arch supports the limestone and allows the heat to pass through and reach the stone in the upper chamber. The stone is introduced at the top of the kiln, and in some kilns also through doors in the walls. When the charge has been fully burned, it is removed through a door just above the arch, and the kiln is allowed to cool sufficiently to allow the workman to put in a new charge.

If the process is continuous, one of the two following forms of kilns is employed. In the first form the fuel-chamber is constructed on the circumference of the limestone chamber near its base; the stone is put in at the top and drawn out at the bottom, being calcined in that part of the kiln where the heat from the fuel-chambers passes into the chamber containing the stone. In the second form, the coal and limestone are placed in the kiln in alternate layers, the whole charge being supported by a grate which is placed across the kiln near its base. The coal in the layer at the grate is ignited, and the layer of limestone above it is calcined and removed through the grate The fire is communicated from the first to the second layer and thus the process becomes continuous. As each layer of coal is consumed it is replaced by a new layer introduced at the top, and as each layer of limestone is removed it is replaced by a new layer of stone, also introduced at the top. The kiln in which the fuel is kept separate from the stone is more easily managed and gives more uniform results than the one in which the fuel and stone are placed in layers; it requires, however, a greater expenditure of fuel.

Preservation.—To protect quicklime, slaked lime, and lime paste from prematurely absorbing moisture and carbonic acid from the atmosphere, they should either be used at once, or they should be kept in closed vessels. In building operations it is customary to use freshly burned lime which, after being slaked, is made into a paste and covered with sand until used.

Tests.—Good lime should be clean, slake readily, and dissolve in water.

Hydraulic Lime.—Hydraulic lime is produced from certain varieties of silicious and argillaceous limestone, notably those of Teil and Seilly, France. It is calcined in the same manner as common lime and is at once reduced to a powder by slaking; it slakes less readily than poor lime, and its increase in volume does not exceed one-third its original volume. After slaking it is packed in sacks or barrels and protected from the air until used.

A pat made of the slaked hydraulic lime and water will harden not only in the air, but also under water. This hardening, called *setting*, differs from that of common lime in taking place throughout the pat and not simply on the surface. The setting is assumed to be due to the crystallization of the hydro-silicates and hydro-aluminates formed when the slaked lime is mixed with water; the crystals bind the material firmly together.

Hydraulic lime is not manufactured in this country, as the limestones are more suitable for the manufacture of cement.

Artificial hydraulic lime may be made by mixing soft chalk or slaked lime with the proper proportion of clay, molding the mixture into bricks, and burning the bricks in a kiln. It is reduced to a powder by grinding.

CEMENT.

Cement is the product of the calcination of certain natural or artificial combinations of calcium carbonate with alumina, carbonate of magnesia, and silica; iron and other elements in small ratios are also usually present. The product of calcination differs from lime in that it cannot be slaked, but must be reduced to a powder by grinding. A pat made of cement powder and water will set under water as well as in the air, and will increase in strength with age.

Cements are divided into three classes: *natural cement*, *portland cement*, and *puzzolans*.

Natural cement is that produced by calcining natural-cement rock at a heat below that of incipient fusion. The terms *quick-setting*, *light*, *Roman*, *American*, and *Rosendale* are all applied to this variety. It is usually of a brown color and has a specific gravity of 2.7 to 3.0.

Portland cement is that produced by calcining up to the point of incipient fusion intimate mixtures, in exact proportions, either natural or artificial, of carbonate of lime and clay. Its name is derived from its resemblance in color to portland stone; the terms *heavy* and *slow-setting* are also applied to this cement. It is of a gray color and has a specific gravity of 3.10 to 3.25.

Puzzolan cement is an artificial cement made by mixing in proper proportions slaked lime and granulated blast-furnace slag. The mixture is not calcined. The name is derived from puzzolana, a naturally burned earth of volcanic origin, found at Pozzuoli near Naples, Italy. This earth and slaked lime will give a mixture having hydraulic properties. Puzzolan cement is of a lilac tint and has a specific gravity of 2.7 to 2.8.

Silica or sand cement is made by mixing portland cement with about an equal amount of sand and grinding the mixture to a fine powder; it is almost as strong as the neat cement.

Manufacture.—Natural cement, being made from the stone itself, is burned in kilns similar to those described for the calcination of lime; the process, however, requires a higher temperature. After being burned the stone is crushed and finally ground into fine powder in suitable machinery.

Portland cement usually results from the burning of a mechanical mixture, hence the first operation is to form this mixture. The processes of manufacture are known as the *wet* and the *dry* processes.

In the wet process only soft materials, such as chalk and clay, are employed. These are dumped in the proper proportions into a *wash-mill* with a large amount of water, and the materials are intimately mixed and reduced to fine powder by means of knives or tines attached to horizontal arms fastened to a vertical revolving shaft. The liquid, or *slurry*, is allowed to flow from the wash-mill through sieves into the settling-basins, or *backs*, where the water is drawn and evaporated from the surface. When the slurry reaches the consistency of soft butter it is removed with shovels and placed on the drying-floor, where it is artificially dried, usually with the waste heat from the kilns. When the slurry is sufficiently dry it is burned in a kiln, usually of the continuous type, in which the fuel and stone are in consecutive layers. The product of the kiln is a very hard and heavy dark green honeycombed clinker, which is reduced to small fragments in a crusher similar to those used in crushing stone. It is finally ground to a powder by millstones. This process is employed in England, where the materials are chalk, and clay from the banks of the Thames and Medway.

The wet process is also modified into the *semi-wet*. In this the settling-basins are omitted and less water is used in the wash-mill. The wet slurry passes from the wash-mill to millstones, where it is ground and the mixing of the materials thus completed. This is also employed in England.

The dry process is employed where the limestone and clay are so hard that they cannot be mixed in a wash-mill. The materials are therefore reduced to a powder separately by crushing and grinding, and are then mixed in the proper proportions in a pug-mill. From the pug-mill they pass to a brick-machine, where they are made into bricks for the kiln. The process of burning the bricks and reducing the clinker to powder may be conducted as in the wet process. The dry process is employed in Germany and in the United States.

The kiln generally employed in this country is the rotary kiln. This is a wrought-iron cylinder 60 feet long and 6½ feet in diameter, which is lined with refractory material. The cylinder is slightly inclined to the horizontal and rotated by proper machinery. The powdered mixture is introduced either wet or dry in a continuous stream at the high end of the cylinder, and is reduced to clinker by the flame of powdered coal or gas which enters under pressure at the low end. The clinker finally drops out at the low end, and the process is thus made continuous.

The manufacture of portland differs from that of natural cement, in that the proportion of ingredients is much more exact, the temperature is higher, and the product of calcination is harder and less easily reduced to powder.

In the manufacture of puzzolan or slag cement, the slag from a blast-furnace is suddenly cooled by means of a jet of cold water and is granulated in a crusher. The lime is prepared in the usual manner, and is slaked before it is mixed with the slag. After mixture the cement is reduced to a powder by grinding.

Preservation.—Cement must be protected from contact with moisture which will produce premature setting; it is usually packed in barrels or sacks lined with stout paper. These must be stored under cover on a raised floor.

Tests.—The process of manufacturing cement consists of three distinct operations: the selection or mixing of the ingredients, the burning of the stone or mixture, and the reduction of the product to powder. Upon the manner in which each of these operations is performed depends the quality of the cement. These qualities can be determined only by making suitable tests. The tests recommended by the Engineer Department of the Army for portland and puzzolan cements are tests for *soundness*, *fineness*, *tensile strength*, *specific gravity*, and for *activity;* in testing natural cement, the tests for soundness and for specific gravity are omitted.

Soundness.—Unsoundness in cement is usually due to an excess of free lime resulting from improper proportions or from underburning. This free lime expands on contact with water and causes disintegration. In this test neat-cement paste is made into pats, which are small cones or spherical segments 3 inches in diameter and a half-inch thick at the center. Faija's test consists in suspending a freshly made pat for six hours over water kept at a temperature of 115° to 120° F.; at the end of this period the pat is lowered into the water, where it is allowed to remain 18 hours. The pat when taken out of the water should show no signs of disintegration. In the Engineer Department tests two pats are made on glass plates, and are exposed to the air for 24 hours under a damp cloth. One is then placed in water which is gradually raised to the boiling-point, and kept there for 6 hours, and the other is immersed in fresh water for 28 days. The pats should remain firmly attached to the plates, and should show no signs of cracking or expanding or otherwise disintegrating.

Fineness.—All other conditions being the same, the adhesive power or the cementitious value of cement, that is, its power to bind together particles of inert sand, depends upon its fineness. A larger percentage of sand may therefore be employed in making mortar with a fine cement than with a coarse one. The Engineer Department requirements are:

$$\left.\begin{array}{l}\text{Natural cement } 80\%\\ \text{Portland cement } 92\%\\ \text{Puzzolan cement } 97\%\end{array}\right\}\begin{array}{l}\text{shall pass through a sieve made of wire 0.005 inch in}\\ \text{diameter, having 10,000 openings per square inch.}\end{array}$$

Fineness may result from the clinker being underburned and therefore more easily reduced to a fine powder; this defect will be developed by the other tests.

Tensile Strength.—As in the case of metals, the test for tensile strength is easily made and furnishes reliable data for determining the value of any given cement, and for comparing the qualities of different varieties. For the purpose of making this test the cement, either neat or mixed with sand, is made into a paste by the addition of water, and is then molded into briquettes. The briquette when taken from the mold is of the form shown in Fig. 117 and is 1 inch in thickness. Its least dimension of cross-section is 1 square inch. To secure comparable results it is necessary to employ constant proportions, by weight, of cement, sand, and water in making the paste for each briquette, and to employ the same amount of pressure in forcing them into the mold.

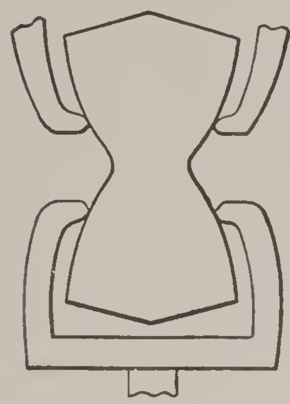

FIG. 117.

The percentage of water by weight in neat tests is given by the Engineer Department specifications as follows: For natural cement 30 per cent, for portland cement 20 per cent, for puzzolan cement 18 per cent.

In mortar briquettes the proportions are 1 cement to 1 sand in testing natural cement, and 1 cement to 3 sand in testing portland and puzzolan cements. In making the mortar the water is added after the cement and sand are thoroughly mixed in a dry state. The percentage of water, by weight, to the mixed cement and sand is: for natural cement 16 per cent, for portland 12 per cent, and for puzzolan 10 per cent.

After the briquette is made it is covered with a damp cloth and left for 24 hours; it is then removed from the mold and immersed in fresh water until tested. The temperature of the water should be between 50° and 70° F. The tensile test is made in a machine which grips the briquette as shown in Fig. 117, and ruptures it at the minimum cross-section by a gradually increasing pull. Several varieties of these machines are in the market.

The Engineer Department requirements for briquettes which are kept 24 hours under a damp cloth and then immersed in fresh water are:

For Natural Cement.—A neat-cement briquette must at the end of 7 days have a tensile strength of 90 pounds per square inch; at the end of 28 days, a tensile strength of 200 pounds.

A briquette of 1 cement and 1 sand must at the end of 7 days have a tensile strength of 60 pounds, and at the end of 28 days a tensile strength of 150 pounds.

For Portland Cement.—A briquette of neat cement must at the end of 7 days have a tensile strength of 450 pounds per square inch, and at the end of 28 days have a tensile strength of 540 pounds.

A briquette of 1 cement and 3 sand must at the end of 7 days have a tensile strength of 140 pounds, and at the end of 28 days a tensile strength of 220 pounds.

For a special quick-setting portland the required strength is reduced 12 per cent for the neat cement, and 15 per cent for the cement and sand briquettes.

For Puzzolan Cement.—A neat-cement briquette must at the end of 7 days have a tensile strength of 350 pounds, and at the end of 28 days a tensile strength of 500 pounds.

A briquette of 1 cement and 3 sand must at the end of 7 days have a tensile strength of 140 pounds, and at the end of 28 days a tensile strength of 220 pounds.

The stress is applied at the rate of 400 pounds per minute; the highest result from a set of briquettes made at the same time is the governing test. A dozen briquettes of the same kind are usually made for testing purposes; the paste is mixed for not more than four at a time, lest the cement set before the last briquette is molded.

Specific Gravity.—This test is recommended for the purpose, of determining whether a portland cement has been adulterated, the materials employed in this process having a less specific gravity than properly burned cement. It is also a test for improper burning.

Activity.—To secure uniform results, it is usual to reject all cements which set either too rapidly or too slowly. For this purpose two limits are established, known respectively as the *initial* and the *final* set. The time of initial set is when the neat-cement paste will just bear without indentation a wire $\frac{1}{12}$ of an inch in diameter, supporting a weight of $\frac{1}{4}$ pound. The time of final, permanent, or hard set is when the paste will just bear without indentation a wire $\frac{1}{24}$ of an inch in diameter, supporting a weight of 1 pound.

The Engineer Department requirements are:

	Initial Set.	Final Set.
Natural cement.........	Not less than 20 min.	Not more than 4 hrs.
Portland "	" " " 45 "	" " " 10 "
Puzzolan "	" " " 45 "	" " " 10 "
Quick-setting portland...	Betw. 20 and 30 "	" " " $\frac{3}{4}$ to $2\frac{1}{2}$ hrs.

The tests are applied to two pats similar to those employed in the test for soundness.

Short-time Tests.—The boiling test for soundness and the tests for time of setting should be made when either time or appliances are lacking for the more complete ones. Masons test the time of final setting by the pressure of the thumb-nail; if no indentation can be made, it is assumed that the cement has attained its final set. When only short-time tests can be made, the Engineer Department limits the choice to cements which have been satisfactorily employed in the locality of the proposed structure for at least three years.

Sand, Gravel, and Broken Stone.

Sand.—Sand is the granular product arising from the disintegration of rocks. *Bank* or *pit* sand is that from inland excavations; *river* and *sea* sand are those excavated along the shores or dredged from the bottom of bodies of fresh and salt water.

Specifications for engineering work usually require the sand to be "*clean* and *sharp*."

By a clean sand is meant one which, if shaken with water in a bottle and then allowed to settle, will leave no scum on the surface of the water and no layer of fine mud on the surface of the sand. River and sea sands, if not affected by the sewage of cities, are usually clean. Sea sand, however, contains salts which attract moisture and produce an efflorescence on the surface of the masonry; if this is objectionable, the sand must be washed in fresh water. A small percentage of non-decaying impurity, about 5 per cent, is usually allowed in sand; the percentage may be readily determined by shaking the sand with water in a graduated glass or bottle, and comparing the depth of the surface layer of mud with that of the clean sand. Dirty sand may be made clean by washing it in a trough of running water.

By a sharp sand is meant one composed of rough, angular grains. Silicious bank sand is usually sharp; river and sea sands have rounded grains. The sharpness may be determined by rubbing it in the hand or by magnifying it with a lens. Sand which is otherwise acceptable is not ordinarily rejected for want of sharpness.

Sand may be classed as *coarse* if it is retained on a sieve having 20 wires to the inch; it may be classed as *fine* if it passes through a sieve having 30 wires to the inch. Coarse sand is preferable to fine or mixed, if there is sufficient paste to fill the voids. The strength of the mortar depends upon the amount of paste. If less paste is used, a mixed sand is better than one composed of grains of uniform size, since the small grains will assist in filling the interstices or *voids* between the large ones, and thus decrease the amount of paste necessary to make a homogeneous mass.

The dust formed in crushing stone which passes through a No. 4 sieve may be employed in the place of sand; it gives a stronger mortar than sand itself.

Gravel.—Gravel is an aggregation of small rounded stones varying in diameter from $\frac{1}{4}$ to $1\frac{1}{2}$ inches. It is employed alone, and also mixed with broken stone, in making concrete.

Broken Stone.—Broken stone is the product produced by breaking quarry stone in a stone-crusher, and separating it from the dust and large fragments, by passing it through an inclined cylindrical revolving screen. Circular holes of different sizes in this screen first remove the dust and then allow the broken stone to fall into bins placed beneath. It is usually specified that the stone shall pass through a two-inch ring. The best stone for this purpose is tough and breaks into irregular cubes with rough surfaces. For concrete the cubes should be of different sizes, so that the small pieces may fill the voids of the large ones.

Lime and Cement Mortar.

Mortar may be employed not only to bind the parts of a masonry structure together, but also as a surface coat of floors, sidewalks, etc. The binding material of the mortar is the paste made of lime or cement, and water; the sand is simply an inert substance, which helps to fill the voids and joints without reducing the strength of the binder below the desirable limits, but greatly diminishes the cost of the masonry. In common lime mortar it also prevents undue shrinkage and increases the compressive resistance of the paste.

Common Lime Mortar.—The process of making lime mortar consists in reducing quicklime to a paste by slaking and then mixing this paste with sand. The quicklime, broken into lumps of convenient size, is placed to the depth of 6 or 8 inches in a water-tight box. With buckets or a hose it is then drenched with just sufficient water to insure its thorough slaking or reduction to powder. During the slaking it may be covered with canvas or a layer of sand to retain the heat and increase its activity, or it may be stirred for the same purpose. When the lime is completely slaked it is covered with the proper volume of sand, evenly spread, and the two ingredients are mixed with a shovel or hoe until every grain of sand is covered by lime paste.

If a box is not available, the sand is placed on a platform and the lime is put in a basin made in the sand.

The slaking is the only operation requiring special attention. If too much water is employed, the lime paste is too thin; if too little water is used, the paste will be full of pieces of unslaked lime. The latter defect may be partially remedied by the addition of more water, but the paste will not be as smooth as it would have been had the proper amount of water been used at the beginning of the process. The proper amount of water is determined by experiment.

Mortar may be mixed in a pug-mill or in a concrete-mixer. The volume of sand employed depends upon the richness of the lime and varies between $2\frac{1}{2}$ and 4 times the volume of lime paste.

Lime mortar is commonly used as soon as made; as it hardens slowly, however, this is not absolutely necessary.

The use of lime mortar is confined to dry places where it is exposed to the air. It is employed in the construction of thin brick walls above ground and in the foundation coats of plaster; it loses its binding properties when exposed to dampness, as in basement walls, and when excluded from contact with air, as in thick walls.

Cement is often added to lime mortar to give it strength. The proportions may then be 1 measure lime, 1 cement, 6 sand.

Cement Mortar.—As the cement is in the form of powder, it is only necessary to mix it thoroughly with the proper proportions of sand and water. There being at least twice as much sand as cement, in mixing, one half the sand is spread in an even layer in a box or on a platform; the cement is spread over this, and the remainder of the sand over the cement. The two ingredients are then thoroughly mixed by turning the mass over at least three or four times with a shovel. A bowl-shaped depression being made in the center of the mass into which the proper amount of water is poured, the process of mixing is then repeated until every grain of sand is coated with cement paste. The strength of the mortar depends largely upon thorough mixing. Cement mortar may also be mixed in a pug-mill or a concrete-mixer. The proportions commonly employed are 1 measure of natural cement to 2 of sand, and 1 measure of portland or puzzolan cement to 3 of sand. Both stronger and weaker mixtures are also used. A perfect mortar is one in which the cement exactly fills the voids in the sand. The volume of voids varies from 30 to 50 per cent of the volume of the sand.

Cement mortar must be used immediately after mixing so as to avoid the setting of the cement before the mortar is in place; it is sometimes specified that the mortar must be in place one hour after it is made.

Uses.—Portland-cement mortar is employed in structures in which great strength is required, as in masonry dams and masonry arched bridges; where the surface is exposed to mechanical wear, attrition, or blows, as in sidewalks and fortifications; and takes the place of natural cement whenever the cost of the work is not thereby increased.

Natural-cement mortar is used in the construction of ordinary walls, sewers, foundations for roadways, etc., when portland is considered too expensive.

Puzzolan-cement mortar is employed wherever natural is used, but should not be employed as a surface coat.

Quick-setting mortars are employed in harbor work between high and low tide, and whenever the masonry must, shortly after it is laid, be exposed to the action of moving water or to frost.

The strength of cement mortar increases with age; in the first year a 1-to-3 portland will attain a tensile strength of over

400 pounds, and a 1-to-2 natural a strength of over 220 pounds. The rate of increase decreases with the age.

Mortar may be made water-proof by adding $\frac{3}{4}$ of a pound of pulverized alum to each cubic foot of sand, and $\frac{3}{4}$ of a pound of soft soap to each gallon of water used.

Cement Concrete.

Cement concrete is an artificial stone in which gravel or broken stone are bound together by cement mortar; the particles united are called the *aggregate*, the uniting material is called the *matrix*.

The aggregate, like the sand in mortar, is introduced to reduce the cost of the artificial stone without reducing its strength below the limits required in practice. Besides gravel and broken stone, broken bricks, cinder, slag, shells, etc., may all be used as aggregate. The character of the aggregate depends largely on the cost and the ultimate strength required. It should always be clean.

If the concrete is made by hand, the aggregate is spread in an even layer, 8 to 12 inches deep, on a platform of boards and thoroughly moistened with water. The mortar, made on an adjoining platform as previously described, is spread over the aggregate and the whole mass is mixed with shovels or hoes until every stone is thoroughly covered with mortar.

If mixed by machinery, the proper proportions of aggregate, cement, and sand are dropped from an elevated platform into a revolving mixer, into which the water may be introduced through the hollow axle. The amount of water and the number of revolutions necessary for complete mixing are determined by experiment. When the concrete is thoroughly mixed it is dropped into cars or carts which are run beneath the mixer. This method is employed on large works. The mixer may be a cubical iron box revolving on a diagonal axle.

Various forms of gravity mixers are also employed.

The amount of mortar should be slightly in excess of the volume of voids in the aggregate. The volume of voids is determined by filling a vessel, whose cubic contents are known, with aggregate, then adding water until the surface of the water coincides with that of the aggregate. The volume of the water poured into the vessel is the volume of the voids in the material when dry and loose. The volume of voids will be less when the concrete is rammed in place. The volume of voids may vary from 35 to 50 per cent of the volume of the aggregates.

The proportions by measure commonly employed in the manufacture of concrete are: natural cement 1, sand 2, aggregate 4 to 5; portland cement 1, sand 3 to 4, aggregate 4 to 8. The proportions vary with the strength required. The most economical proportions can be determined only by varying the proportions of sand and aggregate and studying the result.

Use.—Concrete is employed in the construction of walls, arches, floors, sidewalks, foundation for roadway pavements, and is especially valuable in fortification work on account of its resistance to the penetration of large projectiles; in submarine work, where it may be laid when necessary without excluding the water; and in foundation work, where it is employed in leveling up for the lowest course of brick or stone work.

Strength.—At the end of a year the crushing strength of natural-cement concrete is about 800 pounds, and of portland 2000 pounds per square inch. The tensile strength of concrete cannot be accurately given; if carefully made, it is probably safe to say that its strength per square inch will be about one-tenth its compressive strength as given below.

Experiments seem to indicate that the modulus of flexure of portland-cement concrete is about one and one-half times its tensile strength.

Artificial Cement Stones.—Artificial cement stones are usually carefully made cement mortar which is rendered compact by ramming or compressing the mortar in molds. By making suitable molds the stone may be given any desired form. Beton-Coignet, used in France, belongs to this class.

Ransome stone is an artificial stone made by a different process. The mortar is made of sand, silicate of soda, and water, and is compressed in molds in the usual way. The stone is then immersed under pressure in a hot solution of calcium chloride; a chemical change now takes place resulting in the formation of calcium silicate, which forms an insoluble cement, and sodium chloride, which is removed by repeated washings.

Asphalt and Coal-tar.

Both asphalt and coal-tar are used with sand to make bituminous mortar and concrete.

Asphalt is a natural bituminous substance which is found impregnating stone, as the Val de Travers and Seyssel asphalt; in the form of pitch containing more or less organic and inorganic impurities, as the Trinidad and Bermudez asphalt; and also in the liquid form, as some of the asphalts of this country. The purest deposits, like the Bermudez, contain over 90 per cent of pure bitumen.

The Trinidad Lake asphalt, which has been more extensively used in this country than any other, contains in its natural state about 50 per cent of impurities and is prepared for use in the following manner: The crude asphalt is placed in large stills and there subjected to a temperature of about 300° F. The lighter impurities rise to the top and are skimmed off, while the earthy matter sinks to the bottom; this process is called *refining*. While still hot, the asphalt is mixed with a heavy petroleum oil, which softens it and converts it into *asphalt cement*. The hot cement is mixed with hot sand to make asphalt mortar, and with hot gravel or broken stone to make asphalt concrete. Both mortar and concrete must be put in place while still hot. The percentage of asphalt in the mortar is about ten, which is about the percentage of asphalt in asphalt rocks. These are therefore converted into mortar by reducing them to powder and then subjecting them to heat.

Asphalt mortar is used for roadway pavements; it is also used for roofing, but as it is liable to crack when subjected to changes of temperature, it is not a good material for this purpose.

Pure asphalt, being impervious to water, is used in coating water-tanks and reservoirs and for covering roofing-felt.

Asphalt concrete is also used in roadway pavements and in foundations for machinery when vibration is to be avoided.

Coal-tar and pitch, although not so good as asphalt, are employed for the same purposes.

For further information consult: Merrill's "Stones for Building and Decoration"; Sabin's "Cement and Concrete"; Butler's "Portland Cement"; Taylor & Thompson's "Concrete Plain and Reinforced"; "Professional Papers, Corps of Engineers, U. S. Army, No. 28."

CHAPTER XXI.

MASONRY.

MASONRY is the art of erecting structures in natural and artificial stone; it is ordinarily treated under the heads Stone Masonry, Brick Masonry, and Concrete Masonry.

STONE MASONRY.

Rankine's Rules.—Rankine gives the following general rules to be followed in the construction of stone masonry:

1. *Build the masonry, as far as possible, in a series of courses, perpendicular, or as nearly perpendicular as possible, to the direction of the pressure they have to bear; avoid all long joints parallel to that pressure.*

2. *Use the largest stones for the foundation course.*

3. *Lay all stratified stones in such a manner that the principal pressure which they must resist shall act perpendicular, or as nearly perpendicular as possible, to the planes of stratification.* This is called laying the stone *on its natural bed*, and is of primary importance in strength and durability.

4. *Moisten the surface of dry and porous stones before bedding them, in order that the mortar may not be dried out too fast and reduced to a powder by the stone absorbing its moisture.*

5. *Fill every part of every joint, and all spaces between the stones, with mortar; taking care at the same time that such spaces are as small as possible.*

These rules being followed, the strength of the masonry will depend upon the *character*, *size*, and *shape* of the stone, upon the *accuracy of dressing*, and upon the *bond*.

Character, Size, and Shape.—As has been previously stated, the allowable unit stress on a stone depends upon its composition and texture, and the strength of the mortar upon the character of the cement and the proportion of the ingredients. Fine uniform-grained granite is ordinarily specified when the unit pressure on the masonry is very great; it is laid with strong portland-cement mortar. Where great strength is required the stones are from 2 to $2\frac{1}{2}$ feet thick; in *first-class masonry* they are from 1 to $2\frac{1}{2}$ feet thick; and in *second-class masonry* not less than $\frac{2}{3}$ of a foot thick.

The width of the stones is from *one and a fourth* to *twice* their depth, and their length is from *twice* to *three* times their depth. The stones used in first-class masonry therefore weigh from 1 to 4 tons. The width of the face of a stone is called its *bed;* the depth is called its *build.*

Accuracy of Dressing.—In order that the thickness of the layer of mortar between the stones shall be a minimum, the surfaces of the stones must be carefully dressed. The more careful the dressing the more uniform will be the pressure transmitted from one stone to another. In *first-class masonry* it is therefore specified that the *bedding-joints* shall not exceed $\frac{1}{4}$ to $\frac{1}{2}$ inch in thickness; the *vertical joints* are dressed with equal care, but only to a depth of 12 inches from the surface. In *second-class masonry* less care is taken; the stones are so dressed that the joints shall not exceed $\frac{1}{2}$ to $\frac{3}{4}$ of an inch in thickness.

Bond.—The term *bond* means the method of uniting the masonry so as to form a homogeneous mass without planes of weakness. In forming the bond the stones are divided into *stretchers* and *headers.* A stretcher is a stone whose longest dimension is parallel to the face or back of a masonry wall; a header is a stone whose longest dimension is perpendicular to the face or back of the wall. The bond is formed by making each course of both headers and stretchers, and by laying the stones in the successive courses so that there shall be no continuous vertical joints. The wall is thus bound together both horizontally and vertically, and the pressure on each course of masonry is uniform throughout. The ordinary form of bond is shown in Fig. 118.

In *first-class masonry* the headers extend entirely through the wall if it is less than 5 feet thick; if over 5 feet thick, they must be at least 4 feet long, and extend into the wall 20 inches more than the adjacent stretchers. The distance between vertical joints in consecutive courses is at least 1 foot. In special cases two stretchers are allowed to one header. In *second-class masonry* the minimum length of headers in thick walls is $3\frac{1}{2}$ feet, and three stretchers are allowed to one header. In general

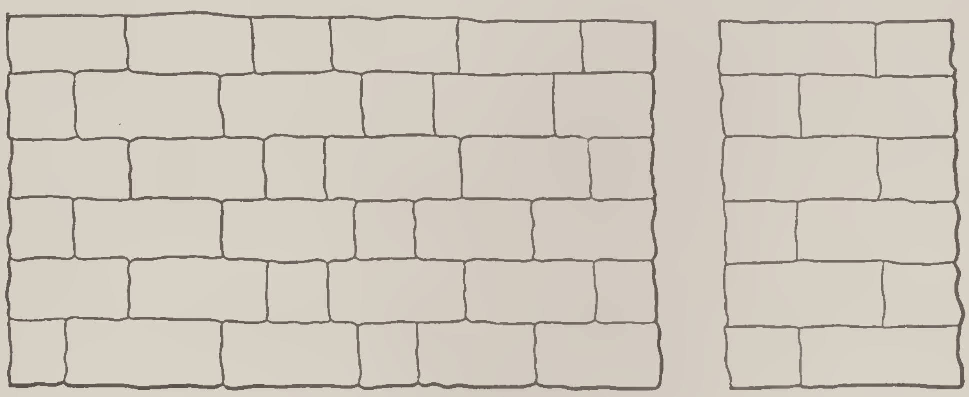

FIG. 118.

the headers should constitute at least *one-sixth the face of the wall.*

When the above methods do not give the desired strength special bonds are made by the use of steel or iron *dowels* and *cramps*. A dowel is a pin or bar which fits in holes drilled or cut in adjacent stones of consecutive courses, and prevents the stones moving separately in a horizontal direction. A cramp is a *Z*, or channel-shaped iron strap, whose long arm is laid in a joint and whose short arms are vertical and are inserted in holes formed in the stones.

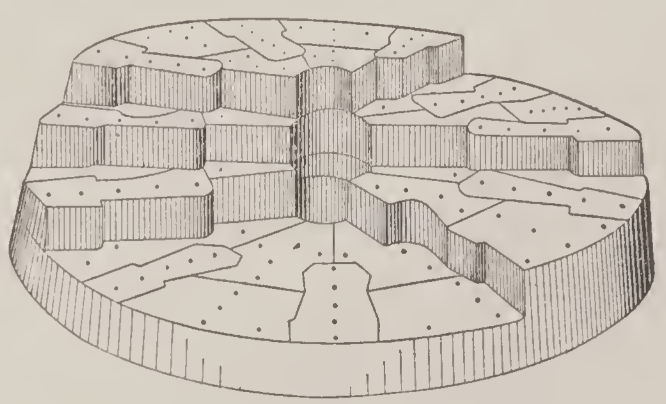

FIG. 119.

In masonry lighthouses on exposed sites, the stones of the same course are also notched into each other to prevent the jumping out of the exterior stones under wave action in a violent storm. Fig. 119 shows the method of doweling and notching employed by the Engineer Department in the construction of the Spectacle Reef Lighthouse, Lake Huron.

According to the amount of dressing, building-stones are divided into three classes: *unsquared* or *rubble stones; squared, hammered,* or *rough-dressed* stones; and *cut stones.* The first class covers all quarry stones that are dressed simply by knocking off acute angles and excessive projections; the second class covers stones that are dressed with a face hammer or an axe until the stones are roughly squared and the vertical and bedding joints are roughly dressed; the face of the stone is usually *rock-* or *quarry-faced*, *pointed*, or *hammered;* the third class covers all squared stones with smoothly dressed vertical and bedding joints; the face dressing may be any of those described under building-stones.

According to the principles of construction masonry structures are of three classes: *simple walls*, *retaining- and reservoir-walls*, and *arches.*

Simple Walls.—*Definitions.*—The *face*, *back*, *batter*, and *profile* were described under Retaining-walls.

Facing.—The stones that have one surface in the face of the wall.

Backing.—The stones that have one surface in the back of the wall.

Filling.—The masonry between the facing and the backing.

Course.—A horizontal layer of masonry; usually one stone in depth.

Coping.—A course of stone placed on top of a wall that is exposed to the weather, to bind the wall and protect its masonry.

Footing.—The courses of masonry at its base, which project beyond the face and back and serve to increase the bearing area.

Quoins.—The stones at the angle when two walls meet.

Joints.—The surfaces of contact of the individual stones.

Bedding-joints.—The joints, usually horizontal except in arches, through which pressures are transmitted.

Builds.—The joints normal to the bedding-joints.

Simple walls are intended to resist vertical forces only; the bedding-joints are therefore horizontal.

The masonry in a wall may be *rubble*, *squared stone*, or *ashlar.*

Rubble masonry is that made of unsquared or rubble stone, and may be laid either dry or in mortar. It is *uncoursed* or *random rubble* when no attempt is made to lay it in horizontal courses; it is *coursed rubble* when it is laid in horizontal courses, although all the stones in a course are not of the same depth. In laying an uncoursed rubble wall, the large stones are bedded as well as possible and then the interstices are filled up with small stone. In laying a coursed rubble wall the wall is constructed in layers a foot or more in thickness, depending upon the size of the stone. *Dry rubble* masonry is used principally for inclosure walls; rubble laid with mortar is used for inclosure walls and for walls of low buildings. Inclosure walls are usually capped by a *coping* which is made of stones of greater width than the wall.

Squared or hammered stone masonry is made of squared or hammered stone and is laid in mortar. If the stones in each course are of the same depth so that the bedding-joints are continuous, it is called *range-work;* if laid in courses not continuous throughout, it is called *broken-range work;* if no attempt is made to lay it in courses, it is called *random work*. Squared stone masonry is employed in the construction of basements and other walls in which appearance is not a governing consideration. At the corners of a building constructed of this masonry cut-stone quoins are frequently used to tie the walls together.

Ashlar masonry is that made of cut stone; it is usually designated by the face finish, as *rock-* or *quarry-face ashlar.* The bedding-joints are ordinarily continuous; when not it is called *broken ashlar.* Ashlar is also called *range*, *broken-range*, and *random ashlar*, depending on the character of the bedding-joints.

Dimension-stones are cut stones all of whose dimensions are specified.

Combined ashlar and rubble walls are constructed when it is desired to secure a pleasing appearance at a less cost than that of a full ashlar wall. The ashlar forms the facing, and coursed rubble the backing; the two are united by ashlar headers which extend through or well into the rubble, or they may be united by cramps. Brick masonry is also used to back ashlar.

The pressure of beams, wall-plates, etc., on masonry should not exceed the following limits:

Rubble masonry with cement mortar......	150	lbs. per square inch
Portland cement concrete...............	250	" " "
First-class masonry, sandstone.....	200 to 300	" " "
" " limestone.....	300 " 500	" " "
" " granite.......	400 " 800	" " "

Weight and Allowable Compressive Stress.—The weight of stone masonry is usually estimated at 160 pounds per cubic foot.

The following have been recommended as allowable stresses in rubble masonry:*

Rubble masonry, lime mortar	5	tons per square foot
" " natural-cement mortar	6	" " " "
" " portland-cement mortar	8	" " " "
Coursed rubble, Portland-cement mortar	10	" " " "

For first-class stone masonry the Boston regulations allow the following:

Sandstone	30	tons per square foot
Marble and limestone	40	" " " "
Granite	60	" " " "

In ashlar faced work no allowance over the strength of the backing is made for ashlar less than 8 inches in thickness; if 8 inches or more in thickness, the excess over 4 inches is allowed.

Retaining- and Reservoir-walls.—The same principles which govern the construction of a simple wall also govern the construction of a retaining- or reservoir-wall. In accordance with Rankine's first rule the bedding-joints of a retaining- and reservoir-wall should not be horizontal, but should be as nearly as possible normal to the resultant pressure. This is rarely ever done in practice because of the extra expense and because of the small angle made by the resultant with the vertical. To resist the shearing action of the horizontal component of the resultant, it is customary in high walls to make the filling between the back and face of the wall of uncoursed rubble of large stones. Such walls have no continuous horizontal joints.

Arches.—*Ovals.* Next to circular, the oval arches of an odd number of centers are the ones most often constructed in engineering practice. Whatever be the span and rise, the tangents to these ovals are vertical at the springing-lines.

To Construct an Oval of any Odd Number of Centers.—In Fig. 120 on the span AR describe the semicircle AIR, and divide it into the desired odd number of equal parts. Draw radii to the points of division. Assume the first center J, so that AJ is less than BC, the assumed rise. Draw DJ parallel to EB. Assume center K and draw OK parallel to FB. Assume center L and draw PL parallel to GB. From P draw PQ parallel

* Proceedings Am. Soc. C. Engrs., Sept. 1904.

to the chord GH, and from C draw CQ parallel to the chord HI. From their intersection draw QM parallel to HI. Then will J, K, L, M, and N be the centers of the circular arcs forming the oval arc $ADPQC$.

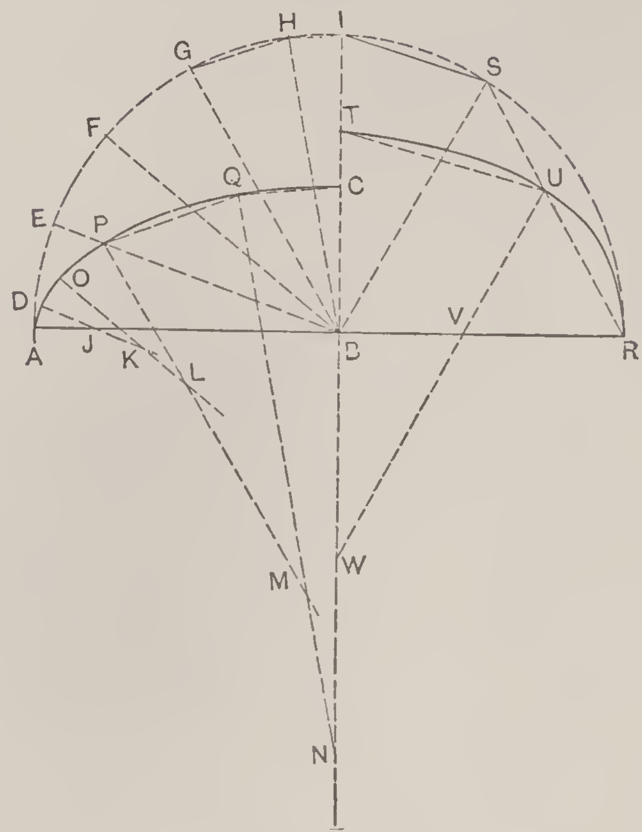

FIG. 120.

The Basket-handled Oval.—This is considered the most graceful form of oval and is employed in the construction of ornamental arched bridges.

In the semicircle Fig. 120 inscribe a regular hexagon of which RS is one side. Draw IS, and parallel to it TU. Draw SB, and parallel to it UW. Then will V and W be the centers of the oval arc TUR. In the basket-handled oval each circular arc subtends an angle of 60 degrees.

Centers.—Since, in a completed arch, each semi-arch is supported by the pressure of the other transmitted through the keystone, the arch must be supported during construction until the keystone is inserted. The frame employed for this purpose is called a *center*. It is composed of a cylindrical platform of planks or boards upon which the masonry rests, and a series of wooden arched or trussed ribs which support the platform. For small arches, such as window-caps, culverts, sewers, etc., a center like that shown in Fig. 121, but without the intermediate struts, may be employed; for somewhat greater spans the struts must be inserted; for masonry bridge arches the platform is supported by a very strong truss somewhat similar to the truss constructed to support a curved roof. The truss must be strong enough to bear the weight of the arch without deformation of the curve of its platform.

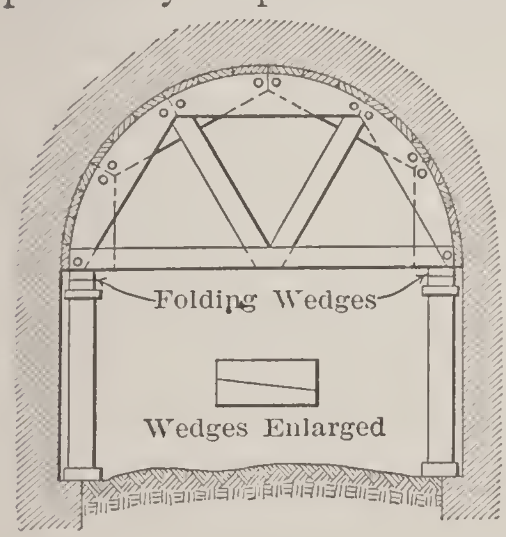

FIG. 121.

The small center shown in the figure rests on folding wedges by means of which it may be slowly lowered when the keystone has been inserted and its mortar has set. A similiar device is employed under the supports of heavy centers, or they may rest on pistons which are supported, in iron cylinders, upon beds of sand; when the center is to be removed, the sand is allowed to escape slowly. Centers of large arches are not removed for several months after the keystone is laid.

In laying the masonry on the center, care is taken to carry up the two semi-arches simultaneously. It is sometimes necessary to load the crown of the center to prevent its rising when the semi-arches have only reached the haunches.

Construction.—In a *right arch* the width of each voussoir, measured along the intrados, is the same; and the coursing-joints are normal to the intrados. Although the voussoirs may increase in depth from the crown to the springing-lines, each stone is a simple right prism and can be readily cut. Five of its faces must be accurately dressed. The arch is constructed in the same manner as a masonry wall; the coursing-joints are continuous, and heading-joints break their continuity at every coursing-joint. If the arch is backed with rubble masonry, as is usually the case between the haunches and springing-lines of large arches, some of the arch-stones are made long enough to bond the arch and its backing.

If the arch is oblique, the construction is not so simple. Three methods of construction have been devised: the *ribbed method*, the *logarithmic method*, and the *helical method*. The first is applied to arches of slight obliquity, and consists in making a number of small overlapping right arches, each with the same rise and span as in Fig. 122. The soffit is therefore not a continuous surface; but the coursing-joints are normal to the pressures, and the arch is easily constructed.

In the *logarithmic method* the heading-joints are all parallel to the head of the arch, as in a right arch. The coursing-joints are warped surfaces generated by constructing lines on the soffit normal to the heading-joints, and using these lines as directrices, and lines normal to the soffit as generators. The projections of the directrices on a horizontal plane are logarithmic curves, and hence the name of the method. The soffit is continuous; the coursing-joints are nearly normal to the pressures; but the arch-stones vary in size and are difficult to cut.

In the *helical method* the intersections of the heading-joints and soffit are helices parallel to the helix which passes through the intersections of the springing-lines and crown with either face of the arch. The intersections of the coursing-joints with the soffit are helices perpendicular to the heading-joint helices. Both systems of joints are approximately perpendicular to the soffit. The soffit is continuous; with the exception of the stones in the faces and along the springing-lines, the arch-stones are all of the same size and are easily cut; the coursing-joints are, however, more oblique to the pressures than in either of the other

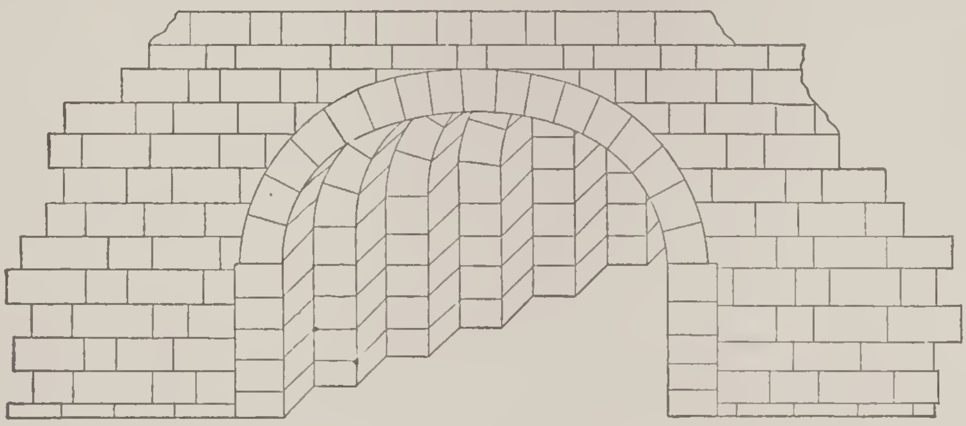

FIG. 122.

methods. In the helical method cut stones must be used in the face and along the springing-lines, but bricks may be used in the rest of the soffit.

Use.—Masonry arches of large span are employed principally in bridge construction. The Cabin John bridge, on the line of the Washington aqueduct, constructed by Captain Meigs of the Corps of Engineers, has a span of 220 feet and was for many years the longest span stone arch.

There are now several in Europe with longer spans. At Plauen, Saxony, there is a five-center oval stone arch which has a rise of 60 feet and a span of 295 feet.

Pointing Masonry.—The joints of stone masonry are *pointed* to prevent water from penetrating the joints, and, when freezing, from forcing out the mortar; and also to secure a pleasing appearance. Pointing consists in cleaning out the joints to a depth of about an inch, brushing them clean, moistening the masonry, and then filling the joints with a rich portland-cement mortar well rammed in. The surface of the pointing is given a smooth finish with a pointing-iron. In order to avoid bringing excessive pressure on the pointing and thus forcing it out of the joint, the operation of pointing is the last work done on the masonry; it is therefore done after the mortar has had time to set and the masonry to settle under the action of its final weight. The pointing should not project beyond the face of the wall.

Settling of Masonry.—Unequal settling is avoided by completing each course of masonry before beginning upon the succeeding one, by bonding the masonry well, and by making the joints narrow and uniform in width.

Effect of Temperature.—In very hot weather mortar is liable to be injured by the too rapid evaporation of its water; this interferes with its normal hardening or setting. To prevent this the stone should be moistened before it is laid, and the masonry itself should be kept moist until the mortar hardens or sets. It is especially necessary to protect the top of an unfinished wall when the work is temporarily suspended.

In very cold weather the hardening or setting of the mortar is retarded by the cold, and in freezing weather the mortar is also disintegrated by the freezing and expansion of its water. It may be used in freezing weather if its temperature can be kept above the freezing-point long enough to allow it to set with sufficient strength to resist the disruptive effect of the frost. This may be accomplished by using a quick-setting portland, by using hot water, by putting salt in the water used in making the mortar, by heating the stones, or by covering the masonry, as soon as laid, with canvas, straw, manure, etc. As workmen are careless, it is usually better, however, to suspend work while the temperature is below the freezing-point.

Brick Masonry.

The general rules given by Rankine (page 387) apply to brick masonry where applicable. Bricks being more porous than stone, rule 4 is more important in brick than in stone masonry.

Brick masonry may be laid with cement, cement and lime, or with lime mortar. The Boston building regulations require first-class or fire-proof buildings to be constructed with cement mortar; second-class buildings to be constructed with cement and lime mortar; and allow only third-class buildings to be constructed with lime mortar.

Bond.—The bond in brickwork is secured, as in stone masonry, by the use of headers and stretchers, and by breaking vertical joints. If each course of masonry consists of alternate headers and stretchers, the bond is said to be *Flemish*. In laying masonry with this bond, the center of each header is in the same vertical line as the centers of the stretchers above and below it. If each course consists wholly of headers or wholly of stretchers, the bond is said to be *English*. In the English bond as commonly constructed there are five to seven courses of stretchers to one of headers. The English bond is more easily laid than the Flemish, and its strength is easily varied by changing the ratio of the number of header to stretcher courses; it presents, however, a less pleasing appearance. If the face of the wall is made of stretchers only, the bond is made by cutting off the corners of the bricks in the usual header-courses and inserting diagonal headers, or by cutting the stretchers of this course lengthwise, removing the rear half and inserting normal headers. This is called a *running* or *blind* bond. Where the above bonds do not give sufficient strength, metal strips may be laid in the joints.

Terra-cotta blocks, used as furring, are bonded to the wall with clamps. Walls which meet at an angle should be bonded, at vertical intervals of 10 feet, with T- or L-shaped iron anchors which extend several feet along each wall.

Walls.—The least thickness of a brick wall is 8 inches; this is the thickness of the walls of a narrow and low dwelling. The approximate rule for high walls is to make the first 25 feet, measured downwards from the roof, 12 inches thick in dwellings, and 16 inches thick in other buildings. The next 25 feet is made 4 inches thicker, and this rate of increase is preserved to the ground-line. From the ground-line to the footing the increase is 4 inches for every 10 feet. The change in thickness is usually made at the floors. The pressure of wall-plates should not exceed 150 pounds per square inch for walls laid with cement.

Weight and Allowable Pressure.—The weight of brick masonry is usually assumed to be 115 pounds per square foot.

The following allowable pressures have been recommended: *

* Proceedings Am. Soc. Civil Engrs., Sept. 1904.

Common brick, lime mortar	7	tons per square foot
" " natural-cement mortar	8	" " " "
" " portland-cement mortar	10	" " " "
Hand-burned brick, " "	12	" " " "

Brick piers are bonded by building into them, at intervals of $2\frac{1}{2}$ to 3 feet, flat bond-stones having the full dimensions of the pier. The allowable load on a pier whose height does not exceed twelve times its least dimension is about seven-eighths of the allowable load on a corresponding section of a wall if the wall has the same thickness as the pier.

Retaining- and Reservoir-walls.—Brick masonry, being lighter than stone masonry, is not so good a material for walls which must sustain a horizontal pressure. If employed in the construction of low reservoir-walls, the wall is usually made impervious by coating it with asphalt or other water-proof material. Sylvester's process has also been employed with success. This consists in applying with a brush a hot solution of three-quarters of a pound of soap dissolved in a gallon of water, and twenty-four hours later a solution of one-half pound of powdered alum dissolved in four gallons of water. This operation must be repeated several times. The soap and alum form an insoluble compound in the pores of the brick. Brick may also be rendered impervious by applying, with a soft brush, finely ground cement moistened with water; several applications are necessary.

The soap and alum process is also employed to prevent efflorescence on walls.

Arches.—Brick is extensively used in the construction of arches for window-cappings, fire-proof floors, sewers, culverts, etc.

There are several ways of laying bricks in an arch. The bricks may all be laid as stretchers; the arch is then composed of concentric rings each 4 inches thick. This is called the *ring-bond* and is employed in the construction of all arches of small span. Where several rings are necessary, it lacks strength in a radial direction; this may be secured by laying metal strips in the radial joints. The arch may be laid with continuous radial joints, the bricks in each radial course being alternate headers and stretchers, and breaking joints parallel to the intrados. This is the *radial bond*. The objection to this method is that the radial joints are very wide at the extrados. This may be avoided by using only wedge-shaped bricks. With ordinary bricks the joints are filled with pieces of slate or with flat stones. The two methods may be combined by dividing the face of the arch into sectors, and constructing alternate sectors of ring and radial bonds; the radial sectors are narrow and the ring sectors wide. This is the *combined ring and radial bond.*

The strength of brick arches was tested in 1895 in Austria with the following results:

FIRST TEST.

	Span, Feet.	Rise, Inches.	Thickness, Inches.	Final Load, Pounds per Sq. Ft., on entire span.
Special brick, lime mortar....	4.42	1.58	3.94	1638
Ordinary bricks...........	4.42	4.91	5.9	1638 applied without rupture

SECOND TEST.

	Span, Feet.	Rise, Inches.	Thickness, Inches.	Breaking Load, Pounds per Sq. Ft., applied on half the span
Special brick, lime mortar....	8.85	5.31	3.94	491
Ordinary brick, lime mortar. .	8.85	9.84	5.51	883

The arches were leveled up over the haunches with concrete.

THIRD TEST.

	Span, Feet.	Rise, Feet.	Width, Feet.	Crown Thickness, Feet.	Springing Line. Thickness, Feet.	Breaking Weight, Lin. Feet.
Brick in cement mortar. .. (300 lbs. cement to 35 cubic feet sand)	74.5	15.0	6.65	1.97	3.6	1.81 tons, or 6001 lbs. per sq. ft.

The load was a uniform live load covering one-half the arch only.

The arch in the third test supported nine-tenths the breaking load of a similar arch of cut limestone laid with the same kind of mortar.

Concrete Masonry.

Walls.—Concrete walls are usually monolithic structures, molded in place, in forms made of planks and frames. The concrete is deposited in layers 6 or 8 inches in thickness, care being taken to disturb the mixture as little as possible in moving it from the mixing-platform to the forms. It is thoroughly compressed by means of a tamp or rammer weighing 30 to 35 pounds. Each layer must not only be put in place before the concrete has had time to set, but should if possible be put in place before the preceding layer has set. This will insure thorough bonding. If the preceding layer has set, it is swept clean, moistened, and covered with a coat of mortar to insure its bonding with the new layer. The tamping is continued until the surface of the concrete is covered with a thin film of water, and the layer is completed by filling up all the depressions in the surface with mortar. If the work is executed in warm weather, the masonry must be moistened from time to time for days, and in large masses for weeks, to prevent the concrete becoming dry and friable before it has thoroughly set.

Long walls are often made in sections, to diminish the probability of developing cracks, due to stresses caused by shrinkage in setting, and by expansion and contraction due to changes of temperature. Each section is of such length that each of its layers may be put in place before the preceding one has set.

The surface of a concrete wall is made of mortar, an inch or more in thickness, which is deposited against the sides of the form; if this coat is put on after the concrete has set and the form is removed, it is liable to separate from the body of the wall.

There is some difference of opinion as to whether the concrete should be mixed with an excess of water or not; with ordinary workmen the results are usually better, and the work is cheaper if an excess of water is used or the concrete is wet.

In the construction of breakwaters, piers, dikes, etc., exposed to wave action, the concrete is commonly made into heavy blocks, these are laid in the same manner as stone masonry if the water can be excluded from the site, but are thrown in at random if the water cannot be excluded. Blocks weighing 40 tons have been molded for this purpose.

Allowable Pressures.—The allowable pressures on concrete walls are: *

Natural cement,	1:2:5	. . . .	8 tons per square foot
" "	1:2:4	. . . .	9 " " " "
Portland "	1:2:5	. . . .	15 " " " "
" "	1:2:4	. . . .	16 " " " "

Retaining- and Reservoir-walls.—Monolithic retaining-walls are made as above described; if the wall is not continuous in a vertical direction, the joint between the sections may either be sloped to the rear in accordance with Rankine's first rule, or it may be stepped.

When concrete is employed in the construction of reservoir-walls only portland-cement concrete should be employed, and care must be taken to prevent leakage. To prevent cracks in the walls, they have been constructed in short sections with continuous vertical joints. Leakage through the joints has been prevented by making a vertical well in the joint and filling it with liquid asphalt or clay puddling. The bottom of a reservoir is made water-tight by making it of two layers of small concrete squares, separated by a half-inch of rich mortar. The reservoir may also be made water-tight by coating it with asphalt applied as a paint to the concrete or to a lining of canvas.

Arches.—Concrete is employed in the construction of floor and bridge arches, culverts, sewers, etc. The arch-ring is usually a monolithic structure; if joints are necessary in its construction they should be radial ones. The strength of portland-cement concrete arches was tested in Austria in 1895 with the following results:

Composition.	Span, Feet.	Rise, In.	Thickness, In.	Breaking Loads, Pounds per Square Foot.
Portland-cement concrete. . .	4.42	4.52	2.95	1638 without rupture
" " . . .	8.85	9.05	3.35	1127 on half the arch
" " . . .	13.3	16.1	3.94	790 " " " "
" " . . .	32.8	39.0	16.4	512 " " " " cracked, 2500 " " " " broke
" " . . .	74.5	15.0	27.6	2.24 tons per running foot over half the arch, which was 6.6 feet wide.

* Proceedings Am. Soc. Civ. Engrs., Sept. 1904.

Reinforced Concrete.

The *allowable tensile* and *shearing stresses* in simple concrete are too small to admit of its use in the construction of structural forms which are subjected to forces of flexure, as floor-beams and slabs, lintels, etc.; or in the construction of those subjected to tensile forces only, as water-tanks, etc.

The *allowable compressive stress* in simple concrete, although about ten times its allowable tensile stress, is also too small to admit its use in the construction of compressive members of small cross-section which are subjected to heavy loads, as the columns of a building.

The requisite strength, in tension and compression, can, however, be secured by imbedding steel or wrought-iron rods in the concrete; these will assist the concrete in bearing the longitudinal and shearing stresses to which it is subjected. Simple concrete thus strengthened is called *reinforced concrete.* In this country the elements employed are good medium steel and carefully made portland-cement concrete. If the cross-section is small, no aggregate is employed in the concrete, which is then simply a cement mortar. Wrought iron may be employed in place of steel.

The possibility of thus making composite structural forms arises from the following considerations.

1. Cement mortar is a preservative of steel; it absorbs carbonic acid and can be applied to the metal in the form of an impervious coating.

2. Both concrete and steel have practically the same coefficient of expansion for changes of temperature.

3. The adhesion of cement mortar to the metal is sufficiently great to require a considerable force to separate them along the surfaces of contact

The composite material is cheaper and resists atmospheric agencies and the effects of a conflagration better than steel alone; it is stronger than concrete alone, and is not so liable to be disfigured or injured by cracks caused by contraction or expansion.

Concrete.—The properties of concrete are dependent upon so many considerations—character and proportion of cement; character, size, and uniformity of grain and proportion of sand;

character, size, and uniformity of particles and proportion of aggregate; thoroughness of mixing, amount of water employed, and amount of compression—that a perfectly uniform product is impossible. It results from these considerations that the constants of concrete deduced by different experimenters are not the same, nor are their proposed formulas.

The following table gives safe unit stresses for steel and portland-cement concrete, as recommended by good authorities.

	Steel.		Concrete 1 . 3 : 6.
Factor of safety	4	5	5
Compression	15,000	12,000	400
Compressive fibers in a beam	......	......	500 to 700
Tension	15,000	12,000	50
Bending	15,000	12,000	75
Shear	12,000	10,000	50 to 75
Coefficient of elasticity	29,000,000		2,900,000
Coefficient of expansion	0.0000064		0.0000064

Safe adhesion of cement mortar to steel 50 lbs. per square inch.

Columns.—Columns such as are employed in buildings are either circular or square and have usually four longitudinal rods near the surface; an additional one may be placed in the axis. These rods are tied together at short intervals by radial wires or metal strips as shown in Fig. 123. The outer rods may, in addition, be surrounded by circumferential wires or by a wire helix. This latter prevents the bulging of the concrete.

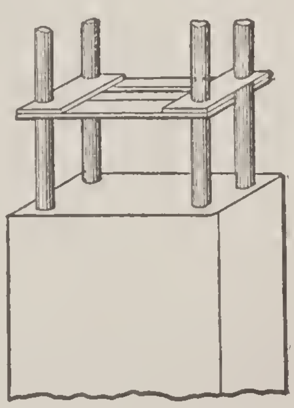

FIG. 123.

The strength of these columns is deduced on the hypothesis that the concrete and the steel rods will be shortened equally as the load increases, and that the columns will fail when the unit stress in the concrete becomes equal to its modulus of crushing.

If W' = load borne by the concrete;

A' = area of cross-section of concrete;

L = length of column;

l = amount of shortening due to W';

E' = coefficient of elasticity of concrete.

Then $\dfrac{W'}{A'} = E'\dfrac{l}{L}$ = unit stress on the concrete.

If W'' = weight borne by the steel;
A'' = area of cross-section of the steel rods;
L = length of the column;
l = shortening of steel rods;
E'' = coefficient of elasticity of steel.

Then $\dfrac{W''}{A''} = E''\dfrac{l}{L}$ = unit stress borne by the steel.

From these equations we have, since $\dfrac{l}{L}$ is the same in both,

$$\frac{W'}{E'A'} = \frac{W''}{E''A''}, \quad \text{or} \quad W' : W'' :: E'A' : E''A'', \quad \text{or} \quad \frac{W'}{A'} : \frac{W''}{A''} : E' : E'';$$

that is, *the unit stress in the concrete is to the unit stress in the steel as the coefficient of elasticity of the concrete is to that of the steel.*

Since the coefficient of elasticity of concrete is one-tenth that of steel, the unit stress in the concrete will also be one-tenth the unit stress in the steel. A square inch of steel will therefore offer the same resistance as ten square inches of concrete.

For the breaking load of a column reinforced by longitudinal rods only, we have therefore

$$W = s_c'(A' + 10A'') = 2000(A' + 10A''), \quad . \quad . \quad (554)$$

in which W = the breaking load;
s_c' = the modulus of crushing of concrete = 2,000 pounds;
A' = the area of cross-section of the concrete;
A'' = the area of cross-section of the reinforcement.

The safe load is one-fifth of the breaking load.

The formula above given has been shown by experiments to give safe results for columns in which the ratio of length to least dimension of cross-section does not exceed twenty.

For a column which has, in addition to longitudinal rods, circumferential wires or a spiral imbedded near its circumference, an additional factor must be added to the above formula to represent the additional load which may be carried, due to this strengthening. From experiment it has been ascertained that if the distance between consecutive bands or turns of the spiral is less than one-sixth the diameter of the column, the circumferential wire will be as effective as 2.4 its area employed as a longitudinal rod.

The breaking weight of such a column is therefore

$$W=s_c'(A'+10A''+24A''')=2000(A'+10A''+24A'''), \qquad (555)$$

in which A''' = area of cross-section of the wire employed in making bands or spiral wrapping.

The circumferential wire cannot replace the longitudinal rods, but must be employed with them.

Rectangular Beams.—A beam may have its reinforcement on the tension side only, or on both the tension and compression sides of the neutral axis. There are many theories on the action of composite beams under flexure. One of the simplest is that which assumes that the coefficient of elasticity of concrete is constant, and places no reliance on the tensile strength of concrete. The errors introduced by these assumptions are on the side of safety if the ultimate strength of the beam is alone considered. The theory adopted is based on the revolution of the plane of cross-section about its neutral axis, as in the common theory of flexure.

With Reinforcement of Tensile Fibers Only.—In the *cantilever beam* whose cross-section is shown in Fig. 124, the con-

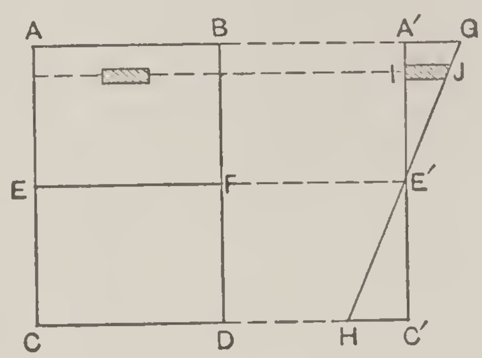

FIG. 124.

crete is assumed to resist compression only. The reinforcement is on the tensile side of the beam.

Let $ABCD$ and $A'C'$ = cross-section of a cantilever;
EF and E' = its neutral axis;
GH = position of cross-section under action of a bending moment M;
IJ = reinforcement of tensile fibers;
y = distance along $E'C'$ measured from E';
$y' = E'C'$.
$y'' = E'I$ or distance from neutral fiber to axis of reinforcement;
s''' = unit longitudinal stress at units distance from E' along $E'C'$;
b = breadth of beam = AB;

d = depth of beam $= AC$;
A'' = area of reinforcement;
s_e = unit stress in the reinforcement A''.

Since the depth of the reinforcement is small, we may assume that the stress in the reinforcement is uniformly distributed over it; the unit stress being the stress at its axis, IJ.

Since the tensile and the compressive stresses on the cross-section must be equal and the compressive stress varies uniformly from the neutral axis to the surface, we have

$$\int_0^{y'} s'''by\delta y = \frac{s'''by'^2}{2} = s_eA''. \quad . \quad . \quad . \quad (556)$$

If $s = s'''y'$ = stress at surface fiber CD, we have

$$\frac{sby'}{2} = s_eA''. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (557)$$

From the equality of moments about the neutral axis, we have

$$\frac{sby'}{2} \times \frac{2y'}{3} + s_eA''y'' = M$$

or

$$\frac{sby'^2}{3} + s_eA''y'' = M. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (558)$$

For the elongation, IJ, of an elementary length of the axial fiber of the reinforcement, we have

$$IJ = l' = \frac{s_e\delta L}{E''}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (559)$$

and for the shortening $C'H$ of the extreme fiber under compression, we have

$$C'H = l'' = \frac{s\delta L}{E'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (560)$$

From similar triangles we also have

$$IJ : C'H :: y'' : y';$$

hence

$$\frac{s}{E'} : \frac{s_e}{E''} :: y' : y'',$$

or

$$s_e = \frac{E''y''s}{E'y'} = \frac{my''s}{y'}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (561)$$

in which $m = \dfrac{E''}{E'}$.

If we represent the distance IC' by d'', we shall have

$$y''=d''-y'. \qquad (562)$$

For any given beam all the quantities in equations (557), (558), (561), and (562) are known except s, s_e, y', and y''. From these four equations we can determine the values of the unknowns.

To determine a value for y' in known terms, the value of s_e from equation (561) is substituted in (557) and (558).

$$\frac{by'^2}{2}=my''A'' \quad \text{or} \quad \frac{by'^2}{2}-my''A''=0. \qquad (563)$$

$$\frac{sby'^2}{3}+\frac{msy''^2A''}{y'}=M \quad \text{or} \quad M=\frac{s}{y'}\left(\frac{by'^3}{3}+my''^2A''\right). \qquad (564)$$

Substituting for y'' in equation (563) its value from equation (562) we have

$$\frac{by'^2}{2}-mA''(d''-y')=0,$$

or

$$y'^2+\frac{2mA''y'}{b}=\frac{2mA''d''}{b},$$

from which

$$y'=-\frac{mA''}{b}+\sqrt{\frac{2md''A''}{b}+\frac{m^2A''^2}{b^2}},$$

or

$$y'=-\frac{10A''}{b}+\sqrt{\frac{20d''A''}{b}+\frac{100A''^2}{b^2}}. \qquad (565)$$

This gives the distance of the neutral axis from the extreme fiber in compression.

Substituting in (564) the value of $my''A''$ from equation (563) we have

$$M=\frac{s}{y'}\left(\frac{by'^3}{3}+\frac{by'^2}{2}y''\right).$$

Substituting for y'' its value from (562)

$$M=\frac{s}{y'}\left(\frac{by'^2d''}{2}-\frac{by'^3}{6}\right)=\frac{sby'}{2}\left(d''-\frac{y'}{3}\right). \qquad (566)$$

In this equation $\frac{sby'}{2}$ is the total compressive stress in the cross-section, and $d''-\frac{y'}{3}$ is its lever-arm with respect to the axis of the reinforcement. If the center of moments is taken at the point of application of the resultant compressive stress, and the total tensile stress in the reinforcement is represented by F, we shall have

$$M = F\left(d'' - \frac{y'}{3}\right) = s_e A''\left(d'' - \frac{y'}{3}\right). \quad . \quad . \quad . \quad (567)$$

In the design of beams and slabs it is necessary to assume values for d'' and the safe values of s and s_e. In beams d'' is made *nine-tenths d*, and in slabs *five-sixths d*. The safe value of s may be taken as 500 pounds, and that of s_e 15,000 pounds for stationary loads, and 10,000 pounds for vibrating loads.

Substituting these values in equations (565), (566), and (567) we may deduce the following table. To make interpolation simple, the values of $\frac{A''}{bd}$ in the first column may be plotted as the abscissas, and those of the second column as the ordinates of a curve which will give the value of y' for any value of $\frac{A''}{bd}$ between 0.004 and 0.050.

$\frac{A''}{bd}$	y'	d''	$d'' - \frac{y'}{3}$	$y'\left(d'' - \frac{y'}{3}\right)$	$A''\left(d'' - \frac{y'}{3}\right)$	$\frac{M}{bd^2}$ Equation (566) $s = 500$.	Equation (567) $s_e = 15,000$.	Equation (567) $s_e = 10,000$.
0.004	$0.23d$	$0.9d$	$0.82d$	$0.1886d^2$	$0.00328bd^2$	47.2	49.2	32.8
0.005	$0.25d$	$0.9d$	$0.82d$	$0.2050d^2$	$0.00410bd^2$	51.3	61.5	41.0
0.006	$0.27d$	$0.9d$	$0.81d$	$0.2187d^2$	$0.00486bd^2$	54.7	72.9	48.6
0.008	$0.31d$	$0.9d$	$0.80d$	$0.2480d^2$	$0.00640bd^2$	62.0	96.0	64.0
0.010	$0.33d$	$0.9d$	$0.79d$	$0.2607d^2$	$0.00790bd^2$	65.2	118.5	79.0
0.020	$0.43d$	$0.9d$	$0.76d$	$0.3268d^2$	$0.01520bd^2$	81.7	228.0	152.0
0.030	$0.50d$	$0.9d$	$0.74d$	$0.3626d^2$	$0.02220bd^2$	90.7	333.0	222.0
0.040	$0.54d$	$0.9d$	$0.72d$	$0.3888d^2$	$0.02880bd^2$	97.2	432.0	288.0
0.050	$0.57d$	$0.9d$	$0.71d$	$0.4047d^2$	$0.03550bd^2$	101.2	532.5	355.0

Since all the values of the seventh column are less than those in the eighth, those in the seventh column determine the allowable values of $\frac{M}{bd^2}$ for steady loads. For vibrating loads the values must be taken from the ninth column if $\frac{A''}{bd}$ is less than 0.007

and from the seventh column if A''/bd is more than 0.007. By plotting the values of A''/bd as abscissas and those of M/bd^2 as ordinates, a curve may be constructed from which the value of M/bd^2 for any value of A''/bd between 0.004 and 0.050.

If in the third column $0.83d$ is substituted for $0.9d$ a table may be constructed for slabs.

Problem.—What must be the width of a reinforced beam 12 inches deep to safely resist a bending moment of 100,000 pounds, if the area of the reinforcement is one hundredth of the area of cross section?

Using the table, we have $\frac{100,000}{b \times 144} = 65.2$, hence $b = 10.7$ inches.

If b is made equal to $\frac{2}{3}d$, as is considered by some engineers the proper form of cross-section of a reinforced beam, $\frac{M}{bd^2}$ becomes $\frac{3M}{2d^3}$.

Area of Reinforcement.—Knowing the cost per square inch of the concrete and the steel, it is possible to select the amount of reinforcement which will make the cost of the beam a minimum. There is, however, another matter which must be considered, and that is, the concrete on the tensile side must not crack and expose the reinforcement. It is usually assumed that reinforced concrete can be stretched $\frac{1}{1000}$ without injuring the beam. This has not, however, been conclusively shown, although the cracks have been shown to be minute. Since the maximum unit stress allowed in the steel is only 15,000 pounds, the elongation produced will be only about $\frac{1}{2000}$ or one-half the allowable elongation.

With Double Reinforcement.—In the cantilever beam shown in Fig. 125 let there be an additional reinforcement KN on the compressive side of the beam. Assume the same nomenclature as before, and in addition let

A''' = a ea of the compressive reinforcement;
y''' = the distance of its axis from the neutral axis;
d''' = the d'stance of its axis from HC;
s_c = its unit stress.

Then will equation (557) become

$$\frac{sby'}{2}+s_cA'''=s_eA''. \quad . \quad . \quad . \quad . \quad . \quad (568)$$

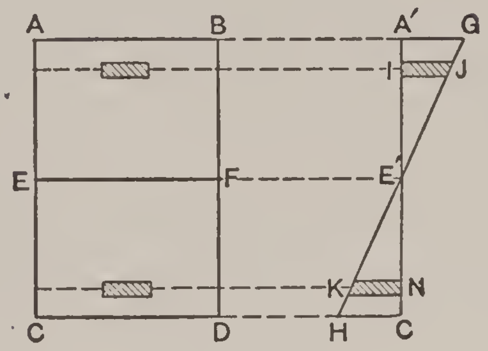

FIG. 125.

Equation (558) becomes

$$\frac{sby'^2}{3}+s_eA''y''+s_cA'''y'''=M. \quad . \quad . \quad . \quad (569)$$

$$HC':KN:IJ::\frac{s}{E'}:\frac{s_c}{E''}:\frac{s_e}{E''}, \text{ also } HC':KN:IJ::y':y''':y'';$$

hence $$y':y''':y''::\frac{s}{E'}:\frac{s_c}{E''}:\frac{s_e}{E''}.$$

Equation (561) becomes

$$s_c=\frac{msy'''}{y'}, \quad s_e=\frac{msy''}{y'}, \quad s_e=\frac{y''s_c}{y'''}. \quad . \quad . \quad . \quad (570)$$

Substituting the values of s_e and s_c from (570) and inserting them in equations (568) and (569), we have

$$\frac{sby'}{2}+\frac{msy'''A'''}{y'}-\frac{msy''A''}{y'}=0$$

or $$\frac{by'^2}{2}+m(A'''y'''-A''y'')=0. \quad . \quad . \quad . \quad . \quad (571)$$

$$M=\frac{sby'^2}{3}+\frac{msy'''^2A'''}{y}+\frac{msy''^2A''}{y'}$$

or $$M=\frac{s}{y'}\left(\frac{by'^3}{3}+my'''^2A'''+my''^2A''\right). \quad . \quad . \quad (572)$$

But $$y'''=y'-d''' \quad \text{and} \quad y''=d''-y'. \quad . \quad . \quad . \quad (573)$$

Substituting these values in (571) we have

$$\frac{by'^2}{2}+m(A'''+A'')y'=m(A'''d'''+A''d'). \quad . \quad (574)$$

Solving with respect to y' we have

$$y' = -\frac{m}{b}(A'''+A'')\pm\sqrt{\frac{2m}{b}(A'''d'''+A''d'')+\frac{m^2}{b^2}(A'''+A'')^2}. \quad (575)$$

This gives the position of the neutral axis if we substitute for m its value, 10.

By substituting the same values in equation (572), we have

$$M=\frac{s}{y'}\left[\frac{by'^3}{3}+mA'''(y'-d''')^2+mA''(d''-y')^2\right]. \quad . \quad (576)$$

To test a given beam under a given load we must proceed as explained above.

If a=distance of resultant of $\frac{sby'}{2}$ and s_cA''' from the extreme fiber in compression, we shall have

$$M=\left(\frac{sby'}{2}+s_cA'''\right)(d''-a). \quad . \quad . \quad . \quad . \quad (577)$$

and

$$M=s_eA''(d''-a). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (578)$$

If A''' is expressed in terms of A'', as $\frac{A''}{4}$ or $\frac{A''}{2}$, and y' and a are expressed in terms of y, and s_c is made equal to s_e, tables may be formed as for a beam with a single reinforcement.

Some engineers treat a beam of double reinforcement as a plate girder of which the reinforcements are the flanges, and alone resist the longitudinal stresses. Under this hypothesis the stress on the compressive rod must not be great enough to cause excessive shortening and consequent crushing of the concrete.

Shear.—The vertical and horizontal shear in a reinforced beam is determined as explained for beams in general; the unit shear must never exceed its safe value, 50 pounds per square inch. The strength of a beam to resist vertical shear is often increased by inserting stirrups, which are imbedded in a vertical or inclined plane of cross section and support the lower reinforcement. These tension-stirrups act in a manner similar to the compressive stiffeners of a p'ate girder.

Adhesion.—To prevent the reinforcement from moving longitudinally in the concrete, the reinforcing bars are usually twisted, made with lugs or with some similar patented device.

Methods of Reinforcing.—There are many different forms of reinforcement devised by engineers and inventors. The typical forms are perhaps the Monier, Melan, and Hennebique systems.

The Monier system, which has been applied to floors, slabs, conduits, and arches, consists in imbedding in the concrete a network of wires. The wires of the network not only resist the tensile stresses in the structures reinforced, but also prevent cracks caused by internal stresses in the concrete.

The Melan system, which has been applied to floors and arches, consists in spacing I beams, either straight or curved, at intervals equal to about ten times the depth of the beams and filling the space between them with concrete. In arched bridges of wide spans the beams are replaced by riveted girders or trusses.

The Hennebique system is the type of the method applied to buildings. Foundation piles are made in the same manner as columns, except that the rods are brought together at the lower end, forming a point which is reinforced by straps and a cast-iron cap. Footings for columns are made of concrete in which are imbedded right-angled systems of steel rods with vertical stirrups. Walls are made of concrete with vertical rods near the interior and the exterior, and horizontal rods at intervals to strengthen the wall and prevent cracks. Columns and floor beams are made as heretofore described; the beams have straight reinforcing rods near the bottom, and also curved rods which are near the top surface at the supports and near the bottom at the middle points. The floor slabs connecting the beams have two sets of parallel rods which are placed perpendicular to each other.

The other systems of construction differ from the above in the forms of the reinforcing rods and metal, and in the exact method of their introduction into the concrete.

Uses.—Reinforced concrete is now employed not only in the construction of foundations, walls, floors, columns, and roofs of buildings, but also in the construction of bridges, retaining and reservoir walls, water-tanks, and conduits, sewers, and other engineering constructions.

Arches.—Since reinforced concrete readily lends itself to the construction of arches, the Austrian engineers made the following tests in connection with those on stone and brick arches heretofore described.

Kind.	Span, Feet.	Rise, Inches.	Thickness, Inches.	Breaking Load, Pounds Square Foot.
Monier mortar and wire. . .	8.85	10.23	1.95	1638 without rupture.
Melan mortar and I beams.	8.85	10.23	1.95	1638 without rupture.
Monier mortar and wire. . .	13.3	16.1	3.94	872 over half the span.
Melan 3.15 I beams, $3\frac{1}{3}$ ft. center to center.	13.1	11.4	3.15	3120 cracked. 3370 broke.
Monier mortar and wire, arch 13 ft. $1\frac{1}{2}$ ins. wide . .	32.8	39.4	5.9 to 7.87	90 tons on half the arch made a crack. 180 tons similarly placed caused failure.
Monier mortar and wire, arch 6.6 ft. wide.	74.5	15.0	19.8 to 23.44	3.09 tons per running foot on half the span caused failure.

For further information consult Baker's "Treatise on Masonry Construction;" Kidder's "Building Construction," Part I; Marsh's "Reinforced Concrete;" Buell and Hill's "Reinforced Concrete;" Taylor and Thompson's "Concrete, Plain and Reinforced."

CHAPTER XXII.

FOUNDATIONS.

General Principles.—In the treatment of foundations, it is customary to consider the material or soil upon which the structure is to rest and the means employed to distribute the weight of the structure over this material. A foundation may be considered satisfactory when the vertical movement or settling of the structure is uniform and hardly appreciable, and there is no horizontal movement of the structure on its base.

If the soil under every part of the structure has the same bearing power, the settling will be uniform if the unit pressure is the same at every point of the base. This is secured by designing the base of every section of the structure so the resultant pressure shall pierce the base as closely as practicable to the center of figure. The amount of settling will then be hardly appreciable if the unit pressure on the base of the structure is but a small fraction of the unit bearing value of the soil. This is secured by enlarging the base of each foundation wall and pier; this enlargement is called the *footing*.

If the soil under the structure is not uniform throughout, the unit pressure on the base at any point must be proportioned to the unit bearing power of the soil at the same point; this is effected by varying the dimensions of the footing. The greatest unit pressure must be where the bearing power is greatest and the converse. If a part of the structure rests upon rock and the remainder upon compressible soil, it will be practically impossible to so adjust the pressure and resistances as to prevent unequal settling. The structure should if possible rest wholly on the rock or wholly on the compressible soil; if this is not possible, a cushion of sand may be placed on the rock.

If the center of pressure of any section of the base does not coincide with the center of figure, the pressure at that part of the base will not be uniformly distributed; and if the soil is uniform, the structure will tend to rotate about some line in its base. In the outer walls of buildings a slight tendency to rotate inwards is not objectionable, since it is resisted by the partition walls and floors and binds the parts together.

The *factor of safety* of a foundation is the ratio of the *unit bearing power* to the *unit pressure;* it should be as large as practicable if appreciable settling is to be avoided.

The soil underneath a structure must be protected from all disturbing influences. On land the foundation walls of buildings should extend below the limits of frost, and in rivers the foundations of piers should be deep enough to be protected from the undermining action of the current. The extreme depth of the frost line depends upon the climate; in New York City the building regulations require the base of foundation walls to be at least *four* feet below the surface of the soil.

Lateral movement of a structure on the soil will be impossible if the horizontal component of the resultant pressure on its base is less than the friction produced by its vertical component, or if the resultant pressure makes with the normal to the surface of the soil a less angle than the angle of friction. Since this angle is about 33 degrees, structures may as a rule be safely constructed on the soil excavated to a horizontal plane.

To avoid excessive excavation when the surface of the soil is much inclined, the base of the foundation wall may be a series of horizontal steps instead of a single horizontal surface.

Classes and Bearing Powers of Soil.—Soils may be divided into two classes, *firm* and *soft*. *Firm soils* are those that will support the weight of any ordinary structure without undue settling; as rock, ordinary earth, dry sand, dry clay, etc. *Soft soils* are those that will not support the weight of an ordinary structure unless their bearing powers are increased by artificial means; as wet clay, wet sand, etc.

The safe bearing power of *rock* is assumed to be about one-tenth of its crushing strength as determined by tests on small cubes. This gives safe bearing values varying from 10 to 120 tons per square foot depending upon the character of the rock. It is rarely necessary to make the pressure on the base of a structure greater than 10 tons per square foot.

The following table of safe bearing values has been recommended.*

Soft clay and wet sand	1 ton per square foot
Ordinary clay and dry sand mixed with clay	2 tons " " "
Dry sand and dry clay	3 " " " "
Hard clay and firm, coarse sand	4 " " " "
Firm, coarse sand and gravel	5 " " " "

Testing Soils.—The values above given are only approximate. When it is desired to measure the *ultimate bearing power* of a soil, a plate covering a square foot or a square yard of the material is loaded until a perceptible settling is observed, accompanied by a rising of the soil in its immediate vicinity. The *safe bearing power* is derived from the *ultimate bearing power* by dividing by a *factor of safety* whose value depends upon the uniformity of the soil.

In testing rock it is desirable to know both the character and the extent of the formation. If the rock is near the surface its character is determined by uncovering a small area; its extent by sounding with a small iron rod or pipe. If the rock is at a considerable distance below the surface, it may be reached by any of the processes resorted to in driving artesian wells. The depth and character of the stratum may be determined by boring.

In testing other soils the bearing power may be determined by loading a limited area after the surface has been excavated to the depth of the bed of the foundation. If the stratum upon which the building is to rest overlies a softer stratum, it is also necessary to determine the extent and thickness of the firm stratum. In Chicago where the surface soil overlies a very soft soil, the building regulations allow a load of 2 tons per square foot on sand and $1\frac{3}{4}$ tons on clay, when the bed is at least 15 feet thick; for thinner strata the loads must be reduced.

* Proceedings Am. Soc. Civil Engrs., Sept. 1904.

CONSTRUCTING FOUNDATIONS ON LAND.

In Firm Soils.—If the soil is rock and generally level, it is necessary only to remove all loose and decayed material under the piers and foundation walls and then fill the crevices and level the bed with a layer of cement concrete. The surface of this concrete becomes the bed of the foundation proper. If the rock is intersected by wide crevices, these may be bridged by masonry arches or by strong iron beams imbedded in concrete; if the surface is inclined, steps are cut in the solid rock for the bed of the foundation.

If the soil is earth, sand, or clay, a trench is dug to a depth below the frost line and its bottom leveled to receive the footings of the foundation walls and piers.

The *footing* (Fig. 126) is a mass of masonry by means of which the base of a wall or pier is widened, so as to decrease the unit pressure upon the soil and at the same time move the center of pressure farther from the edges. A footing is necessary whenever the safe bearing power of the soil is less than the unit pressure on the base of the wall or pier. A deep footing is usually stepped, since, with less masonry, it can be made as strong as a rectangular footing.

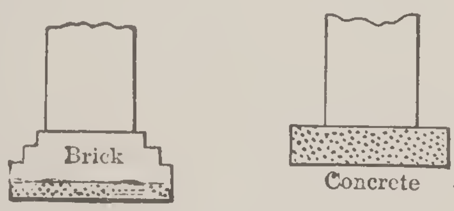

FIG. 126.

The vertical thickness of the footing may be deduced by assuming that the projecting part is a cantilever acted upon by a uniform upward pressure per square inch, equal to the unit downward pressure upon the entire base produced by the weight of the wall and its load.

The width of the footing in feet at any cross-section is determined by dividing the total pressure per running foot of the base of the wall, by the safe bearing power of the soil per square foot.

According to standard building regulations the thickness of a concrete or stone footing should be at least one foot. Each stone must extend the full width of the wall. If the footing is stepped, each step of a concrete or stone footing is made one foot thick. The steps of a brick footing are either one or two bricks thick; if one brick thick the steps are $1\frac{1}{2}$ inches wide, if two bricks thick the steps are 3 inches wide. A brick footing always rests on a bed of concrete.

For the piers or columns of high buildings the footings are made either of several layers of steel I beams, rods, or bars laid at right angles to each other and imbedded in concrete,* or the bases of the piers are connected by strong inverted masonry arches which in turn rest on beds of concrete. In the New York regulations the arches must be 12 inches and the beds 18 inches thick.

A foundation bed of sand or clay under buildings is protected from the action of the water by the construction of suitable drains; when necessary the foundation beds are protected from washing away by inclosing them in a suitable barrier of sheet piling or other material.

If springs are encountered, or water from any source runs into the foundation trench, the water is drained to a hole dug at some convenient place and then removed by pumping.

Soft Soils.—The bearing power of the soft soils most nearly approaching in consistency to the firm soils may be increased sufficiently to bear the weight of an ordinary structure by increasing the area over which the weight is distributed. This may be done by making the footing wider, or by placing under the footing of the wall a deep cushion of compressed sand much wider than the footing itself.

If the soil is marshy and the cushion is continually wet, a *grillage* made of several layers of beams in close contact may be employed instead of the sand. Temporary weights, such as the abutments of small military bridges, are often supported by mattresses of poles and brush tied together by wire.

If the soil is too soft to admit of this construction, the trench under each foundation wall must be excavated until a layer of firm soil is reached, or the foundation must be strengthened by the use of piles. The soft soil should be removed by excavation and the foundation laid on the firm soil beneath whenever this can be done without great cost; this form of foundation is more reliable than a foundation on piles.

Piles.—A *pile* is a column which is driven or forced into

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* Cambria Handbook, p. 299.

the soil. Piles are divided into several classes according to their purpose and the method of driving.

Common Piles.—A common pile is a wooden pile driven into the earth by means of blows delivered on its head; it is intended to resist pressure either from above or from one side, or simply to compress the soil and increase its bearing power. The term *bearing pile* is also applied to a common pile which supports a load placed upon it. Common piles are trunks of trees, usually oak, elm, pine, or cypress.

Short Piles.—If intended simply to compress the soil, common piles are from 8 to 15 feet long, 8 to 12 inches in diameter, and are driven close together over the bottom of the footing trench or over the area of the foundation. The weight of the structure rests on both the compressed soil and the piles. A modification of this method is to make a number of holes by driving and withdrawing a short pile; as soon as made each hole is filled with compressed sand. The structure then rests on the compressed soil and the pillars of sand.

Long Piles.—If intended to support the weight of the structure or to resist great lateral pressure, the common piles are from 12 to 18 inches in diameter; their length depends upon their load and the character of the soil, and may be as great as 100 feet.

The upper or larger end of the pile is a plane surface normal to its axis, and is protected from *brooming*, or splitting, by surrounding it with a thick wrought-iron ring 1 to 3 inches wide. If the soil is soft, the lower end is also a plane surface; if the soil is hard, it is a conical surface, or a truncated cone with a base of from 4 to 6 inches in diameter; if the soil is very hard, the end is protected by a solid conical shoe which is strapped or bolted to the pile. In very soft soils the piles are also sometimes driven with the larger end downwards. The bark is usually removed before driving.

Bearing-piles are also made of cast and wrought iron and of reinforced concrete. These are employed in places where wooden piles would not be durable or where the pile acts as a column without lateral support and great strength is required.

Sheet Piles.—Common sheet piles are thick planks driven edge to edge in a vertical position so as to form a sheet or wall.

Sheet piles are employed when it is desired to support the walls of a trench or make a temporary dam in a stream or pond not exceeding 20 feet in depth.

If the material is soft, the lower ends of the sheet piles are so beveled that the resultant pressure of the earth will force each pile against the preceding one driven. The joints between the different piles may also be closed by driving the piles so as to overlap each other. In the Wakefield sheet-piling, each sheet consists of three planks so fastened together as to form a tongue and groove similar to flooring. Interlocking sheet-piling is also made of rolled or riveted steel plates of various shapes.

Pile-driving Apparatus: Common Pile-driver.—The common pile-driver consists of two similar right-angled triangular frames of strong timber. The height of the driver is about 30 or 40 feet; its base is about half its height. The vertical posts are called the *leaders* or *guides* and are fastened to the inclined pieces of the triangular frame by horizontal braces. The two frames are placed side by side, about $2\frac{1}{2}$ to 3 feet apart, and are united by horizontal struts which connect the horizontal braces and beams and the two inclined beams; the pile-driver rests on rollers or is bolted to the deck of a scow or the floor of a car and is braced laterally.

The pile is placed upright between the leaders and is held in place by temporary pieces which rest in brackets attached to the leaders. It is driven by a heavy mass of iron, called the *hammer* or *ram*, which slides on rails fastened to the inner surfaces of the leaders.

The power employed in raising the hammer is applied to a rope which is fastened to the hammer and passes through a pulley attached to a cross-beam connecting the tops of the leaders. This power may be man, horse, or steam power. Man power is employed only in light work such as military bridging. The men pull on small ropes which are attached to the main hoisting-rope. This is called a *ringing engine*.

When horse power is used, the hoisting-rope passes through a second pulley called a *snatch-block*, which is attached to the base of the pile-driver and thence to an ordinary capstan.

If steam is employed, the hoisting-rope passes from the snatch-block to a hoisting-drum, which is held to its axle by a friction-clutch. When the clutch is tightened the drum rotates with the axle and the hammer is raised; when the clutch is released the drum rotates in the opposite direction and the hammer is dropped.

The rope may also be firmly attached to an axle whose rotation may be reversed, and to a *nipper*. The nipper is a weighted pair of tongs which slides between the leaders. The tongs are engaged in a ring in the ram when it is desired to raise the latter, and are disengaged when the hammer is to be dropped. The tongs may be disengaged by bearing against inclined plates near the top of the leaders or by means of a dropping device operated by a cord.

Steam-hammer.—In the steam-hammer the blow is delivered by a heavy hammer attached to a vertical piston which has a stroke of about 3 feet. At the bottom of the driving-apparatus is a heavy ring which rests on the head of the pile and is held in place by the leaders and supports the steam-cylinder. The hammer is a heavy mass, weighing from $1\frac{1}{2}$ to 2 tons, attached to the piston, and also held in place by the leaders; its head passes through the base ring and strikes the pile. The steam-cylinder is at the top of the driving-apparatus, where it is supported by pillars attached to the base ring, and is connected to the boiler by a flexible tube. The driving-apparatus is attached to a rope which passes over a pulley at the top of the leaders; by means of the rope it may be raised or lowered.

The steam-hammer drives piles more rapidly than the ordinary pile-driver; the blows, although of less energy, are delivered with greater rapidity.

The Water-jet.—If the soil is sand, silt, or mud without boulders or other obstructions a pile may be forced into the ground by placing a load on its top and then forcing a stream of water to its lower end. The water softens and stirs up the material so that the pile sinks under the load placed upon it. In sinking a wooden pile the stream passes through an iron pipe which is attached to the pile by means of staples. When the pile is in place the pipe is withdrawn.

Piles which would be shattered by the hammer and reinforced concrete piles may be driven in this manner.

For piers, wharves, lighthouses, etc., the piles sunk by this method are usually hollow cast- or wrought-iron piles about 8 to 12 inches in diameter. The lower end of the pile terminates in a horizontal disk, about 3 to 4 feet in diameter, which greatly increases the bearing power of the pile. This is called a *disk pile* (I, Fig. 127). The water is forced into the pile through a cap and escapes through holes in the bottom of the disk. Iron disk piles are made in sections which are fastened together as the pile is driven.

The Capstan.—If a common pile is converted into a screw by attaching near its bottom spiral blades, the pile may be screwed into the ground by fastening horizontal bars or a horizontal wheel near its top and thus converting it into a captsan. Such piles are called *screw piles* and are usually made with a shaft of solid

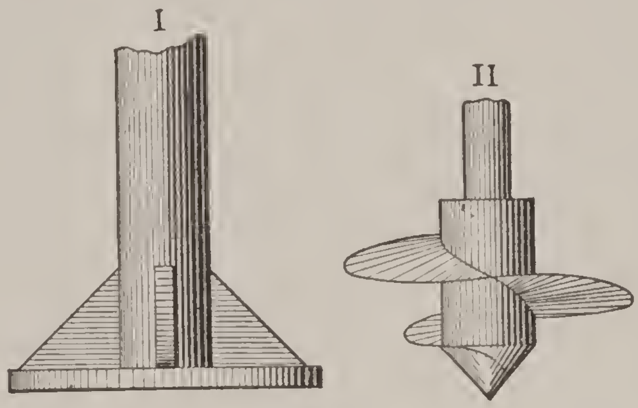

FIG. 127.

wrought iron 3 to 5 inches thick and blades of cast iron 3 to 4 feet in diameter (II., Fig. 127). Screw piles may be forced through soils in which the water-jet cannot be employed and are used on shoals along the coast where the water is so rough that the ordinary pile-driver cannot be employed. Like the disk pile, the screw pile has a large bearing area to resist not only the downward pressure but also the lifting power of the waves.

Load on Piles.—The resistance of a driven pile to further movement could be accurately determined at any time were the intensity, direction, and law of distribution of the earth pressure upon its sides and bottom known. As these cannot be definitely ascertained, all formulas for the safe load upon a pile must be theoretical, based upon hypotheses, or simply empirical formulas based upon observation.

Theoretical Formulas.—Captain Sanders, Corps of Engineers, deduced a formula on the hypothesis that the work of the pile's resistance developed by the last blow of the hammer is equal to the theoretic kinetic energy of the hammer at the instant of striking,

or $$Rd = \frac{Mv^2}{2} = Wh \quad \text{or} \quad R = \frac{Wh}{d},$$

in which R = ultimate resistance of pile or its ultimate load, in pounds;

M = mass of hammer;

v = striking velocity of hammer, in feet per second;

W = weight of hammer, in pounds;

h = height of fall of hammer, in feet;

d = penetration of pile at last blow, in feet.

Recognizing that the value of R in this formula was too great because some of the kinetic energy of the hammer was expended in overcoming the friction of the hammer in the guides, in deforming the pile and hammer, in producing heat, etc., he proposed as the safe load of a pile one-eighth of R. His safe load was therefore

$$R' = \frac{Wh}{8d}. \qquad (579)$$

To provide for the losses due to friction, etc., in the original formula it has also been proposed to increase the denominator of the theoretical formula by a constant determined from experiment. If the constant $\frac{1}{12}$ is adopted, the formula becomes

$$R = \frac{Wh}{d + \frac{1}{12}}. \qquad (580)$$

If h is left in feet and d is expressed in inches, this becomes

$$R = \frac{Wh}{\frac{d}{12} + \frac{1}{12}} = \frac{12Wh}{d+1}. \qquad (581)$$

If the factor of safety 6 is introduced, the formula becomes

$$R = \frac{2Wh}{d+1}. \qquad (582)$$

This is known as the Engineering News formula.

Captain Mason, Corps of Engineers, deduced a formula based on the hypotheses—

1. The work of the pile's resistance during the last blow of the hammer is equal to the theoretical kinetic energy of the pile and hammer at the instant following the striking.

2. The momentum of the pile and hammer at the same instant

is equal to the momentum of the hammer at the instant of striking. Let the nomenclature be as on page 423, and in addition let w and m = weight and mass of the pile;

V = velocity of hammer and pile the instant after striking, in feet per second. Then

$$Rd = \frac{MV^2}{2} + \frac{mV^2}{2} = \frac{(W+w)V^2}{2g} \quad . \quad . \quad . \quad (583)$$

and

$$Mv = (M+m)V \quad \text{or} \quad \frac{Wv}{g} = \frac{(W+w)V}{g}, \quad \text{hence} \quad V = \frac{W}{W+w}v.$$

Substituting this value in the first equation,

$$Rd = \frac{W^2v^2}{(W+w)2g}, \text{ or, since } v^2 = 2gh, \ Rd = \frac{W^2h}{W+w} \text{ or } R = \frac{W^2h}{(W+w)d}.$$

For safety a factor of four is employed, or $R' = \dfrac{W^2h}{4(W+w)d}$. (584)

If the weight of the pile is equal to that of the hammer, which is approximately true for long piles, this formula gives the same value as Captain Sanders' formula.

Other theoretical formulas have also been recommended with the losses due to friction and deformation introduced.

The theoretical formulas above deduced give an approximate result for the ultimate and safe loads of piles driven in firm soil immediately after the pile is driven, provided no great deformation of the pile has taken place. The bearing power of such piles probably diminishes, however, with the lapse of time. The formulas cannot be applied to piles driven in very soft soils, in which the resistance is due to the friction on the sides alone. The safe loads of piles driven in very soft soils are greater than those given by the formulas.

These facts led to the adoption of purely empirical rules for safe loading. The New York building regulations state simply that the piles shall not be less than 5 inches in diameter at the small end, nor shall any pile be loaded with more than 20 short tons. A common rule is to limit the load for long piles to 200 pounds per square inch in very soft soil and 800 pounds per square inch in very firm soil. Probably the only satisfactory way of determining the ultimate and safe load is to drive several piles on the site selected and some time after they are driven load them until they begin to sink. This will give the ultimate bearing power from which the safe bearing power may be obtained by the introduction of a factor of safety.

The resistance of screw and disk piles is due principally to the resistance of the soil under the disk and screw. This is increased, however, by the resistance along the sides, which varies from 200 to 800 pounds per square foot of this area, depending upon the firmness of the soil.

Method of Constructing Common Pile Foundations.—When the weight of the structure is to be borne by the piles, the piles are driven along the bottom of the foundation trench in rows $2\frac{1}{2}$ to 3 feet apart; the piles in each row are similarly spaced. The tops are sawed off in a horizontal plane and are covered with a *grillage* or with a layer of concrete.

The grillage is made by capping each row of piles with a heavy longitudinal beam, which is fastened to the piles by mortise and tenon joints or by drift-bolts. A plank platform may be spiked to the capping or one or more layers of beams placed at right angles to each other and in close contact may be laid between the capping and the platform, each layer being fastened to the layers above and beneath. When a concrete bed is employed, the earth about the tops of the piles is replaced by a bed of concrete which holds the piles in place and distributes the weight over the entire area as well as over the piles.

If the soil to a considerable depth is very soft before the grillage is constructed, the piles are braced laterally by throwing loose stone in the spaces between them. This is usually done when the piles are driven into a firm stratum which is covered by a thick layer of soft material.

FOUNDATIONS UNDER WATER.

The foundations of bridge piers and abutments, wharves, dock walls, breakwaters, etc., are constructed on sites covered with water. The difficulty of construction depends upon the depth of the water, the velocity of the current, the variation in depth of the water, wave action, and the care which must be taken to prevent settlement.

There are two general methods of executing the work: in the first the foundation is laid without draining the water from the site; in the second the water is drained from the site so that the work may be executed practically as in open air. Whatever the method of execution, if the structure is an important one, the soil must be previously examined by borings or soundings.

WITHOUT DRAINING THE WATER FROM THE SITE.

Random Rock.—The simplest method of constructing foundations under water is to throw in layers of quarry stone until the stone reaches the surface of the water. The foundation is then leveled with a bed of concrete which becomes the base of the superstructure. This method is employed in the construction of breakwaters, jetties, etc. The foundation is rendered compact by the action of the waves and slight settlement is not objectionable. On exposed sites very large blocks of concrete are preferred to natural stones.

Cribs.—If the soil is firm, the foundations for light structures may be made by sinking common cribs. These are constructed of round or squared logs halved at the ends and firmly fastened together with bolts or iron cramps. The crib is divided into a number of pockets by strong cross-walls halved into the sides. Some of the pockets are floored near the bottom and others are left open. The crib is floated to the site, held in place by piles or anchors and sunk in place by filling the closed pockets with quarry stone. When the crib reaches the bottom, all the pockets, both open and closed, are filled with stone and the crib is thus securely anchored. The stone in the open pockets settles under those parts of the crib which do not touch the bottom. Cribs require less stone than random rock foundations. If cribs are to be used for foundations of masonry structures the crib work should be terminated below the low-water line; for piers of temporary bridges the crib extends above the water level.

Concrete.—If the bed is of exposed rock and the current is not strong, the foundation may be made of concrete. Since concrete dropped through water is injured by the mortar being washed from the stones, the following methods have been employed to get it into place with as little injury as possible. The mortar is lowered to the bottom in open mesh bags which are carefully laid by divers; it is lowered in water-tight boxes with doors in the bottom which can be opened by means of a cord when the box reaches the bottom; or it is passed through tubes which reach from the deck of a scow to the bottom. The concrete is placed and rammed in layers. Since concrete is always more or less injured by being thus laid, and is not as uniform or compact as when laid on land, this method of construction is employed only when the other methods are too expensive.

Piles.—If the bed of the river is of earth, and the water is not too deep, some form of pile foundation may be employed.

If common piles are used as a foundation for a masonry structure, the piles are cut off and capped by a grillage all below the surface of the water. The piles may be cut off by a cross-cut saw fastened to a triangular frame as shown in Fig. 128 and

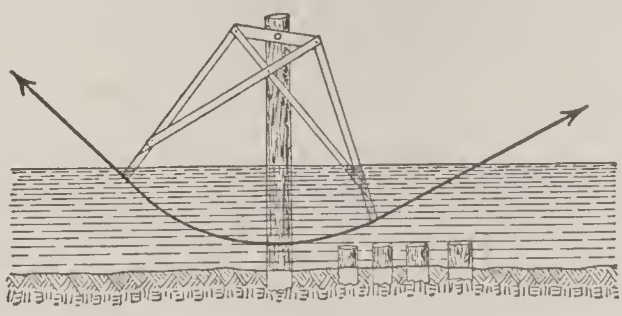

FIG. 128.

worked by ropes, or with a circular saw fastened to a vertical shaft which is attached to a scow and rotated by proper machinery. The piles are strengthened laterally by the grillage and by broken stone or other material deposited between them. If the water is too deep to give the proper lateral support to the piles, the caisson method as described below is used.

Iron disk and screw piles are made long enough to extend above the water and may be leveled by telescoping caps. They are stiffened laterally above water by struts and ties connecting each pile with at least three others.

The Cushing Cylindrical Pier.—This is a concrete pillar resting on a pile foundation and surrounded by a cylinder of wrought or cast iron. Its diameter varies from 4 to 10 feet. To construct this pier, a cluster of piles covering as large an area as can be conveniently surrounded by the cylinder selected is driven well into the bed of the stream. The piles of the cluster are in close contact. The piles may be cut off just below the water line, although this is frequently omitted. The iron cylinder is lowered over the piles in sections 5 or 10 feet long, which are bolted or otherwise fastened together. The cylinder is forced by weights into the soil of the bottom as deep as is judged advisable, and built up to the desired height of the pier. The cylinder is then filled with concrete with or without first pumping out the water. The piers are usually constructed in pairs or groups and are united by braces. These piers are used in the construction of highway and short-span railway bridges.

In firm soil and still water the cluster of piles may be omitted and the concrete made to rest on the soil itself; in soft soil the same method may be employed, but the diameter of the pier must be increased.

The Common Caisson.—The common caisson is employed in laying foundations of a limited area, as bridge piers when the water does not exceed 20 to 25 feet in depth.

The caisson (Fig. 129) is a flat bottom boat with vertical sides.

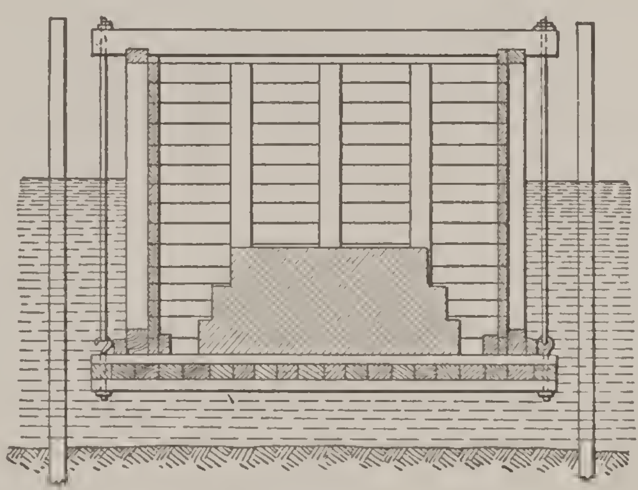

FIG. 129.

The bottom of the caisson is a grillage made of two or more layers of heavy timbers placed in contact with each other and fastened together. Its sides consist of frames to which planks are spiked. The sides are held in place by blocks spiked to the bottom, and by iron rods which at the bottom hook into iron rings fastened to the grillage beyond the sides, and at the top pass through heavy cross-beams which are notched into the sides. When the caisson is completed it is thoroughly calked to make it water-tight.

Each horizontal dimension of the caisson is from 4 to 6 feet greater than the pier, and its height is several feet greater than the depth of the water. The caisson is loaded with a few courses of masonry to give it steadiness and is then floated to the site which has in the mean time been prepared to receive it. It is accurately put in position, and while the masonry is being laid is held in place by common piles, anchors, or cribs. When the caisson has sunk nearly to the bottom it is gradually filled through valves with sufficient water to sink it in place. If it does not rest evenly on the bottom the water is pumped out, the caisson floated, and the foundation of the caisson readjusted. When it rests satisfactorily on its foundation, the masonry is continued until it reaches the top of the caisson. The iron rods are then removed and the sides float to the top and are taken away.

The bottom of the caisson may rest upon the soil itself if it is firm, especially in still water, where there is no danger of undermining. The bed is prepared by dredging the soft material and then leveling it for the grillage.

If there is danger of undermining, the caisson may rest on piles. These are driven as in preparing pile foundations on land, and are then cut off in a horizontal plane near the bed of the river.

If the bottom is rock the bed may be leveled with concrete as explained heretofore.

The Well or Open Dredging Process.—This term may be applied to the process by which a masonry or iron cylinder or a wooden crib is, by means of interior excavation, lowered to the stratum upon which the foundation is to rest. The method is applicable to foundations both on land and in water; in the former the excavation is made by hand and in the open air, in the latter the excavation is made by dredging machinery which works through the water. The ordinary form of wooden crib for deep foundations under water (Fig. 130) is rectangular in cross-section and has vertical or slightly battered walls. The lower section of the crib *AA*, which may be called the *dredging chamber*, is 10 to 20 feet high and is inclosed and divided by wedge-shaped walls whose lower edges form the cutting edge of the crib. The wedge-shaped walls are made of 12″×12″ beams in close contact and their cutting edges are covered with iron plate.

The upper part of the crib is divided by vertical walls into a number of rectangular cells; like the exterior walls, the walls of the cells are made of 12″×12″ timber, laid either in close contact or with small intervals. The cells immediately above the wedges *BB* have their bottoms closed by them, and are used to hold the material by which the crib is sunk in place. All other cells *CC* are open at the bottom and are the wells through which the material is dredged.

The iron cylinder is similar in construction, having a dredging chamber with a conical roof pierced by the dredging-wells.

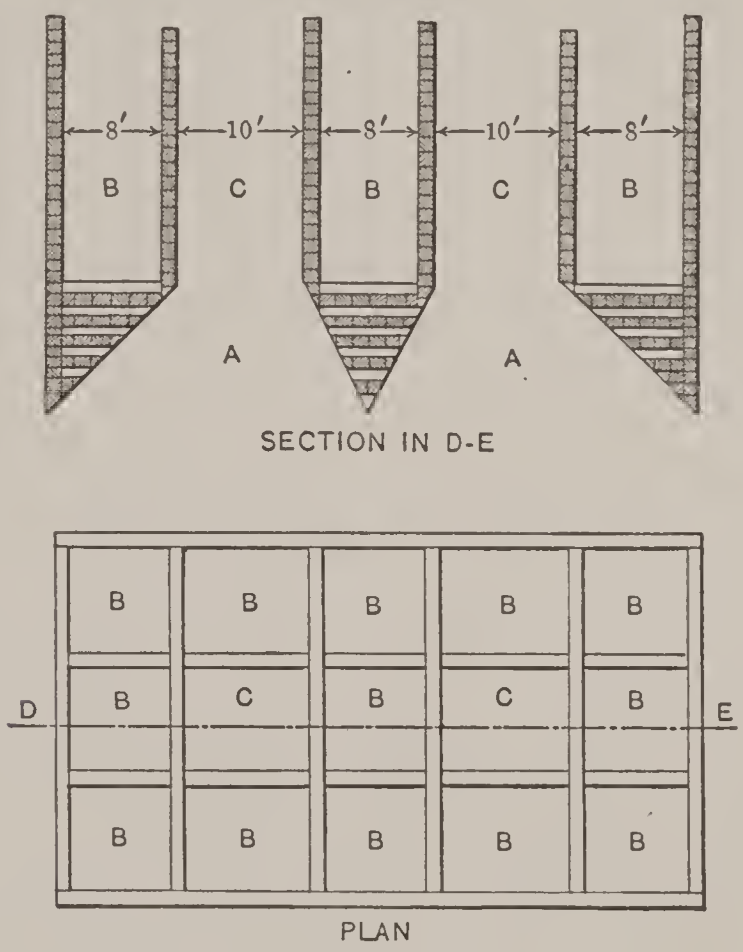

FIG. 130.

The walls of the cylinder and of the dredging-wells are of steel plates which are fastened to a connecting skeleton steel framework of the proper form.

Simple iron cylinders to be filled with concrete, or masonry piers resting on a strong grillage, have been sunk in the same manner. In the piers circular dredging-wells must be left for dredging the material.

The same process is used in sinking shafts on land. A circular ring of masonry resting on a wooden curb is slowly sunk into the earth by gradually removing the material from the interior and building up the ring. A depth of 250 feet has been reached in this manner.

Methods of Sinking.—The lower part of the crib is constructed on shore, and, when launched, it is floated into place and there held by piles, anchors, or common cribs.

The closed cells are then gradually filled with gravel, broken stone, or concrete. When the crib rests on the bottom, the dredging-buckets are lowered and the chamber is dredged; as the resistance below is diminished and the load above is increased, it settles under its weight until its base finally reaches the desired stratum. All the cells are now filled with concrete laid under water, and the foundation is complete.

The upper part of the crib may be constructed as a wooden coffer-dam with water-tight walls; when the crib is in place and filled with concrete, the water in the upper part may be pumped out and the masonry laid as in open air.

Dredging Apparatus.—The character of the apparatus depends upon the depth of the water and the material to be excavated. The most common apparatus is the *clam-shell* bucket attached to the hoisting-chain of a crane. As its name suggests, it is a bucket made of two leaves each shaped like an ordinary scraper. When open their cutting-faces are vertical; when closed they form a semi-cylindrical bucket. A modification of this bucket, called the *orange-peel* bucket, is hemispherical in form and opens out into four sections. These buckets are made of steel and when dropped open penetrate the soil, due to their own weight. By a suitable device the buckets are closed while in the soil and the material is hoisted by the crane.

Sand-pumps, buckets on endless chains, and other forms of dredging apparatus are also used.

Cribs and cylinders can be sunk by this method to a great depth in a soil free from large logs and boulders. To ascertain whether the soil is free from obstructions, it is examined by soundings or borings sunk to the level of the proposed foundation. There is however always some danger that an unexpected obstacle will prevent further settlement. Up to a depth of about 100 feet the work may be aided by divers, but beyond that the work must be trusted more or less to chance. There is also some danger of the crib becoming wedged, due to the unequal settling of the sides. For these reasons the pneumatic caisson, as hereafter described, is preferred for depths less than 100 feet.

Poughkeepsie Bridge.*—The cribs used in the construction of the cantilever bridge over the Hudson at Poughkeepsie were 100 feet long, 60 feet wide, and 100 to 115 feet high. The height was such that when in place the top of each crib was 20 feet below high water. The dredging-chamber was 20 feet high and was divided into two equal parts by a central longitudinal partition. The wedge-shaped side walls of the chamber were 10 feet thick at the top and its central partition 16 feet thick. The dimensions of the upper cells were generally $10' \times 12'$, and their walls were 2 feet thick.

The deepest crib rests on a stratum of gravel 135 feet below high- or 127 below low-water level; the masonry pier, $25' \times 87'$, was constructed in a separate caisson which was lowered on the crib when the latter was in place and filled with concrete. The masonry itself begins at 7 feet below low water.

The Hawksbury Bridge.—The piers of the Hawksbury bridge in New South Wales were constructed by means of large iron cylinders sunk in the same manner. The deepest pier rests 140 feet below low-water level of the river; it is the greatest depth that has been reached in pier construction. The plan of each cylinder is a rectangle with semi-circular ends whose extreme dimensions are $24' \times 52'$. The dredging tubes were three in number, equally spaced along the longer axis and each 8 feet in diameter. The dredging-wells and sides of the cylinder were connected by vertical trusses.

The Diving-suit.—In all the above methods the services of the diver are required. The diver wears a strong air-tight suit surmounted with a copper helmet which rests on his shoulders. The helmet is connected by a flexible tube with a small hand air-compressor above water. To enable him to see, the helmet has a glass face, and to keep him in an upright position the soles of his boots are heavily weighted. The diver is raised and lowered by means of a rope which passes under his arms.

The pressure of the air in the helmet must be sufficient to withstand the pressure of the water on the flexible tube, or 15 pounds above the normal for every $34\frac{1}{2}$ feet of submergence. The extreme depth to which a diver has gone is about 150 feet.

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* Trans. Am. Soc. of Civil Engrs., vol. 18, pp. 199–216.

EXCLUDING THE WATER.

Common Dams.—When the depth of the water above an impervious stratum does not exceed four feet and there is no current, a common earthen dam may be constructed around the site to be occupied by the foundation and the water excluded from the site by pumping. The top of the dam is two to three feet wide and the sides are allowed to take their natural slope under water. It cannot be employed in a current which washes away the material of the dam.

Coffer-dam.—In water between four and twenty feet in depth with a good bottom, the coffer-dam may be employed. Its sides are sheet piling, which reduces the amount of material required in the dam proper and protects the material from the current.

To construct a coffer-dam (Fig. 131), two parallel rows of

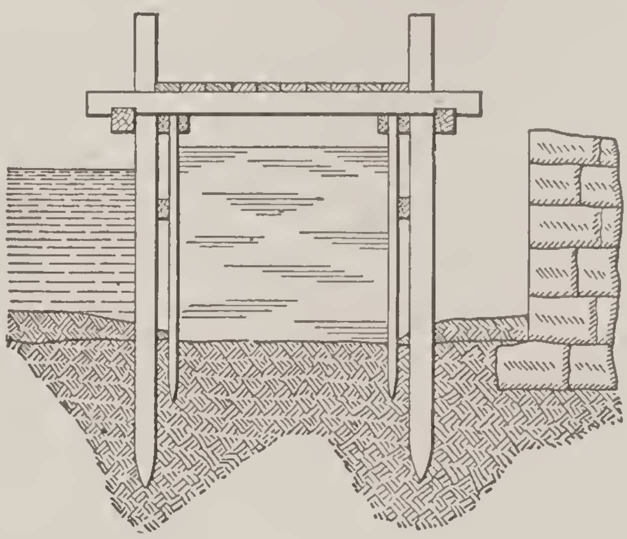


FIG. 131.

common piles are driven around the site of the proposed structure.

To strengthen them longitudinally and laterally, the piles of each row are connected by heavy horizontal beams called *wales* bolted to the piles above the water level, and the wales themselves are connected by rods or cross-beams. If the wales are between the rows of piles they will serve also as guides to the sheet piling; if on the opposite sides of the piles, as shown in the figure, additional horizontal pieces are bolted to the piles to serve as guides. If the water is deep, additional guides are placed between the water level and the bottom, which are held in place by battens spiked to the upper guides.

The sheet piles are driven into the impervious soil and form a *coffer*, which holds the puddling or dam proper. The sheet piles may be spiked to the guides or held in place by riband pieces as shown.

The *puddling* should consist of material which is impervious to water and which will not be washed away if a leak is developed in the dam. A mixture of clay, sand, and gravel is the best material for the purpose; the finer material fills the interstices of the coarser, and the coarser material resists the movement of the water in case a leak is developed in the dam. Common loam, clay, and fine gravel are also employed as puddling material. The puddling is placed and tamped in such a manner that there shall be no distinct layers.

Experience has demonstrated that a thickness of two feet of good clay puddling is sufficient to prevent the percolation of the water through a coffer-dam of the usual height. As a dam of that thickness has a small resisting moment and brings great stress on the common piles, the thickness of low dams is made equal to their height and that of high ones equal to half their height.

The *sheet piles* must support the pressure of the puddling which is counteracted on the outside row, by the pressure of the water; the outer sheet piling may therefore be thinner than the inner. The thickness of the sheet piles depends upon the distance between the guide pieces. The ordinary piling is two and one-half to three inches thick, but this must be increased if the dam is high and the distance between the guides is great. The piles must be driven sufficiently deep to prevent the puddling from escaping underneath them.

The *common piles* must, with the weight of the dam, resist the tendency of the dam to overturn or slide, under the water pressure against the outside sheeting. This will determine the length of the piles and the distance between the piles in each row; the latter is usually from four to eight feet.

The distance between the inner row of piles and the foundation is at least three feet if no inner excavation is necessary; if the soil must be excavated, this distance is increased by one and one-half times the depth of the excavation. When necessary the stress in the common piles may be relieved by inner braces which rest against the foundation itself or against adjacent sides of the dam.

The *water is removed* and the interior space kept dry by the use of common, centrifugal, or force-pumps, depending upon the area to be drained and the amount of leakage. To prevent interruption duplicate pumps are provided.

Modifications of the Coffer-dam.—When the bottom of the river is rocky, it may be impossible to drive common piles to support the dam. Under such conditions each row of common piles may be replaced by a row of iron rods inserted in holes drilled in the rock, or by cribs or box-shaped boats which are floated to the site and sunk by loading them with stones. The puddling is placed between the lines of sheet piling supported by the rods, cribs, or boats as in the case of the common coffer-dam.

Rectangular cribs made of heavy logs, halved into each other at the corners and fastened together by bolts or iron clamps, are employed to protect the dam when it is exposed to floating ice in a swift current. The cribs are properly cross-braced and have a floor to hold the rock used to sink them.

When the bottom is rocky and the water is deep, great difficulty is often experienced in closing the joint between the rock and the puddling and in draining the interior, because of the seams in the rock. To close these seams an apron of concrete is often laid around the dam.

Pneumatic Caisson.—The *plenum pneumatic process* consists in excluding the water from a limited area by means of compressed air. The apparatus first employed was the *diving-bell*. This was a strong metal box open at the bottom which was lowered to the bed of the river by means of cranes. The workmen entered the bell while it was above water and were lowered with it. The water was prevented from entering by the pressure of the air which was increased as the bell sank. The air was compressed by some form of air-pump and forced into the interior of the bell through a flexible tube.

The pneumatic caisson is simply a diving-bell whose sides are prolonged so that they project above the surface of the water. The workmen now enter and leave the bell through doors made in its roof. To prevent the sudden escape of the compressed air the workmen first enter a small air-tight vestibule in which the air pressure can be increased or diminished at will. The caisson above the diving-bell is simply a common caisson or coffer-dam with water-tight walls within which the masonry of the foundation is begun.

Construction.—The caisson (Fig. 132) is divided into an air-tight *working-chamber* and a water-tight *coffer-dam* extending

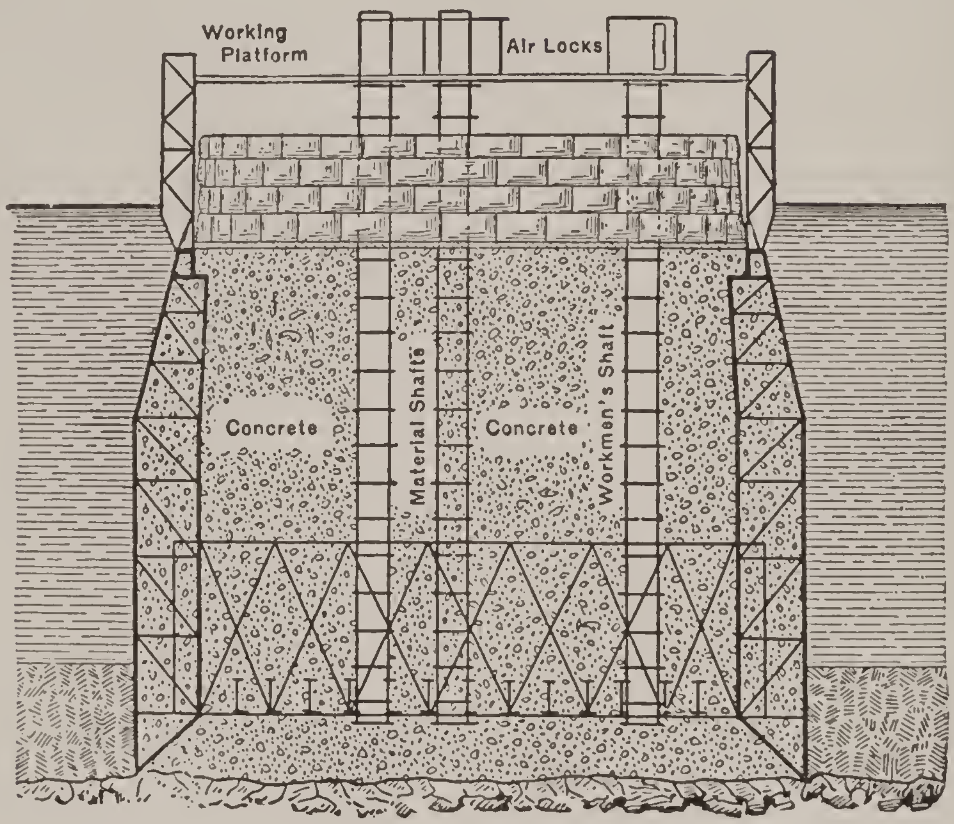


FIG. 132.

from the roof of the working-chamber to some distance above the water surface. It is provided with *air-locks* and *vertical shafts* for communication and the removal of material, and *pipes* for forcing compressed air and water into the chamber, removing sand and mud, and carrying the lighting wires.

The *working-chamber* is usually 8 to 10 feet high and is surrounded by wedge-shaped walls with cutting edges similar to those of the crib and the cylinder heretofore described. The partition walls, if any, have flat bearing surfaces to support the weight of the roof and its load. The roof of the chamber must be air-tight and strong enough to support the pressures to which it is exposed. If made of wood, it consists of several layers of $12'' \times 12''$ beams laid in close contact; if of iron or steel, it is composed of trusses or girders connected by I beams and covered with plates.

The *coffer-dam* may be made of wood properly calked to make it water-tight or it may be made of iron properly strengthened. To decrease the height of a wooden coffer-dam, a grillage of many layers of 12″×12″ timber is placed upon the roof of the working-chamber. In the grillage the logs may be placed in close contact or may be separated by intervals filled with concrete.

The vertical *shafts* are usually of iron plate, and if at any time filled with compressed air, must be air-tight. Those for communication are about $3\frac{1}{2}$ to 4 feet in diameter and are provided with air-locks. Those for dropping concrete into the working-chamber are smaller and have air-tight doors at the top and bottom.

The *air-locks*, or vestibules, are air-tight chambers which have doors for communication with the outer air and the working-chamber, and are provided with air-cocks and escape-valves for increasing or diminishing the air pressure. As the doors open towards the interior of the caisson, they are kept closed by the air pressure whenever there is a difference of pressure on the two sides of the wall in which they are made. An air-lock may be constructed at the top of the shaft, at the bottom, or at any intermediate position. The first is the best position for the workmen in case of accident, but the locks must be moved as the caisson is built up. The second is the most convenient, but also the most dangerous. An intermediate position is sometimes secured by making the shaft discontinuous; the upper part contains the air-lock at its base, the lower part is tangent to the upper and overlaps it to the height of the lock. A section of the shaft itself may be converted into an air-lock by inserting horizontal air-tight doors.

The material at the bottom of the river through which it is necessary to lower the caisson in order to reach a firm stratum is usually mud, sand, or silt. These materials can be most readily removed by utilizing the compressed air in the chamber or by means of a sand- or mud-pump.

If the air itself is used, the discharge-pipe is continued to the bottom of the working-chamber and a curved elbow inserted. The short arm of the pipe is flexible and its end is partially inserted in a pool of water formed at the bottom. By opening a valve, the air rushes up the pipe and carries with it the water with which earth is mixed. The sand-and mud-pumps are constructed on the principle shown in Fig. 133.

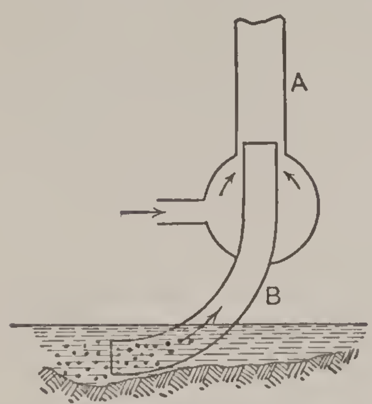


FIG. 133.

Water under high pressure is forced into a chamber at the base of the discharge-pipe *A*, just below its junction with the suction-pipe *B*. As the water escapes between the pipes it creates a partial vacuum in pipe *B*, sucks up the mud or sand and water and discharges it over the coffer into the water or into scows. Large boulders may be broken up and removed through the air-locks, or may be stored near the roof of the chamber and finally buried in the concrete when the chamber is filled.

Incandescent electric light is the best method of lighting the air-chamber, as ordinary lights burn rapidly in compressed air and vitiate it.

Sinking the Caisson.—The pneumatic caisson is commenced on shore, launched, partially loaded with masonry, floated into position, anchored and lowered to the bottom much in the same manner as the crib and cylinder sunk by the well process. When it comes to rest, compressed air is forced into the chamber, and the workmen descend into it and begin to remove the material. When sufficient material has been removed they either leave the chamber or take refuge in the shaft while the air is allowed to escape gradually. The weight of the caisson then causes it to sink a few feet. If more weight is needed, concrete may be added to that already placed on the roof of the working-chamber. The difficulty is usually, however, the reverse, that is, to prevent the caisson from sinking too rapidly. This can be regulated only by carefully excavating under the partitions or piers of the working-chamber.

This process is continued until the caisson finally reaches the stratum upon which it is to rest. If this is gravel or approximately level rock, it is leveled with concrete and the chamber filled with concrete. The concrete is lowered through the small shafts. The lower doors of these shafts are closed and supported, and the material poured in; the upper doors are then closed, the pressure regulated, and the lower doors opened. Water is poured into the tubes before and after the concrete to prevent the premature setting of the concrete due to the heat of the compressed air.

If the bed is an inclined stratum of rock, it may be blasted to a horizontal surface, blasted into steps, or, if the inclination is not great, the foundation may be prevented from sliding by steel rods let into the rock. The caisson is stopped as soon as one of its edges rests upon any part of the rock, and if the foundation is built upon steps or an inclined surface, it is leveled with concrete laid in water.

As the pressure in the working-chamber increases fifteen pounds for every $34\frac{1}{2}$ feet of submergence of the cutting edge, the depth at which this method can be utilized depends upon the amount of pressure which workmen can stand. This is about forty-five pounds above the normal, which makes the limiting depth about 100 feet.

A combination of the pneumatic caisson and the open dredging caisson has been devised and utilized for very deep foundations.

The Caissons of the New York Suspension Bridges.—The caisson of the New York pier of the Brooklyn Bridge is constructed of timber and is 172 feet long, 102 feet wide, and 32 feet high; its sides have a batter of 10/1. The working-chamber is about 10 feet high, its side walls have a slope of 45 degrees, and its roof is a solid wooden grillage 22 feet thick. During construction the roof was supported by temporary partitions and piers. The caisson rests on rock at a depth of 78 feet below mean high tide; its working-chamber is filled with concrete. Among the novel features of the caisson was a central shaft $7\frac{3}{4}$ feet in diameter extending $2\frac{1}{2}$ feet below the cutting edges. This shaft was kept full of water and was utilized to remove large boulders. Besides ordinary calking the chamber was made air-tight by a continuous sheet of tin in the body of the roof, and was protected against fire by a metal lining. A detachable wooden dam was employed above the grillage. The caisson of the Brooklyn pier was similar in construction; as the top of the masonry was always above water, no dam was employed.

The caissons of the Williamsburg Bridge, New York, are of timber, but are only 76 feet long and 60 feet wide. Two caissons were employed at each tower. The walls of the caisson and the roof of the working-chamber are of solid construction, but the greater part of the caisson is of an open crib work strengthened by several longitudinal and transversal Howe trusses. The roof of the working-chamber rests on the lower chords of the Howe trusses, and the platform for the masonry on the upper chords. The trusses themselves rest on the walls and the partitions of the working-chamber. Where necessary a dam with timber walls was employed above each caisson. When finally in place the caissons were filled with concrete.

The Forth Bridge.—One of the iron caissons employed in the construction of the Forth Bridge, Scotland, is shown in Fig. 132. The base of the caisson is 70 feet and the top 60 feet in diameter. The caisson proper is filled with concrete and extends to low-tide level. The iron detachable dam extends from the low-tide level to some distance above high tide, the range of the tide being 18 feet. On account of the exposed position of the piers, the sides were made of two concentric cylinders of iron plate connected by strong bracing. The roof of the working-chamber was supported by several riveted trusses and I beams. The caisson was provided with iron shafts each $3\frac{1}{2}$ feet in diameter, one for men and two for materials. Tubes for lowering concrete were inserted in the latter when the caisson was in place. The air-locks were at the top. One of the piers of this bridge rests on rock 96 feet below the level of high tide. This bridge is shown in Fig. 140.

Foundations below Quicksand.

None of the methods heretofore described is practicable when the bed of the foundation lies below a thick stratum of quicksand, or fine sand and water which runs easily. The Poetch method of overcoming this difficulty consists of substituting, for the wooden or masonry walls of the foundation-shaft as heretofore described, walls of frozen quicksand.

To form the walls a number of iron pipes, closed at the bottom, are sunk or driven through the quicksand to the desired depth. These pipes are placed at small and regular intervals along some polygon surrounding the site of the shaft. Smaller pipes, open at the bottom, are inserted in the first set, and both sets are connected with a refrigeration apparatus so that the freezing mixture flows into the smaller and returns by the larger pipes. A cylinder of frozen soil is thus formed about each pipe, and in time these cylinders interlock and form a wall within which the shaft may be safely excavated. Pipes open at the bottom may be first driven when the closed pipes cannot be easily forced into the soil; these are called *pilot* pipes and are large enough to contain the closed pipes.

Another method which has been utilized for overcoming the difficulties of working in quicksand is the forcing of cement grout into the sand by means of force-pumps The cement in setting converts the sand into a solid mass.

For further information consult Patton's "Practical Treatise on Foundations," Fowler's "Ordinary Foundations," and Wellington's "Piles and Pile Driving."

CHAPTER XXIII.

BRIDGES.

Definitions.—A *bridge* is a structure erected over a water-course to connect lines of communication upon opposite banks. A *viaduct* is a similar structure designed to carry a line of communications above the surface of the ground.

The essential parts of a bridge are the *substructure*, by means of which the roadway is elevated, and the *superstructure*, which carries the moving load and transmits its weight to the substructure. The substructure is composed of *abutments* and *piers;* the superstructure, of the *floor* and the *floor-carriers*.

An *abutment* is one of the end supports of the superstructure.

A *pier* is one of the intermediate supports of the superstructure.

The floor is composed of the *flooring*, over which the travel passes; the *longitudinal joists*, which support the flooring; and the *floor-beams* or *cross-girders*, which support the joists and transmit the weight of the floor to the floor-carriers.

The *floor-carriers* are the *plate girders*, *trusses*, *arches*, or *cables* which support the floor and transmit its weight to the abutments and piers.

A *span* is that portion of a bridge between the centers of adjacent piers and abutments.

A *skew bridge* is one whose axis is oblique to the longer dimension of its piers and abutments.

Wooden Trestles.—The simplest form of support for a viaduct is the wooden trestle-bent (Fig. 134). It is employed where the viaduct is on firm soil. The bent is made up of a *capsill*, *AB*; a *groundsill* or *mudsill*, *CD*; two *vertical posts;* two *batter-posts*, *AC* and *BD*; and two *cross-* or *sway-braces*, *AD* and *BC*. The sway-braces are omitted in bents less than 10 feet high.

The interior posts may also be inclined and make with the batter-posts an inverted W. In railway construction all the members except the sway-braces are $12'' \times 12''$ timbers; the sway-braces are 3-inch planks. In highway viaducts lighter pieces are employed. The height of the simple bent is limited to 25 feet. If bents of a greater height are required, they are made in the form of superposed bents as shown in Fig. 135.

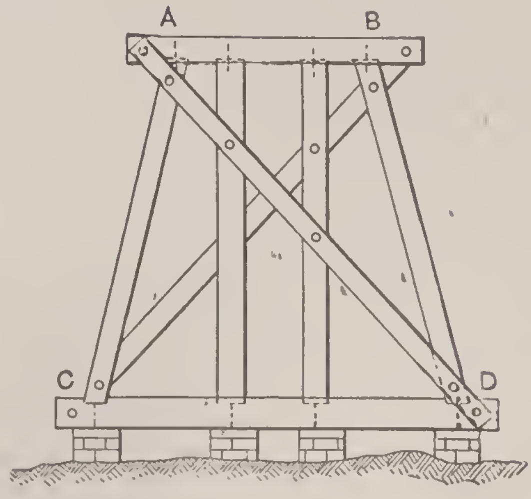


FIG. 134.

The space between consecutive horizontals is called a *story* and does not ordinarily exceed 20 feet in height.

In high bents the posts may be continuous and be made of single or spliced pieces, or the bents may be constructed separately and superposed. In the former type the stories are made by bolting timbers in pairs to the posts. In the second type the superposed sills may be in absolute contact, or they may be separated by the longitudinal bracing.

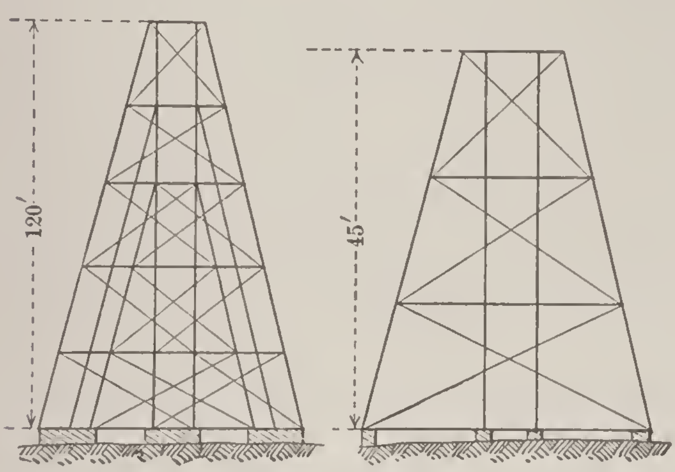


FIG. 135.

Simple bents are placed at intervals of 12 to 14 feet; high bents, at intervals not exceeding 25 feet. They are connected horizontally by the stringers which connect adjacent caps, and support the roadway. To give additional longitudinal strength, the batter-posts of simple bents are connected by horizontal or diagonal bracing similar to the sway-bracing. In high viaducts or trestles the intermediate sills of the different bents are connected by horizontal beams and the batter-posts of each story by horizontal or diagonal bracing.

The floors may also be strengthened by a diagonal system of bracing connecting the consecutive capsills.

In constructing the bents, the posts are mortised or notched into the sills and the joints strengthened by drift-bolts, dowels, or plates; the sway-bracing is bolted or spiked to the posts.

In a viaduct constructed on soft soil the common trestle-bents are replaced by *pile-bents.* The simple pile-bent consists of a capsill supported on three or more piles. The cap is usually but a short distance above the surface; where high bents are required the pile-bent is covered by one or more stories of common trestle.

Steel Trestles and Towers.—A steel trestle-bent has but two posts, which are usually inclined. The posts are built-up columns which rest on masonry bases; the capsill is a plate girder, and the groundsill is replaced by a laced member attached to the posts; the sway-bracing is made of steel ties. In low viaducts on city streets the bents consist of posts and caps only; the former are bolted to immovable bases.

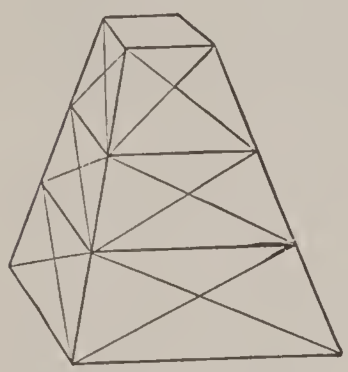


FIG. 136.

High steel viaducts consist of a series of steel towers connected by the superstructure. This requires less material than a series of trestle-bents, as no longitudinal bracing is required between towers. A trestle tower (Fig. 136) is formed of four vertical or inclined posts. Each face of the tower is divided into stories and braced like a simple trestle-bent. At intervals the diagonally opposite posts are connected by horizontal braces. If the longitudinal dimension of the tower is much greater than its lateral dimension, an additional post is inserted on each side.

Cribs.—Crib piers are constructed of logs or beams fastened together and weighted as previously explained. They are employed in the construction of hasty or unimportant bridges. If the current is strong, the cribs are so placed that they present the vertex of an angle to the current.

Masonry Piers.—Masonry piers, extending at least to the height of flood-level, are employed in the construction of all important bridges. The plan of the pier is usually rectangular, with the longer dimension parallel to the current (Fig. 137). If the current is strong, the shorter faces of the pier, at least below the level of high water, are rounded or pointed to deflect the water and thus reduce both the pressure on the pier and the undermining action of the current. These deflectors are called *starlings.* If exposed to floating ice in a strong current, a portion of each up-stream starling is inclined at an angle of 45 degrees with the horizontal, so that the ice will be forced up the inclined surface until it breaks of its own weight. This prevents the formation of an ice-gorge and the consequent pressure on the pier. The inclined surface is called an *ice-breaker* and extends from a few feet below low water to a few feet above high water.

The longer dimension of the pier is fixed by the width of the superstructure; the shorter dimension is made sufficiently large

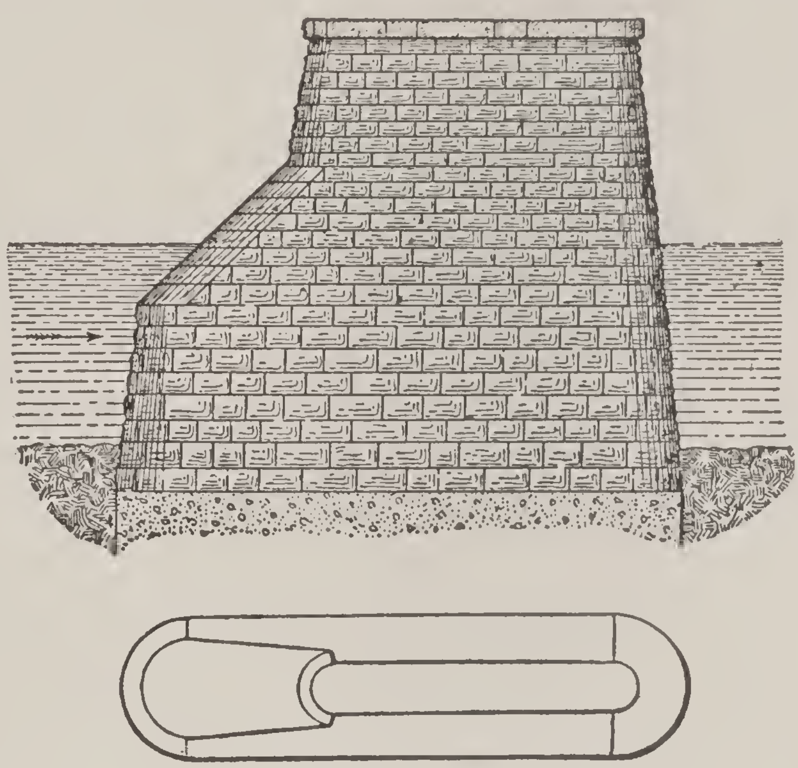


Fig. 137.

to give an ample bearing surface for the superstructure and a large factor of stability to the pier itself. For spans over 100 feet the piers are not usually less than 6 feet thick.

The top of the pier is crowned by a heavy overhanging coping, and the faces have a slight batter which does not ordinarily exceed 12/1. The dimensions of the base depend on the direction and intensity of the resultant pressure and the bearing power of the soil. If the base is much larger than the top, offsets with water-tables are made in the faces.

Piers are usually constructed of rubble or concrete with a facing of quarry-faced ashlar.

Approaches and Abutments.—The approach to a bridge may be either a trestle or an embankment. If the former, the abutment is similar in construction to a pier; if the latter, the abutment is a combined pier and retaining-wall. It is designed to resist both the pressure of the embankment alone, and the pressure of the embankment with the weight of the superstructure.

If the side slopes of the embankment need not be supported, the abutment is a straight retaining-wall with a rectangular face, whose length is determined by the width of the superstructure. If the side slopes must be supported, walls triangular in elevation, called *wings*, are added in prolongation of the straight wall. Both the top and bottom surfaces of each wing are usually made in steps. If the base of the abutment is liable to be submerged at high water, the wings are bent back so that the contraction of the waterway is gradual. The wings of the abutment are also made perpendicular to the face; this is called a U abutment. In the U abutment the wings become counterforts, and if the embankment is not very wide, they relieve the abutment proper of much of the earth pressure on it. The length of the wings is about one and a half times the height of the abutment. A straight abutment may also be strengthened by a wide central counterfort; it then becomes a T abutment. The T abutment is employed in railway bridges. The width of the counterfort is that of the track, and its length one and a half times the height of the abutment. It relieves the abutment of much of the pressure due to a heavy engine about to move on the bridge.

Abutments are usually constructed of masonry, but for temporary or unimportant bridges they may also be of wood. A wooden abutment is usually made of squared logs placed on top of each other and held in place by anchor-logs fastened to the face-logs, and to transverse logs buried in the embankment. The transverse logs must be beyond the plane of natural slope passing through the foot of the abutment face. If the abutment is U-shaped, the wings should if possible be connected by anchor-logs.

The Floor.—If a simple highway bridge (Fig. 138) is taken as a model, the *flooring* consists of yellow-pine or white-oak planks about three inches thick, which are laid transversely and securely spiked to the joists. The opening between planks is $\frac{1}{4}$ inch; for convenience in making repairs, a continuous longitudinal joint is made along the center line of all long bridges having a roadway 16 or more feet wide. The sidewalks are of

2-inch planks, and are usually separated from the roadway by the roadway carriers; if not thus separated, they should be elevated a few inches above the roadway. If the roadway extends to the carriers, longitudinal beams, at least 6 inches high, called *wheel-guards*, are spiked to the floor to prevent the carriers from being injured by the hubs of the wheels. A *hand-rail* is also constructed along each side of the flooring. The minimum clear width of roadway should be 10 feet for a single vehicle and 16 feet for two vehicles abreast.

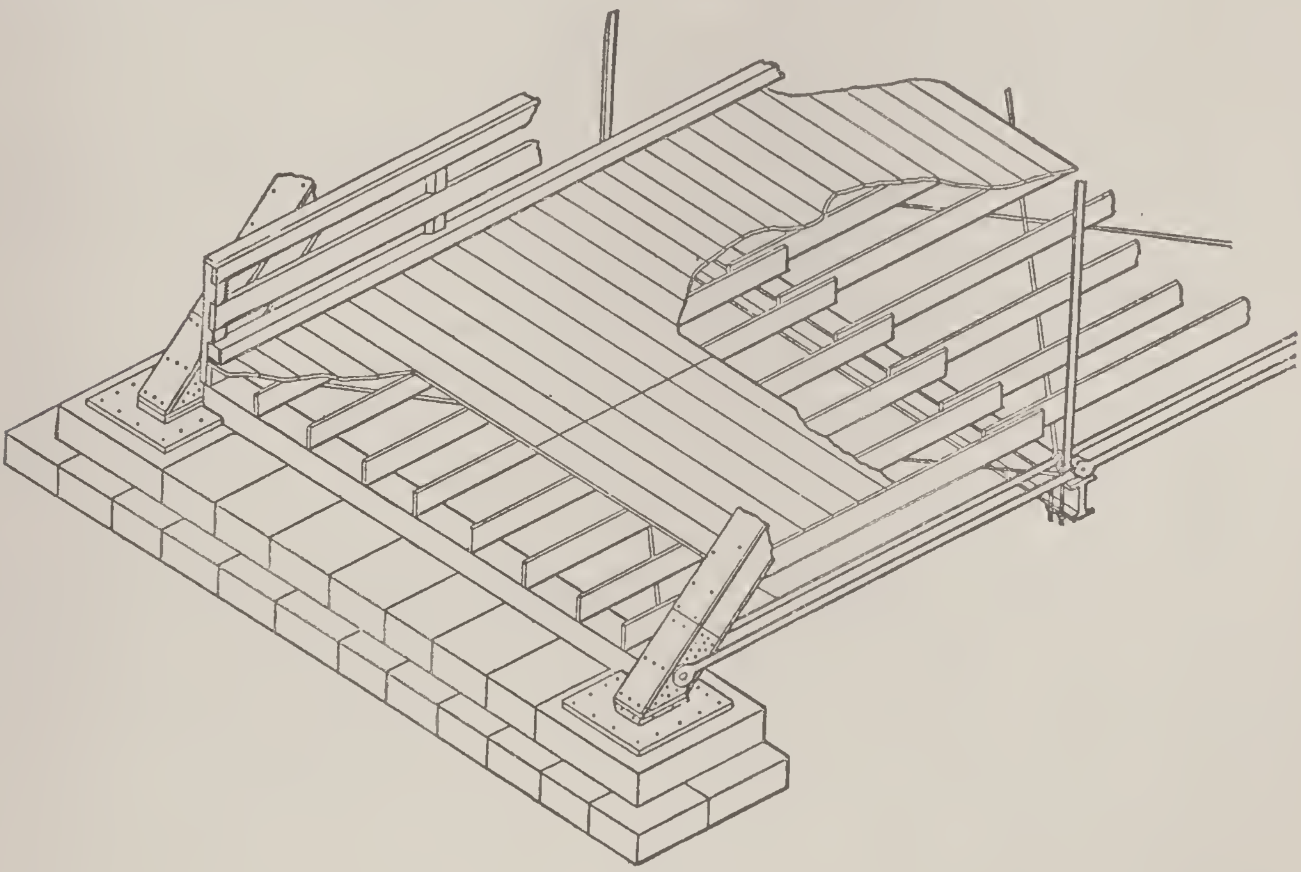


Fig. 138.

The *joists* are of yellow pine or other wood, not less than 3 inches thick; they are spaced about 2 feet center to center. Joists of adjacent panels either overlap or abut against each other on the floor-beams. An air-space is left between their ends. If the spans are less than 25 feet, the joists rest directly upon the piers and abutments and become the roadway carriers; the joists may then be rectangular wooden beams or steel I beams.

The *floor-beams* or *cross-girders* connect the roadway carriers and support the floor; rectangular wooden beams, singly or in pairs, rolled steel I beams, and plate girders may be employed for floor-beams.

Metal Floors.—When the roadway is to be paved, the pavement is supported on a *buckle-plate* floor which is riveted to steel

I-beam joists. A buckle-plate is a steel plate, about 4 or 5 feet square and $\frac{5}{16}$ of an inch thick, which is so bent that if placed horizontally its central point is about 2 inches higher than its edges. Metal floors are also made of channel-shaped plates.

In railway bridges of less than 15 feet span the cross-ties are usually fastened to longitudinal stringers which rest on the piers and abutments; in bridges of longer span these stringers rest on and are fastened to the floor-beams. In bridges with metal floors the ties are embedded in the ballast, which is supported by the floor. For short spans reinforced concrete floors of the Melan type are also constructed.

Camber.—The floor of a bridge is slightly arched in order that it may not become concave under the heaviest live load to which it may be subjected. The rise of the arch is called the *camber* of the bridge.

Floor-carriers.—According to the method of supporting the floor and transmitting its weight to the piers and abutments, bridges are classified as *plate-girder*, *truss*, *arch*, and *suspension bridges*.

Plate-girder Bridge.—A *plate-girder bridge* is composed of two or more plate girders to which are riveted the floor-beams. These beams rest either on the flanges or on brackets riveted to the web. The ends of the girder rest on steel bedplates which have sufficient area to distribute the weight and reduce the pressure on the masonry within safe limits. In all plate-girder and truss bridges, if the friction is not sufficient to resist the lateral pressure from the wind, the girders and trusses are so fastened to the abutments that lateral movement is impossible. To prevent longitudinal movement one end is anchored to the abutment; to allow longitudinal expansion the other end is supported on a smooth plate if the span is short, and on steel rollers if the span is long. Plate girders are most economical when utilized for spans of 20 to 100 feet.

Truss-bridge.—A truss-bridge (Fig. 139) is composed of two or more parallel trusses to which the floor-beams are attached, usually at the panel points of one of the chords. Trusses are ordinarily made of wood, wood and steel, or of steel alone. The drawing is that of a light steel highway-bridge truss for a span of 100 feet.

Steel- and wrought-iron trusses may be either riveted or pin-connected. The riveted truss is employed when great stiffness or freedom from vibration is desired. The ordinary types of trusses are the *Warren truss* or *riveted girder* for spans from 20 to 100 feet, the *Pratt truss with parallel chords* for spans from 20 to 200 feet, and the *Baltimore* or similar trusses for spans of 200 to 800 feet.

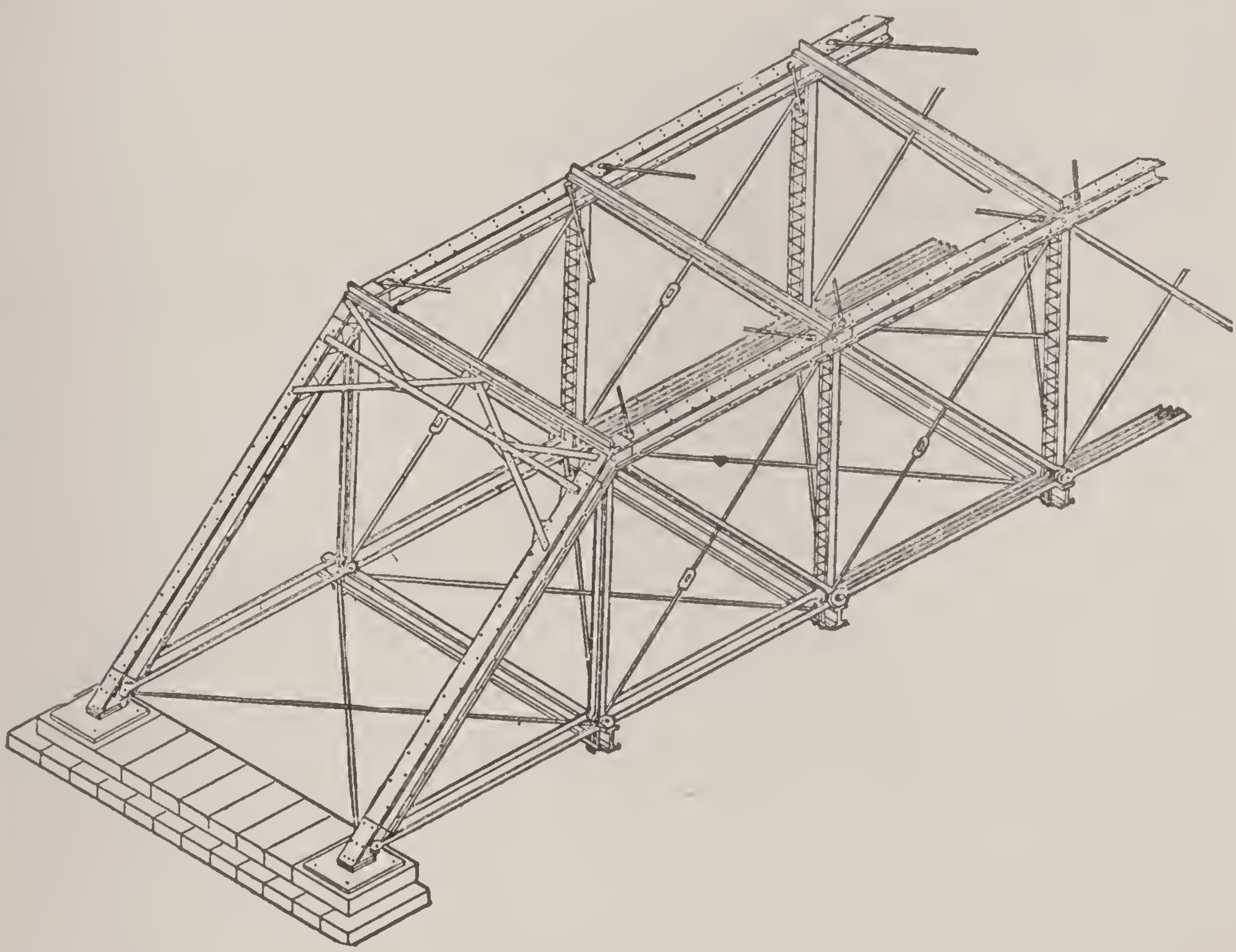


FIG. 139.

The flanges of the Warren truss are usually made of two angles with or without cover-plates, like those of a plate girder, or of two channels. If the span is short, the flanges are connected at their ends by narrow rectangular plates; between the plates the triangular bracing is composed of angles either singly or in pairs. The angles and plates are either riveted to the flanges or both angles and flanges to a connecting plate. If the span is long, all the members of the Warren truss are laced members.

In a pin-connected truss the compression chord is made of two channels, either rolled or built up, connected by a top horizontal cover-plate. The inclined end- or *batter-posts* of the type shown in Fig. 139 are of similar construction and are rigidly connected with the compression chord. The tension chord is made of eye-bars. To stiffen the truss near the abutments the use of laced members instead of eye-bars is recommended in the first two panels. The vertical struts are made of two laced or latticed channels; in short trusses they are riveted to the upper chords so as to secure greater stiffness. The diagonals are eye-bars; in light bridges the counters have turnbuckles so that they may be adjusted. The vertical tie near the abutment may be an eye-bar or, if stiffness is desired, a laced member. The floor-beams are either riveted to the verticals or are suspended from the pins by eye-bar connections; the former is preferable. The lateral bracing is composed of laced angles and eye-bars.

In the riveted truss the general construction is the same, but the members are usually all laced or latticed pieces and are riveted together.

The length of the truss is usually governed by local conditions; it is ordinarily assumed that in a bridge consisting of a number of spans the arrangement is economical, if the cost of substructure and superstructure is approximately the same. In medium spans the height of the truss is about $\frac{1}{5}$ to $\frac{1}{8}$ the span; in very long trusses this ratio is reduced.

Cantilever Truss.—Cantilever bridges are of two general types. In the first type, shown in Fig. 140, each pier supports

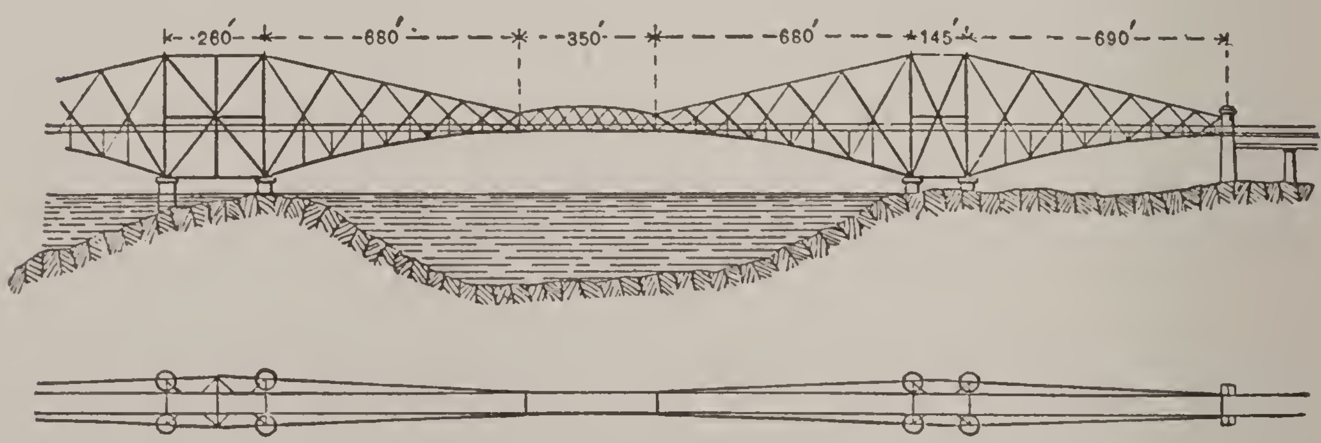


FIG. 140.

a double cantilever tower with long arms, and the ends of the arms of adjacent towers are the supports for a simple or *suspended* truss which connects them. In the Forth Bridge, Scotland (Fig. 140), there are three cantilever towers each about 330 feet high. The central tower rests on a base 120 by 260 feet, and is 1620 feet long between the ends of its arms; it is connected with the other towers by riveted trusses 350 feet long. The shore ends of the other towers rest on the abutments. The suspended trusses are 150 feet above high-water level. Each tower rests on four masonry piers; the distance between adjacent piers of consecutive towers is 1710 feet center to center. The St. Lawrence cantilever bridge near Quebec has a central span of 1800 feet between piers, center to center.

In the second type, shown in Fig. 141, a simple truss resting on two points of support has attached to its ends cantilevers

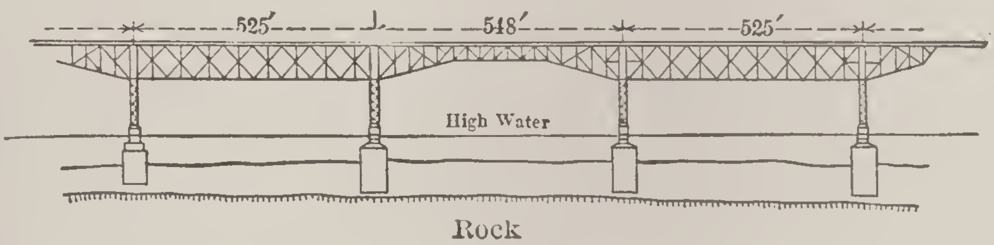


FIG. 141.

whose extremities form the supports for a simple suspended truss. The bridge shown in the figure is that over the Hudson River at Poughkeepsie. It has two simple-truss spans of 525 feet and three cantilever spans of 548 feet. The bridge over the Mississippi River at Memphis of the same general type has a simple-truss span of 621 feet, a single-cantilever span of 621 feet, and a double-cantilever span of 790 feet.

Howe Truss.—The Howe truss is the type of bridge which is usually constructed when timber and simple forms of iron are employed. It is the principal truss employed for hasty work in military operations. The chords, the main diagonals, and the counters are made of timber, and the vertical braces of iron rods. The floor-beams rest on and are supported by one of the chords. The king-post type is employed when the span does not exceed 30 feet; the queen-post with counterstruts, when the span does not exceed 40 feet; and the type shown in Fig. 142 for longer spans. In long trusses the main diagonals are in pairs and inclose the counter-diagonals; to secure rapidity in construction all the diagonals are of the same size. The verticals are in pairs, and for the same reason are of uniform size. The chords are made of three beams of uniform size and of the same width as the diagonals, which are laid side by side and bolted together. The diagonals abut against hard-wood blocks or iron castings which are notched into the chords; the diagonals are held in place on the blocks by dowels. Washers are placed under the heads and nuts of the rods to prevent them from crushing the chords.

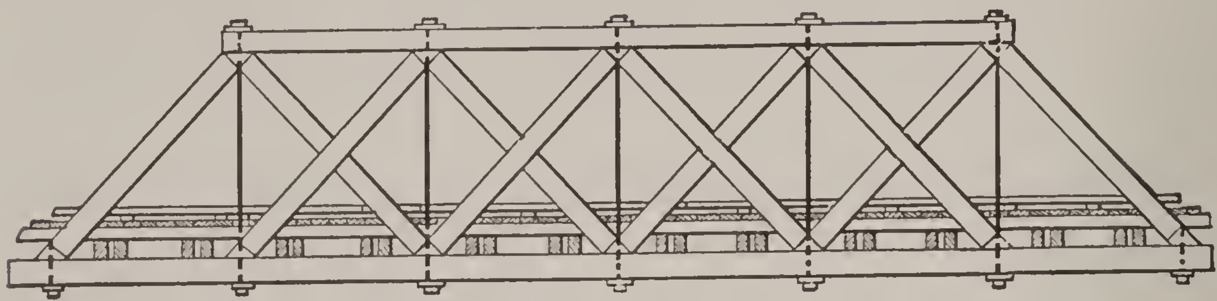


FIG. 142.

Arched Bridges.—Arched bridges are constructed of masonry, of reinforced concrete, and of steel alone. The arch is the type of bridge which is constructed whenever it is desired to secure a structure of fine architectural appearance; it is the only type that can be made wholly of masonry.

Masonry Arches.—The masonry arch has been employed in bridge and viaduct construction since it was first developed by the Romans. It is preferable to all metal structures, since it is practically indestructible if well founded, and requires little expense for maintenance. It is, however, more expensive than a metal bridge, and its employment is limited to shorter spans. Within limits as to span, arched bridges admit of great variation in construction.

According to their spans all existing masonry arch bridges may be divided into three classes: those exceeding 200 feet, those between 100 and 200 feet, and those less than 100 feet. In the first class are less than ten, and in the second less than fifty; nearly all are therefore in the third class. When bridges over 100 feet are to be constructed, it is usually found cheaper to construct a series of arches than to construct a single-span bridge.

Form of Intrados.—The full-center arch is easily constructed, since the voussoirs all have the same cross-section; it is very stable, since the resultant pressure at the springing-line makes a small angle with the vertical. Its field of usefulness is limited, however, by the fact that the ratio of span to rise must be constant. To employ the full-center arch the height of the roadway above the tops of the piers must be greater than one half the span. The full-center arch is therefore limited to the construction of high masonry bridges or viaducts. When these are of great height and length, the bridge is made of high piers connected by a series of full-center arches, or of two or more tiers of arches. A viaduct of this latter type in Saxony is 1900 feet long, 264 feet high, and has four tiers of arches.

The segmental arch is also easily constructed, and the ratio of span to rise admits of great variation. The horizontal component of the thrust on the abutment increases, and the area between the intrados and the chord connecting the springing-lines decreases with the length of the radius of curvature. A segmental arch can therefore be adjusted to any span and rise, but for long spans requires very stable abutments; in the long-span segmental arches of the first and second classes the abutments are usually natural ledges of rock. When a bridge over a river consists of a series of segmental arches, their springing-lines must be high enough above the water surface to give the desired clearance for navigation and the requisite waterway for floods.

The three-center ovals (page 393) and the elliptical arches partake of some of the properties of the full-center and segmental arches, but are not so easily constructed. The ratio of the span to the rise admits of great variation, and the horizontal component of the thrust at the springing-lines is less than in a segmental arch of the same span and rise. These arches have a greater area between the intrados and the chord connecting the springing-lines, and a greater clearance at every point, than a segmental arch having the same rise and span, and present a more pleasing appearance. Elliptical and three-center oval arches are usually constructed when the springing-lines are low and it is desired to secure the maximum clearance and waterway. This type is employed for single arches of moderate span, and also for a series of arches.

In the oval of many centers (page 393) not only the ratio between the span and rise admits of great variation, but the curvature may be changed as often as seems desirable between crown and springing-line. This permits of the construction of an arch of the elliptical type which is very flat near the crown. Its clearance and waterway are even greater than the three-center oval. Arches of wide span and slight rise are usually of this type.

Spandrel.—The face-walls of the spandrel may be continuous, or they may be pierced by transversal arches which rest on the arch-ring and support the roadway (Fig. 143). The latter arch presents the more pleasing appearance and has a smaller load on the arch-ring; the dimensions of the arch-ring and the abutments may therefore be reduced.

Capping.—To prevent the percolation of water through the arch-ring it is coated with an asphalt or other water-proof coating. The drainage from this coating is received in a cross-drain at the lowest point of the extrados, and conveyed through the walls by a suitable conductor.

Materials.—Masonry arches are made of stone, brick, or concrete. Stone arches have been constructed with spans of about 300 feet, brick arches with spans of about 150 feet, and concrete arches with spans of about 140 feet. Concrete arches of long spans are usually of the segmental type with steel hinges at the crown and springing-lines.

Luxemburg Bridge.—This bridge (Fig. 143), which spans a deep valley adjacent to the city of Luxemburg, Europe, shows the

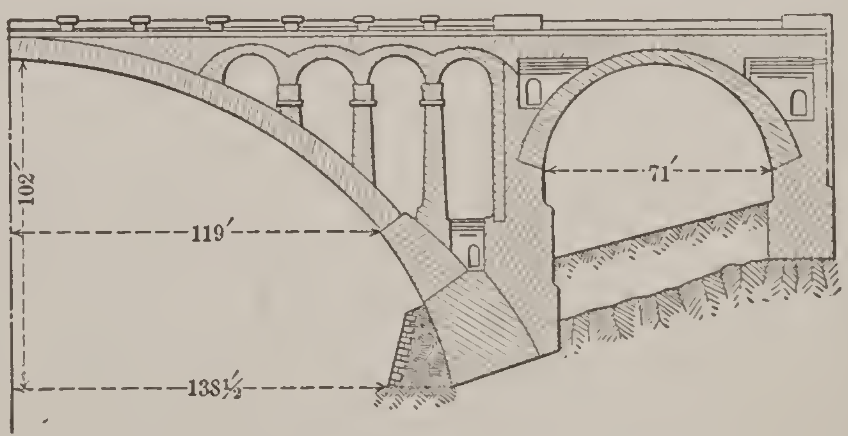


FIG. 143.

present development of arch construction. It consists of three stone arches; the outer arches are full-center arches with spans of 71 feet, and the middle one is an oval of 277 feet span. As the abutments of the main span are underneath the surface of the ground, the visible span is about 260 feet. The arch-ring is 4.7 feet thick at the crown and 7 feet at the springing-lines. The spandrel arches are full-center arches of 18 feet span. To save expense, the bridge, which is 60 feet wide, is made of two arched ribs, each 20 feet wide, separated by an interval of 20 feet. The roadway bridges this opening. By constructing one rib at a time the same center was available for both ribs.

Arches of Reinforced Concrete.—Arches of concrete reinforced by steel are of modern construction. Those of the Melan type are made by imbedding parallel steel ribs in concrete. The ribs are placed about 3 feet center to center, are curved to the form of the arch, and extend well into the abutments. In small arches they are made of I beams, and in large ones of riveted girders of the Warren type. In the Monier type wire nets are imbedded near the intrados and extrados. Modifications of these methods are to imbed near the intrados and extrados either flat or twisted bars spaced like the ribs of a Melan arch. Arches of the Melan type have been quite extensively employed in this country. An arched bridge at Topeka, Kansas, has one span of 125 feet, two of 110 feet, and two of 97 feet. This type of arch can be constructed more rapidly and cheaply than a masonry one and will probably wholly replace the masonry arch in highway-bridge construction.

Steel Arches.—The roadway of a steel-arch bridge may be supported by two or more plate-girder ribs or by two or more open braced ribs. In a plate-girder arch the roadway is usually supported by a trestle which rests on the ribs. The open braced rib may be made by uniting a straight with a curved chord by riveted triangular bracing (Fig. 144). In this construction the floor-beams are usually supported at the panel points of the upper and straight chord. The open braced rib may also be made of two curved chords connected by riveted triangular bracing (Fig. 145). The roadway may then be supported on a trestle resting on the rib, or it may be suspended by rods attached to the rib. The arch presents a more pleasing appearance than the simple truss, and is more easily erected over deep depressions.

Arched bridges are of three general classes: those with hinges at the crown and springing-lines, those with hinges at the springing-lines only, and those without hinges. The hinge is a heavy steel pin against which the arch or its segments abut, so that the arch can accommodate itself to expansions and contractions caused by variations of temperature.

In the three-hinged arch the reactions at the crown and springing-lines can be determined with the same accuracy as those of the simple truss; for this reason it is prefererd by many engineers to the other types; its maximum deflections under heavy loads are, however, greater than in the other types. One of the most notable arches of this type is the railway bridge over the Viaur River in France (Fig. 144). The span of the arch

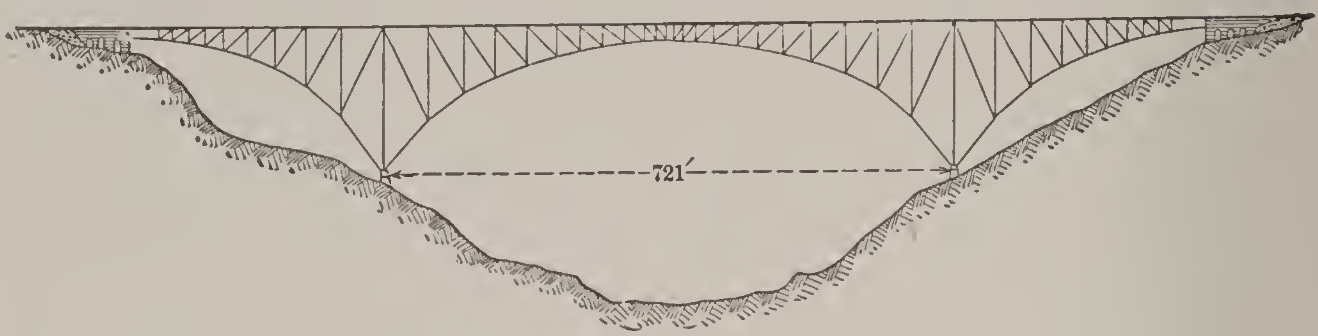


FIG. 144.

is 721 feet, its rise 176 feet, and the height of the roadway above the river 380 feet. The total length between abutments, which is 1344 feet, is made up of the central span, two cantilevers each 226 feet, and two suspended trusses each 85 feet. The cantilevers and trusses reduce the crown thrust. The two ribs of which the bridge is made are 109 feet apart at the springing-lines and 19 feet at the crown.

In the two-hinged arch the reactions cannot be determined with as great accuracy as in the three-hinged one, but its maximum deflections are less. It has also its advocates. Two of the most notable bridges of this type are those over the Niagara River below the falls. The double-track railway bridge which replaced the suspension bridge has a central arch connected with the abutments by two simple trusses. The arch has a span of 550 feet, a rise of 114 feet; the railway is 225 feet above the river. Its general form is similar to the arch span in Fig. 144. The roadway bridge has a two-hinged arch of the type shown in Fig. 145, which has a span of 840 feet, rise 137 feet, and carries a 46-foot roadway 240 feet above the water level.

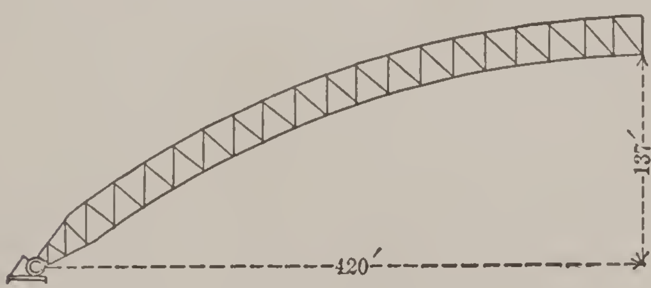


FIG. 145.

The arch without hinges is not now employed for long spans; the bridge over the Mississippi at St. Louis, constructed in 1874, is one of the most notable examples. This bridge has one span of 520 feet and two spans of 502 feet each.

Suspension Bridges.—A suspension bridge (Fig. 146) is one in which the floor is suspended from two or more cables or

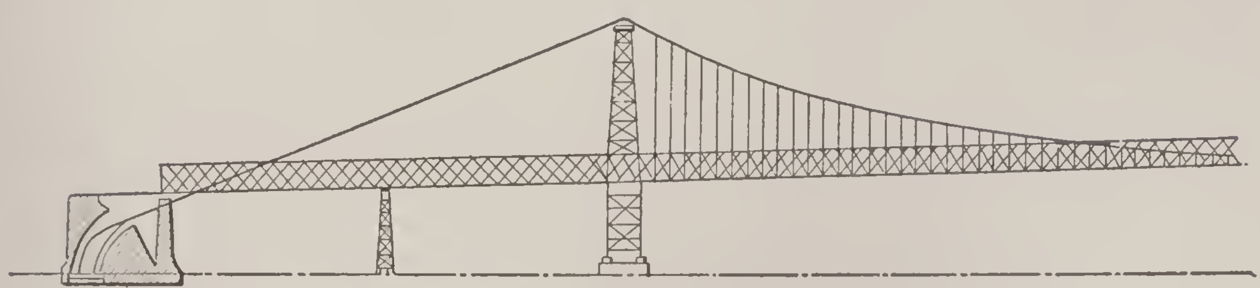


FIG. 146.

chains which are stretched between the piers or abutments. This type is employed for very long spans, and for short spans over deep gorges when great stiffness is not essential. It is probably the only type which can be applied to spans exceeding 2000 feet; it has been proposed to utilize the suspension principle for bridges of 3000 feet span. For spans between 1000 and 2000 feet the suspension bridge has been largely replaced by the cantilever.

The essential parts of a suspension bridge are the *main cables* or *chains*, from which the floor is suspended; the *towers*, upon which these cables rest; the *anchorages*, which hold the ends of the main cables; the *suspenders*, by which the floor is attached to the main cables; the *trusses*, or other devices, by which the oscillations are checked; and the *floor system* itself.

Main Cables.—The main cables are hemp or wire rope, wire or eye-bar cables. Hemp rope is used in the construction of temporary bridges only; twisted wire rope is employed in the construction of bridges for short spans or light loads; wire and eye-bar cables are employed for long spans and heavy loads.

Steel-wire cables are formed in place and consist of parallel wires. In the Williamsburg suspension bridge over the East river the wire used is No. 6, 0.19 inch in diameter. The cross-section of each cable shows 7696 wires, divided into 37 strands of 208 wires each. The wire in each strand is so spliced that it forms a continuous thread; the strand is held by a pin at each anchorage. The strands are bound together in the form of a cylindrical cable. Each wire is protected from moisture by a coating of hot linseed-oil, and the whole cable is covered by a sleeve of steel plate. The wire has a minimum tensile strength of 200,000 pounds per square inch.

The eye-bar cable is preferred by some engineers to the wire cable because the tensile strength at any point of the cable can be proportioned to the stress at that point. The allowable unit stress of steel in the form of bars is, however, much less than in the form of wire; the weight of the cable must therefore be greatly increased. A notable bridge of this class is at Budapest, Hungary.

The curve of the unloaded cable due to its own weight is by definition a catenary. If its weight, which is small compared with the uniformly distributed load, is neglected, the curve of the cable due to the latter load is a parabola.

In Fig. 147, let the curve represent the cable as deflected by a load uniformly distributed along *abc*.

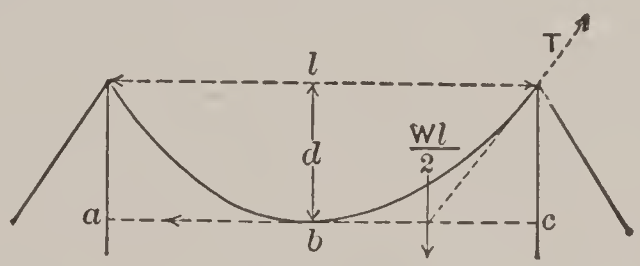


FIG. 147.

Let b = the origin of coordinates;
bc = axis of X;
bd = axis of Y;
l = span;
d = deflection of cable at middle point;
W = weight per unit length of roadway;
T' = the tension in cable at b;
T = the tension in cable at any other point;
x and y = coordinates of any point.

The forces acting on the cable to the left of any point (x, y) between b and T are T' and Wx. Since the cable is at rest, their moments must be equal, or

$$\frac{Wx^2}{2} = T'y, \quad \text{or} \quad x^2 = \frac{2T'y}{W}. \quad . \quad . \quad . \quad . \quad (585)$$

This is the equation of the curve assumed by the cable; from its form we know it is the equation of a parabola referred to rectangular coordinate axes through its vertex.

To find the tension at b, in the equation of the curve substitute for x and y the coordinates of any known point, as that at the top of the tower; the resulting equation will be

$$\frac{l^2}{4}=\frac{2T'd}{W}, \quad \text{whence} \quad T'=\frac{Wl^2}{8d}. \quad . \quad . \quad . \quad (586)$$

Since there is only tension in the cable, the value of T at any point between b and the tower, as x, must be equal to the resultant of the other forces acting on the cable between b and x. These forces are T' and Wx; hence $T=\sqrt{W^2x^2+T'^2}$. Since T increases as x^2 increases, the tension in the cable will be a minimum at b and a maximum at the tower. By making $x=0$, we find the minimum value of T is T'. By making $x=\frac{l}{2}$ and substituting for H its value found above we have

$$T=\sqrt{\frac{W^2l^2}{4}+\frac{W^2l^4}{64d^2}}=\frac{Wl}{2}\sqrt{1+\frac{l^2}{16d^2}}. \quad . \quad . \quad . \quad (587)$$

From the values of T' and T we see that if the span and loading remain constant, the tension in the cable may be decreased by increasing its deflection.

Towers.—The towers are constructed of masonry or steel. The height of the towers is fixed by the clearance which must be left for navigation under the bridge, and by the maximum tensile stress which is to be allowed in the cables.

Each tower is subjected to a vertical force which is equal to the sum of the vertical components of the tensile stresses in the two branches of the cable at the top of the tower, and to a horizontal force which is equal to the difference between the horizontal components of the tensile stress in the two branches at the same point. The horizontal force reduces to zero whenever the two branches of the cable make the same angle with the vertical through their point of intersection. As the horizontal force tends to overturn the tower, it is desirable to reduce its intensity to a minimum. This is effected by resting the cable on a *saddle* supported by rollers. This saddle has a slight longitudinal motion, which restores the equality between the angles whenever this equality is disturbed by the elongation or contraction of the cable due to changes of temperature or to the oscillations produced by a moving load. The same effect may be produced by a rocking tower.

Anchorages.—Each anchorage (Fig. 148) is usually a heavy mass of masonry which holds the anchor-plate to which the cable is attached. Its dimensions must be sufficient to resist the vertical and horizontal components of the stress in the cable. The anchor-plate is laid in a horizontal position and is connected with the cable proper by a curved eye-bar chain. The pins of this chain are held in place by heavy blocks of stone imbedded in the masonry. A tunnel is left in the anchorage for these chains. The distance of the anchorages from the towers increases with the height of the tower, and decreases with the inclination of the cable at the anchorage; if the cable between the tower and anchorage is not loaded, the anchorage will be nearer the tower than if this part of the cable supports a portion of the roadway. The anchorages are also the abutments of the superstructure.

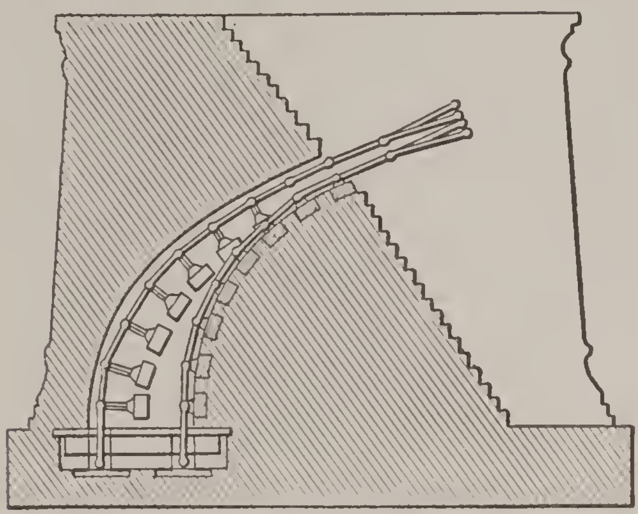


FIG. 148.

Suspenders.—The suspenders are the vertical cables which connect the floor system and the cables. When two cables support each end of the floor-beams, compensating devices are frequently introduced, so that, whatever be the relative positions of the two cables vertically, each will bear one-half the total load.

Stiffening Devices.—If a concentrated load is moved over a suspension bridge in which each floor-beam is attached to a suspender, but the floor-beams are not rigidly connected, the load will cause each floor-beam to oscillate in a vertical plane. The amount of oscillation will be affected, the relative weight of the dead and live load, the dip of the cable, and the velocity of movement.

Various methods have been devised for checking these oscillations. One of the most effective devices is the *stiffening-truss*. This is a vertical truss which is fastened to the floor-beams. The truss may be a single truss extending from tower to tower, or it may be made of two halves which have a joint at the middle point. It is so designed that it can resist pressure acting vertically either upwards or downwards. A stiffening-truss is placed along each side of the roadway. When a concentrated load comes on a suspender the load is transferred from the cable to the truss as soon as the former yields at the point of juncture of suspender and cable; the truss distributes the load over the floor-beams and suspenders and thus over a long section of the cables. Wide bridges have a number of such trusses.

A second method of checking the oscillations is to truss the cable itself. In this method the cable forms one of the chords of a vertical truss and is connected by suitable web bracing to a second chord. The auxiliary chord is designed to resist oscillation stresses only. This method has been applied to bridges with eye-bar chains.

A third method is to connect the roadway on opposite sides of each tower by cables running over the tower. A fourth is to attach guys to the cable at its points of maximum deflection under a moving load; these points are at the quarter points of the span. The guys should be in the same vertical plane as the cable and should be as nearly as possible perpendicular to it. A fifth is to attach each main cable to a small cable which lies in the same vertical plane as the main cable, but is below it. The small cable is attached at the foot of each tower and is convex upwards. A sixth is to anchor the roadway by cables fastened to rings in the rock below the bridge; this method may be employed when the bridge spans a rocky gorge.

Floor.—The floor system differs in no essential details from the floor system of truss-bridges. Its camber is greater than that of a truss-bridge.

East River Bridges.—Two of the principal suspension bridges are the Brooklyn and the Williamsburg bridges over the East river, New York City. Both bridges are wire-cable bridges of about the same span, 1600 feet, but differ from each other in many details. The latter is shown in Fig. 146. The towers of the Brooklyn bridge are of stone 280 feet high, those of the Williamsburg bridge of steel 330 feet high. The clearance being the same, 135 feet, the deflection of the cable in the latter bridge is the greater. With the same maximum stress it will therefore carry a heavier load. In the former bridge the shore spans are suspended from the cable; in the latter the cable between the tower and anchorage is unloaded. In the former the stiffening-trusses are made in two halves jointed at the middle point of the bridge; in the latter the stiffening-trusses are continuous between points outside of the towers; they form cantilevers for the support of the shore trusses.

Loads in Bridge Design.

Highway Bridges.—Standard specifications require highway bridges to be designed to support the following *dead* and *live* loads.

The dead load on a member is the weight of the member itself and that part of the weight of the bridge which is transmitted to it by the other members. Thus a joist supports not only its own weight but also the weight of an area of planking whose length is that of the joist and whose width is the distance between centers of joists. A floor-beam supports in addition to its own weight the weight of the flooring and joists in an area whose length is the distance between centers of floor-beams and whose width is the width of the flooring. This area is called a *panel* of the floor.

Each member of the *floor system* is also designed to resist the greatest stress which can be produced in it either by the greatest uniformly distributed live load or by the greatest concentrated live load which will probably be placed on one of its panels. The concentrated load is placed in such a position that it will produce its greatest possible stress in the member considered. The uniformly distributed and concentrated loads are assumed to act separately or together; if they act simultaneously the uniform load is confined to the area not covered by the concentrated load. The uniform load is that of a crowd of men. If confined in a contracted space inclosed on all sides, this may reach 180 pounds per square foot; upon a bridge it would probably never exceed 150 pounds. This latter weight should be substituted for that given below when conditions warrant it. The concentrated load is that of a steam road-roller or of a crowded electric street-car.

Bridges are divided by Cooper* into four classes and are de-

* Cooper's Specifications for highway and railway bridges.

signed for the loads given in the table. The first is for the heaviest city loads and the fourth for country highway travel.

WEIGHT OF LIVE LOADS ON FLOOR.

	Uniform.	Concentrated.
1st class:	100 lbs. per sq. ft.	24 tons uniformly distributed on two 12-ft. axles, 10-ft. centers, any part of roadway.
2nd "	100 " " "	24 tons uniformly distributed on two 12-ft. axles, 10-ft. centers, on railway tracks; 12 tons on two 5-ft. axles, 10-ft. centers, any part of roadway.
3d "	100 " " "	18 tons uniformly distributed on two 12-ft. axles, 10-ft. centers, on railway track; 12 tons on two 5-ft. axles, 10-ft. centers, any part of roadway.
4th "	80 " " "	6 tons uniformly distributed on two 5-ft. axles, 10-ft. centers, any part of roadway.

Each member of *the truss* is designed to resist the greatest stress which will result from the combination of the dead and the following live loads.

WEIGHT OF LIVE LOADS ON TRUSSES.

Class of Bridge.	Span.	
1st and 2d	up to 100 feet	1800 pounds per lin. foot on car tracks.
1st and 2d	over 200 "	1200 " " " " " " "
1st	up to 100 "	100 " " sq. " outside car tracks.
1st	over 200 "	80 " " " " " " "
2d	up to 100 "	80 " " " " " " "
2d	over 200 "	60 " " " " " " "
3d	up to 100 "	1200 " " lin. " on car tracks.
3d	over 200 "	1000 " " " " " " "
3d	. . . .	Same as 2d class, outside car tracks. Interpolate for intermediate spans.

The weight of the live load per linear foot decreases as the span increases because the probability of the bridge being subjected to a live load covering every linear foot decreases with the span.

Merriman's formula for the dead load per linear foot of a highway bridge is

$$\text{Weight in pounds} = 140 + 12b + 0.2bl - 0.4l, \quad . \quad . \quad (588)$$

in which b = width of floor in feet,

l = length of span in feet.

Railroad Bridges.—The dead load is the weight of the track and the weight of the structure itself.

Merriman's formulas for the live load per linear foot of a railroad bridge are

$$\text{Weight in pounds} = 560 + 5.6l \text{ single track}, \quad . \quad (589)$$
$$\text{"} \quad \text{"} \quad \text{"} \quad = 1070 + 10.7l \text{ double track}, \quad . \quad (590)$$

in which l = length of span in feet.

The live load is the weight, on all tracks, of trains as long as the bridge, composed of two of the heaviest engines on the road hauling a train of loaded cars. The train or trains are placed so as to produce the maximum stress on the member considered. The concentrated load is an assumed load which will produce as great stresses on the floor system as any real load.

The maximum live loads for railroad bridges are, according to Cooper:

Engine................	9,800	lbs. per lin. ft.
Engine and tender......	8,125	" " " "
Train load..............	5,000	" " " "
Concentrated load	120,000	" on two axles 6 ft. apart.

Impact.—In some specifications the uniformly distributed live load is increased by a certain percentage to provide for the increased stresses due to impact. This percentage for railroad bridges is derived from the following formula:

$$P = \frac{40,000}{(L + 500)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (591)$$

in which P = the percentage,

L = the length in feet of the uniform live load which produces the maximum stress in the member considered.

Wind Stresses.—The pressure of the wind upon a bridge or viaduct tends to buckle the superstructure, move the structure on its supports, and overturn the piers, towers, and trestles. All these effects must be considered in bridge design.

Intensity and Direction of Pressure.—The intensity and direction of wind pressure on any plane surface normal to its direction is approximately expressed by the formula

$$p = 0.004v^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (592)$$

in which p = the pressure in pounds per square foot,
v = the velocity of the wind in miles per hour.

According to this formula the pressure of a wind whose velocity is 100 miles per hour is 40 pounds per square foot.

As the maximum pressure on small surfaces is much greater than on large ones, it is customary in this country to assume a pressure of *30 pounds* per square foot on large surfaces, and to increase this to *45 or 50 pounds* whenever it seems desirable to increase the factor of safety, as on unloaded railroad bridges.

The wind pressure is greatest when its direction is normal to the vertical plane through the axis of the bridge, as it then acts on the largest surface and produces all the effects above described. If the wind blows parallel to the axis, its effect is felt principally by the supports.

Wind-bracing.—Let the truss shown in Figs. 139 and 149, in which the floor rests on the lower chord panel points, be exposed to the pressure of the wind blowing normal to the planes of its trusses. The actual area exposed to pressure is the area of the vertical projection of its floor system plus twice the area of the vertical projection of one of the trusses. This area in square feet multiplied by 30 will give the total pressure in pounds upon the bridge. As the abutment ends of the truss are prevented from moving laterally, this pressure will buckle the trusses and bend the vertical posts unless these are braced to resist this pressure. To prevent deformations in the trusses under wind pressure it is customary to connect the vertical trusses by one or two horizontal ones as shown in Figs. 139 and 149. These horizontal trusses transmit the horizontal pressures to the supports in the same manner that the vertical trusses transmit the vertical loads.

At the loaded chord the horizontal truss is made by simply connecting the floor-beams by diagonal ties (Figs. 139 and 149). The loaded chords of the vertical trusses thus become the tension and compression chords of the wind truss and the floor-beams become its struts. At the unloaded chord horizontal struts as well as a system of diagonal ties are introduced between the chords of the vertical trusses. The effect of the wind pressure is therefore to increase the stress in the main tension chord on the leeward side and in the main compression chord on the windward side. In determining the stresses on the wind trusses the pressures are assumed to act horizontally at the panel points of the chords.

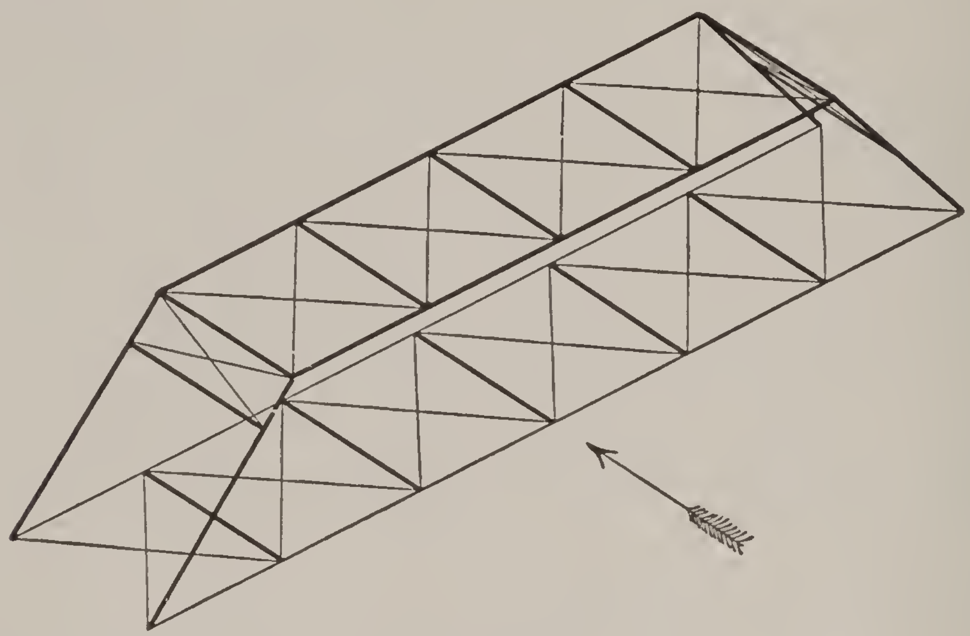


FIG. 149.

In Figs. 139 and 149 the wind pressures are transmitted by the lower wind truss immediately to the abutments, but the upper truss transmits them only to the top of the end posts, from which points they must be transferred to the abutments by the shearing resistance of the end posts. To prevent the deformation of the portal of the bridge by the accumulated pressures at the top of the end posts, portal brackets, or *portal bracing*, are introduced. The brackets or bracing prevent any change in the angle between the end posts and the strut connecting their tops.

To simplify the calculation of the stresses in the wind-bracing of a highway bridge, it is customary to assume that the area of a linear foot of the vertical projection of a truss is uniform, and the same for all spans up to 300 feet; for longer spans there is a small percentage of increase in the area per linear foot. The area assumed for the short spans is 10 square feet per linear foot, or 300 pounds pressure per linear foot, which is divided equally between the trusses. The same area is assumed for railroad bridges of less than 200 feet span. The truss connecting the loaded chords has in addition a moving load due to the pressure of the wind on a moving train or other load. This pressure is assumed to be that on a surface 5 feet high or 150 pounds per linear foot in a highway bridge, and on a surface 10 feet high or 300 pounds per linear foot in a railroad bridge. The effect of a horizontal moving load on a wind truss is similar to the effect of a vertical moving load on one of the main trusses.

A bridge which has a wind truss connecting its upper chords is called a *through bridge;* the least distance between this truss and the floor, or the clear headroom, should be 14 feet in a highway and 20 feet in a railway bridge. If the floor rests on the upper chord the bridge is a *deck bridge.* If the floor rests on the lower chord, but the trusses are not high enough to admit of lateral bracing between the upper chords, it is a *half-through* or *pony-truss* bridge. In this bridge the floor-truss transmits all the wind pressure; the floor-beams are usually prolonged and their ends are connected with the upper chords by inclined braces.

The wind-bracing of a plate-girder or a steel-arch bridge is designed in a similar manner.

The bridge is prevented from moving laterally by friction and by bolting it to the abutments. A strut, omitted in Fig. 149 for clearness, connects the bases of the end posts resting on the same abutment and divides the wind pressure between the bedplates.

The wind pressure on piers, towers, or trestles is determined in the same manner as the pressure on the trusses; the pressure per square foot is, however, assumed to be 45 to 50 pounds per square foot. To simplify calculation it is also assumed as uniform per vertical foot and to vary from 150 pounds per vertical foot in highway-bridge towers to 250 pounds in railway-bridge towers. The towers must have a factor of safety of two against overturning under the most unfavorable conditions; in a railway bridge this will be when an empty train covers the entire bridge.

A high cantilever tower is rendered stable by making the base much wider than the top. In the Forth bridge (Fig. 140) the base of each tower is 120 feet wide and the top 33 feet. A high steel arch is made stable by making the width at the springing-lines much wider than at the level of the crown. The Viaur bridge (Fig. 144) is 109 feet wide at the springing-lines and 19 feet wide at the roadway.

In a suspension bridge the wind pressures are transmitted to the towers by wind trusses connecting the chords of the stiffening-trusses, by the tension in the cables when these are not in vertical planes normal to the piers, and by auxiliary guys.

In truss- and girder-bridges built on a curve the wind braces must also resist the lateral pressure due to a moving train.

Sway-bracing.—If the vertical posts of a through bridge are much higher than is necessary to give the desired headroom, they may be connected by sway-bracing as shown in Fig. 150. This bracing prevents the deformation of the panels formed by the vertical posts, floor-beams, and top connecting struts, by wind pressure or by a load not symmetrically disposed with respect to the two trusses. A train on one track of a double-track railroad bridge would be an example of such loading. Since no headroom need be provided, in a deck bridge each diagonal connects the top of each post with the base of the one opposite.

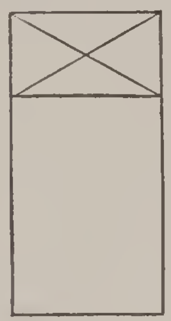


FIG. 150.

Temperature Stresses.—Allowance is made in bridge design for a variation of temperature of 150 degrees.

Bridge Erection.—The term *plant* is applied to the derricks and other machinery employed in erecting the bridge; the term *false works* to the trestling that supports the bridge during erection.

Trestle-bents if not too high are framed on the ground and then lifted into place. High bents and towers are constructed in position by the use of the ordinary hoisting-apparatus.

Plate girders and riveted Warren girders are delivered on the site in sections as long as can be conveniently transported and handled. These sections are riveted together on the site and the girder is then put into place by moving it on rollers which are supported by stringers resting on trestle-bents. A large girder is hauled into position by a rope attached to a capstan. Small girders may also be placed in position by means of derricks alone. A Howe truss like that shown in Fig. 142 is assembled on shore and placed in position in the same manner as a girder.

A pin-connected truss requires the erection of a platform at the level of the lower chord. All the members that are attached to the pins of the lower chord are assembled on this platform.

A second platform may be constructed for the upper chord or the pieces of this chord may be raised and united to the web members by means of a *traveler*. This is a car which runs on a track on the lower-chord platform and supports a longitudinal cantilever or crane at a level above that of the upper chord. The cantilever is provided with hoisting-apparatus.

Cantilever spans are erected without false works. The cantilever which is being constructed is supported by a similar one constructed at the same time on the opposite side of the tower, or by a truss which has been previously constructed. The suspended truss connecting two cantilevers is constructed as a part of the cantilevers; when united at the middle point of the span the upper chord is severed between the cantilever and truss. The construction stresses in the chords of the suspended truss are therefore the reverse of their final stresses. This change must be provided for in the design. As the cantilever requires no false works during construction it is particularly suitable for sites where, on account of the depth of water or other conditions, false works would be very difficult to erect or maintain.

Masonry and reinforced concrete arches are constructed on centers. Steel arches are usually constructed without false works, the character of the site being such that the semi-arches can be held by cables until they are united at the crown. If the site does not admit of this method of construction the arch is supported on a center.

Suspension bridges are constructed by means of platforms, cars, and travelers suspended from the main and auxiliary cables.

Drawbridges.—A drawbridge is one in which one or more spans can be temporarily raised or removed to allow the passage of vessels. They are usually constructed over navigable rivers with low banks.

The draw-span may be raised vertically or moved horizontally, or it may be rotated about a horizontal or vertical axis.

In the first type, or *lift-bridge*, the floor is lifted vertically until the clearance is sufficient for passing vessels. The floor is attached to counterweighted cables which pass over pulleys supported by steel towers. The force required to lift the bridge may therefore be reduced to that necessary to overcome the friction in the hoisting-apparatus.

In the second type the draw-span is supported by a car which runs on a track whose axis coincides with that of the span, or it is supported on a boat or *ponton* which is floated out of position.

The above types are employed only where the conditions are extremely favorable for their construction; draw spans, as a rule, rotate either about horizontal or vertical axes.

In the third type, or *bascule bridge*, the draw-span rotates about either a fixed or a movable horizontal axis and may consist of one or two leaves. If the span is short, hinges are fastened to the shore ends of the floor; these rotate about pins or bars attached to the abutments. The leaf is rotated by means of counterweighted chains or cables attached to its outer end, which pass over pulleys in walls or towers above the abutments. The drawbridge over the moat of an old fortification is of this type. If the span is long, the hinges are so placed that the center of gravity of the revolving leaf shall be at or near the hinge; the leaf is then a double cantilever with a short heavy shore arm which rotates in a well made in the abutment. The London tower bridge, whose draw-span is 290 feet, is of this type. Fig. 151 represents a bascule bridge with a movable axis. The short

FIG. 151.

arm supports a heavy counterweight W. This is the type commonly constructed in this country and has been applied to a span of 275 feet in Chicago, Ill. In large-span bascule bridges the cables are dispensed with and the power is applied to the leaf at the abutments. The position of equilibrium of the leaf may be horizontal or inclined; the latter facilitates its rapid opening.

In the fourth type, or *swing*-bridge, the draw is in the form of a double cantilever with arms of equal or unequal length. If the span is short and a single opening is desired, the axis of rotation may be on the abutment and the shore cantilever may be short and heavy. The ordinary construction, however, is to place the axis on a pier, make the cantilever arms equal, and thus secure two equal openings. There are several bridges of this type in this country in which the double cantilevers are over 500 feet long from end to end. The longest, 520 feet, is at Omaha, Neb. The draw-spans of swing-bridges are supported by rollers which move on a circular track.

The cantilevers may be either plate girders or trusses. If plate girders, they are designed first as unloaded cantilevers when the spans are open, and as continuous beams resting on three points of support when the spans are closed and loaded. If the bridge is a truss-bridge, the truss is usually designed as a double cantilever when the spans are open and unloaded, and as two trusses resting on two supports each when the spans are closed and loaded.

Swing-bridges are usually preferred in wide rivers where the central pier is no serious obstruction to navigation; bascule bridges are preferred where the channel is narrow.

For further information consult Waddell's "De Pontibus," Foster's "Wooden Trestle Bridges," Cooper's "Specifications for Railway and Highway Bridges," Thacher's "Specifications for Railway and Highway Bridges," Greene's "Arches," Merriman and Jacoby's "Higher Structures."

ILLUSTRATED DESCRIPTIONS OF BRIDGES.

Luxemburg stone arch.	Eng. News, vol. 47, p. 179
Plauen stone arch	" " " 51, p. 74
Topeka reinforced concrete	" " " 37, p. 105 and vol. 39, p. 99
Viaur steel arch	" " " 44, p. 158
Niagara steel arch	" " " 37, p. 252
Forth cantilever	Engineering, vol. 49, p. 213
Quebec "	Eng. News, vol. 37, p. 252
Memphis "	" " " 27, p. 470 and vol. 28, p. 251
Poughkeepsie cantilever	Eng. Bldg. Record, May 1888
Brooklyn suspension	Eng. News, vols. 5 and 6
Williamsburg "	" " vol. 50, p. 535
Bascule drawbridges	" " " 45, p. 18
Lift drawbridge	" " " 31, p. 320
Swing drawbridge	" " " 30, p. 448
London Tower bridge	" " " 31, p. 43
Pecos River steel tower bridge	" " " 27, p. 125

CHAPTER XXIV.

TRUSSED ROOFS AND FLOORS.

Definitions. — Fig. 152. The *roof-covering* is the outer or water-proof coating of the roof. The materials employed for

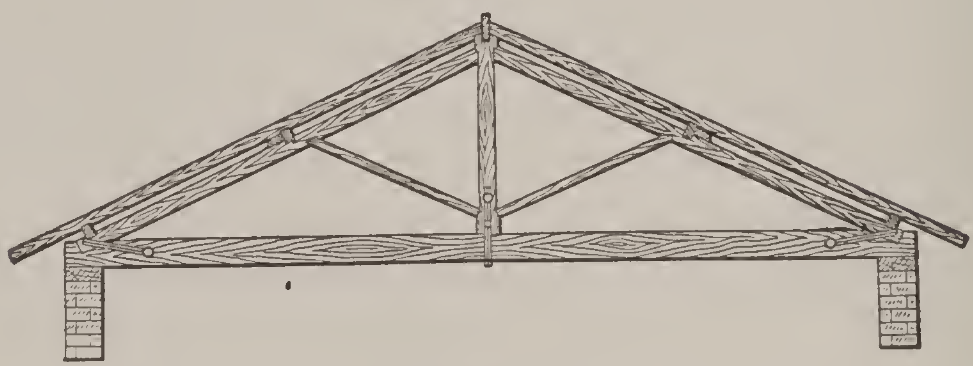


FIG. 152.

this coating are shingles, slate, tiles, asphalt, tin, copper, lead, and corrugated iron.

The *sheathing* is the layer of boards or other material to which the covering is attached.

The *rafters* are the inclined beams which support the sheathing; they correspond to the joists in bridge floors.

The *purlins* are the horizontal beams which support the rafters and correspond to the floor-beams of bridge floors.

The *roof-trusses* are the frames which support the roof and transmit its weight to the walls or columns of a building.

The *wall-plates* are the wooden beams which are laid on the top of the wall to distribute the weight transmitted by the trusses; if rectangular stones are used for the same purpose, they are called *templates*.

The *pitch* of a roof is the angle which its plane makes with the horizontal; in simple roofs the pitch is also expressed as the ratio of the rise to half the span. The most common pitch is $26\frac{1}{2}$ degrees, or $\frac{1}{2}$.

The *ridge* is the highest horizontal line of the roof.

The *eaves* are the lowest horizontal lines of the roof.

A *shed* or *lean-to* roof has a single plane surface (*a*, Fig. 153).

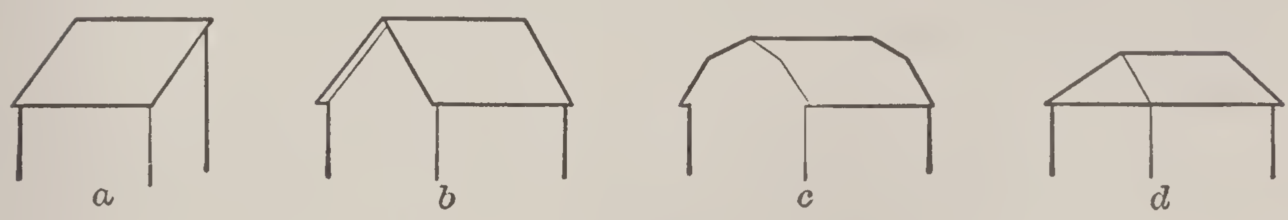


FIG. 153.

A *gable* roof has two plane surfaces intersecting in the ridge (*b*, Fig. 153).

A *curb* or *gambrel* has four plane surfaces; the two on each side of the ridge have different inclinations and intersect in a line parallel to the ridge, called the curb (*c*, Fig. 153). The *mansard* is a curb roof with steep sides and a rather flat top.

A *hipped roof* has the same inclination at the ends of the building as at the sides; the inclined lines of intersection of its slopes are the *hips* (*d*, Fig. 153).

A *valley* is the line of intersection of roof surfaces making a reentrant angle, as the line of intersection of two gables which are perpendicular to each other.

Roofs may also be arched, conical, and dome-shaped.

Construction.—The kind of roof-covering varies with the character of the building, the architectural effect desired, and the pitch of the roof. If the angle of pitch is small, the covering should be a continuous sheet, as asphalt, tin, etc.; otherwise the covering may be composed of overlapping plates, as slate, tiles, etc.

The sheathing is usually inch boards, covered by layers of tarred paper or similar materials; when a fire-proof construction is desired, the sheathing consists of slabs of terra cotta or other fire-proof material supported by a metal framework.

Wooden rafters are beams 2 to 3 inches thick and 4 to 10 inches deep. They are spaced from 16 to 24 inches, center to center, for ordinary loads; these distances may be increased or diminished. If the wooden rafters rest directly upon the wall-plates and not on purlins, the opposite rafters are connected about midway between the wall-plates and ridge by a horizontal brace called a *collar-beam.* In metal roofs the rafters are I beams or other structural steel forms. The I beams are spaced at wider intervals than wooden rafters, and in fire-proof construction are connected by inverted tees which support the fire-proof sheathing.

The purlins are usually spaced about 8 to 10 feet, as wider spacing requires the rafters and purlins to be inconveniently large. They are either square wooden beams or one of the forms of structural steel beams. The rafters are sometimes omitted and the sheathing fastened to the purlins themselves; the purlins must then be spaced as rafters.

The trusses are made of wood, of wooden compression and steel tensile members, or wholly of steel. The compression

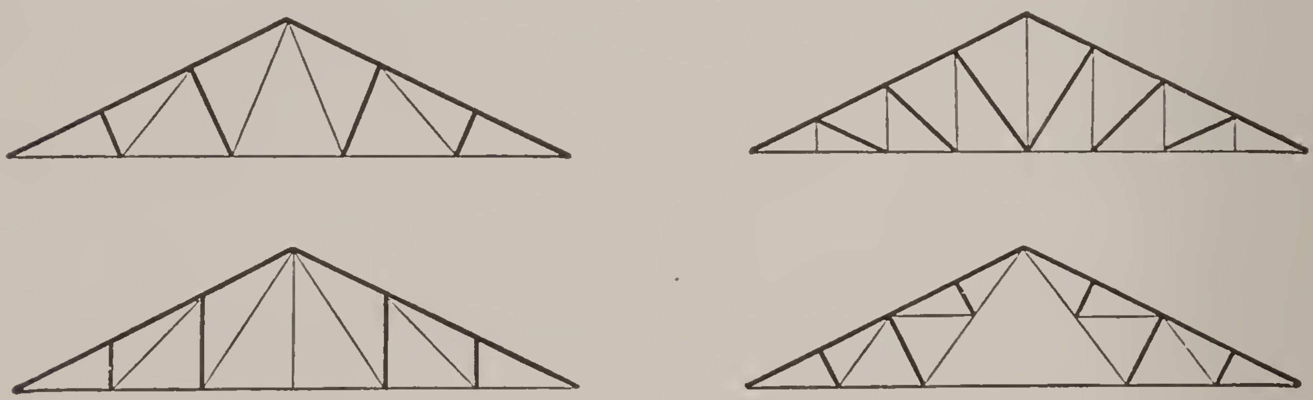


FIG. 154.

chord corresponds in general outline to the roof itself; the tension chord may be a right or broken line. The web members may all be inclined, as in a Warren truss; the compression members may be vertical and the tension members inclined, as in the Pratt truss; the compression members may be inclined and the tension members vertical, as in the Howe truss; or the upper chord may be trussed, thus forming a Fink truss (Fig. 154). The panel points of the upper chord are usually at the purlins.

In wooden trusses the members are all of the same thickness and differ only in depth; they are united by mortise and tenon or notched joints, strengthened by straps or bolts. Steel trusses may have pin-connected joints along the lower chord or they may be riveted throughout; the latter is the more common construction. In the riveted truss all the members are usually angles in pairs riveted to connecting plates and fastened at intervals to each other.

Like the bridge-truss, one end of the roof-truss is anchored firmly to its support; the other end rests on a smooth plate or on rollers which permit longitudinal motion but do not permit lateral or vertical motion.

Arched roofs may have their trusses in the form of the bow-string truss, or they are in the form of an arch with parallel chords, and are usually hinged at the springing-lines and crown.

Loads.—The *dead load* of a roof is the weight of the material of which it is made and in addition any permanent weights suspended from its trusses. The *live load* is the pressure of the wind, the weight of the snow which may lie on it, and any temporary weights which it may have to support. The *dead load* is approximately as follows for roofs whose span does not exceed 75 feet. The weight is that of the actual roof area.

Shingles, tin, or equivalent.	10 lbs. per square foot.
Slate on boards.	15 " " " "
Fire-proof construction of steel and terra-cotta tile.	40 " " " "

The *live load*, excluding the wind, is usually assumed as 25 pounds per square foot of roof surface. Flat roofs used as floors are designed for floor loads.

In computing the wind pressure on a roof the action line of the wind is assumed to be horizontal and its intensity is generally taken as 30 pounds per square foot.

Let F = total pressure of the wind on a vertical plane normal to its action line;

N = normal pressure on the same plane when inclined to the horizontal at an angle ϕ;

H = the horizontal component of N, or $N \sin \phi$;

V = vertical component of N, or $N \cos \phi$.

The relation of F and H was determined by experiment to be

$$*\, H = F \sin \phi^{1.842 \cos \phi} \quad \text{or} \quad \dagger\, H = F \frac{2 \sin^2 \phi}{1 + \sin^2 \phi}.$$

* Hutton. † Duchemin.

The first gives the ratios of N, H, and V to F for different values of ϕ as follows:

ϕ in degrees.	10	20	30	40	50	60	70	80	90
$\frac{N}{F}$	.24	.45	.66	.83	.95	1.00	1.02	1.01	1.00
$\frac{H}{F}$	.04	.15	.33	.53	.73	.85	.96	.99	1.00
$\frac{V}{F}$	.24	.42	.57	.64	.61	.50	.35	.17	0.00

The normal pressure N must be resisted by the roof construction, which transmits its effects to the walls.

The horizontal pressure H tends to move the roof horizontally and must be resisted by its anchorages. It also tends to overthrow the walls which support the roof.

The vertical component V must be added to the dead and other live loads in computing the strength of the walls.

Merriman gives the following formula for determining the *approximate* weight of a roof-truss. It is employed in computing approximate dead load.

$$W = \tfrac{1}{2}al + \tfrac{1}{10}l \text{ for a wooden truss;}$$
$$W = \tfrac{3}{4}al + \tfrac{1}{10}l \text{ for a steel truss;}$$

in which W = weight of one truss in pounds;
a = truss interval in feet;
l = span of truss in feet.

Floors.

The floor of a building is constructed in the same general manner as the floor of a bridge and consists of *flooring*, *joists*, and *girders*. Its essential qualities are strength and stiffness.

According to the method of construction floors may be divided into wooden, I-beam, and reinforced concrete floors.

Wooden Floor.—In the wooden floor the flooring is made of narrow-tongued and grooved boards which are supported by wooden joists.

To secure stiffness as well as strength, the width of the joists is small as compared with their depth; the width varies from 2 to 3 inches and the depth from 6 to 14 inches. They are spaced from 12 to 16 inches. To hold the joists in position they are *bridged* at the middle point of short spans and at intervals of 8 feet in long ones. The bridging consists of vertical cross-bracing composed of 1 by 3 inch pieces, which is inserted between consecutive joists. The joists are supported by the walls of the building or by girders. If the upper surfaces of the joists and girder are to lie in the same horizontal plane, the joists are notched into the girders or are supported by iron stirrups which are fastened to the girders; the latter method is the better, as the total cross-section of both joists and girder are preserved.

The girders are rectangular beams which rest on the walls of the building or upon columns or pillars. If the span is great, wooden girders are replaced by steel I beams.

I-beam Floor.—In this floor the joists are steel I beams, and the girders steel I beams, plate or box girders. The joists and girders are fastened together by angles riveted to the webs of the girders. The joists are connected by arches or plates of incombustible material such as bricks, tile blocks, or concrete. Above the arches or plates the floor is leveled with cinder concrete and in this concrete bed are placed wooden strips to which

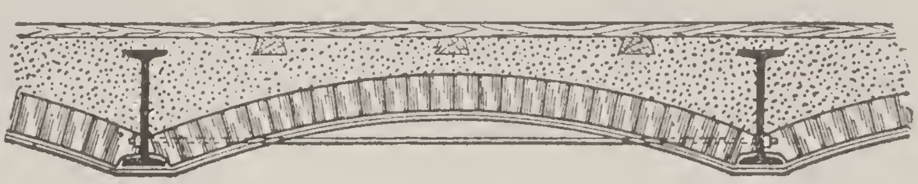


FIG. 155.

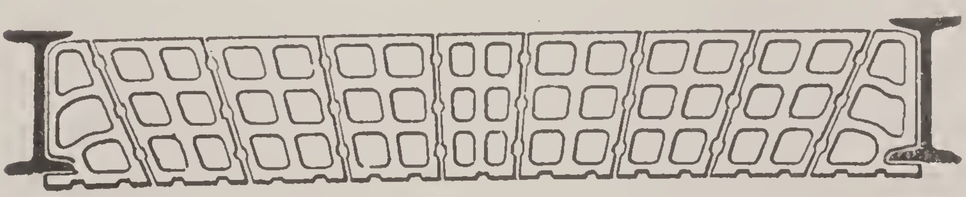


FIG. 156.

the flooring is nailed. The flooring may also be made of tiles or other fire-proof material. The fire-proof floor differs from the semi-fire-proof in having its joists and girders protected from the flames of a fire underneath it, by a ceiling or other shield of fire-proof material. (Figs. 155 and 156.) The construction of the brick and tile-block floors is shown in Figs. 155 and 156; the depth of the brick arch for spans of 5 feet is about 4 inches; the flat tile-block arch is deeper. The thrust of the arch is neutralized by ties connecting the consecutive joists at intervals of 8 feet. When the blocks are laid as in Fig. 156 the method is called *side construction;* when laid as in Fig. 157 it is called *end construction.* The side construction is usually preferred, although the block is stronger longitudinally than laterally, because the arch is more easily constructed. Holes may be made through these arches for the passage of flues and pipes.

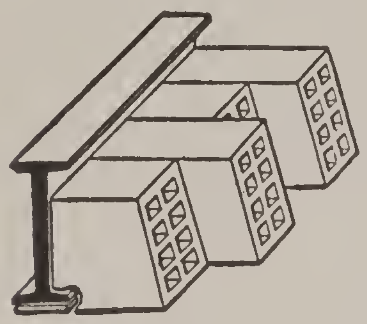


FIG. 157.

The joists may also be connected by arches of concrete or plates of reinforced concrete (Figs. 158 and 159). The arches

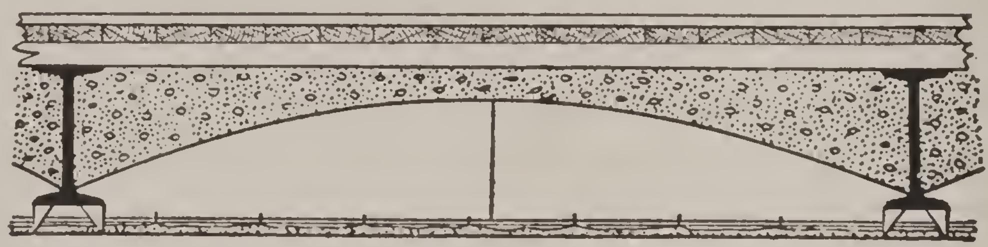


FIG. 158.

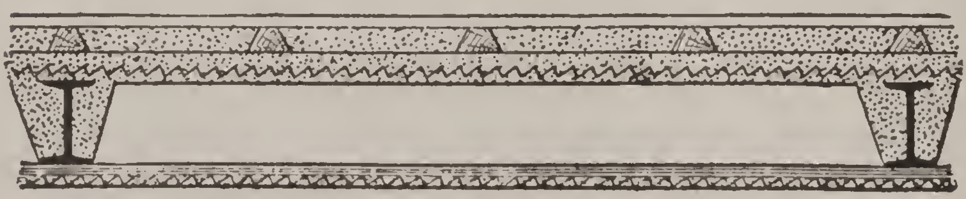


FIG. 159.

and plates are supported during construction by platforms suspended from the steel-floor joists. All these forms of floors will safely support the loads given below. The fire tests required of fire-proof floors are given in standard building regulations.

Reinforced-concrete Floors. — In fire-proof buildings the I-beam floor is now being replaced by the reinforced-concrete floor because of its cheapness and fire-resisting qualities. It differs from the I-beam floor in having no rigid framework of steel as its basis; only a minimum amount of metal is employed in reinforcing the floor-beams, joists, etc., usually not exceeding one per cent of the cross-section. The floor proper is a large slab of reinforced concrete which is supported on the joists and covered by any desired floor surfacing. By means of the reinforcing bars or wires and the concrete, all the parts of the floor system are firmly attached to the supporting columns and to each other and made into a monolithic mass. Its cheapness results from the small amount of reinforcement required, and its fire-resisting qualities from the thorough coating of the metal by concrete. During construction a reinforced-concrete floor must be supported throughout by a platform. Many different designs of these floors have been patented and are being constructed.

Loads.—Each member of the floor system must be designed to resist a dead load equal to the weight of the member and the weight of so much of the floor as is transmitted to it by the other members, and a live load which depends on its probable loading. The live load is assumed to be uniformly distributed.

Dead Load.—The weight of a wooden floor including its joists is about 10 pounds per square foot; with the attached ceiling 20 pounds.

The weight of a brick or block floor arch in pounds is about four times the depth of the arch in inches, and may vary from 25 to 45 pounds per square foot. If the weight of the I beams, flooring, and ceiling are added, the weight will be increased to a total of 75 to 100 pounds. The floors of the government printing-office in Washington, which were designed for live loads of 300 pounds per square foot, were computed for a dead load of 125 pounds per square foot. In this building the supporting columns are in rows 35 feet centers, and the columns in each row are at intervals of 12 feet.

Live Load.—According to standard building regulations the live loads for which floors must be computed are:

Dwellings.	40 to 60 lbs. per square foot.
Schools.	75 " 100 " " " "
Stables.	75 " 100 " " " "
Public assembly halls.	90 " 120 " " " "
Warehouses.	120 " 250 " " " "

Some of these loads are considered by competent engineers as not complying with the conditions of practice and the following table (page 480) has been recommended to replace it.*

* Schneider in Proceedings of Am. Soc. of Civil Eng'rs, Sept. 1904.

Classes of Buildings.	Live Loads, in Pounds.		
	Distributed Load. 1	Concentrated Load. 2	Load per Linear Foot of Girder. 3
Dwellings, hotels, and apartment-houses.	40	2,000	500
Office buildings.	40	5,000	1,000
Assembly rooms with fixed seats, like theaters, churches, schools, etc. . .	40	5,000	1,000
Assembly rooms without fixed seats, like ballrooms, gymnasia, armories, etc.	80	5,000	1,000
Stables and carriage-houses.	70	5,000	1,000
Ordinary stores and light manufacturing.	40	8,000	1,000
Sidewalks in front of buildings.	100	10,000	
Warehouses and factories.	from 120 up	Special	Special
Charging floors for foundries.	" 300 "	"	"
Power-houses, for uncovered floors. .	" 200 "	The actual weights of engines, boilers, stacks, etc., shall be used, but in no case less than 200 lbs. per square foot.	

The live load to be employed is the one in columns 1, 2, or 3, which produces the greatest stress in the member considered; the distributed load is the weight of people, furniture, etc.; the concentrated load is that of a safe, which may be omitted in computing the dimensions of floor members of simple dwellings. In estimating the live load transmitted from the floors to the columns of high buildings a reduction of 5 per cent is made for each floor below the top one until this reduction reaches 50 per cent. This corresponds to the reduction made in the live load for long-span bridges.

For further information consult Fowler's "Specifications for Steel Roofs and Buildings"; Schneider's "Structural Design of Buildings"; Transactions American Soc. of Civil Engineers, 1905; Kidder's "Building Construction," Part I; Merriman and Jacoby's "Higher Structures."

CHAPTER XXV.

HIGHWAYS.

Definitions.—A *highway* is a line of communication over which the public has a right of way. The ordinary highway is designed for the use of wheeled vehicles.

A *street* is a highway of a village or city designed for general use; an *alley* is a narrow highway designed for the use of the owners of abutting property.

A *road* is a country highway which connects villages and towns.

The *carriageway* is that part of a highway designed for the use of vehicles.

The *sidewalks* are the parts of the highway reserved for pedestrians.

A *pavement* is a layer of hard material placed on the natural surface of a carriageway or sidewalk. The term is usually confined to block or sheet coverings.

A *macadam* or *broken-stone* carriageway is one which is covered by a layer of stones broken into small fragments.

Gutters are shallow depressions along the sides of the carriageway of a street. They are usually paved to catch the surface drainage of the street and conduct it to the sewers.

A *side ditch* is a deep depression along the side of the carriageway of a road. It catches the drainage of the road and of the adjoining lands which slope towards the road and conducts it to the nearest natural watercourse intersecting the road.

A *curb* is a line of narrow stones which supports one side of the pavement or gutter of a street and separates the carriageway and sidewalk.

The *axis* of a highway is the line of its surface which bisects it longitudinally. The axis of the carriageway is a similar line which bisects the carriageway.

The *trace* of a highway is the projection of its axis on a horizontal plane.

The *profile* is the development of the axis and trace on a vertical plane.

The *grade* of any part of the axis of the carriageway is its inclination to its trace. It is usually indicated by its rise per mile or per hundred feet, assuming the grade to be uniform for this distance. A grade of $\frac{1}{100}$ is called a 1 per cent grade; a grade of $\frac{2}{100}$ a 2 per cent grade, etc.

A *transverse* or *cross section* of a highway is the intersection of its surface by a plane normal to its axis.

The *crowning* of the carriageway is the vertical height of its axis above its sides.

Profile.—The profile of a highway is the natural profile of the country over which the road passes modified by *cuts* and *fills*. The cuts and fills together reduce both the irregularities in the profile and the inclination of its slopes or grades. The *maximum desirable grade* on a highway is such that a horse can pull his ordinary load up the grade without loss of speed and without undue fatigue. This grade will depend therefore on the tractive power of the horse and on the load and resistance of the vehicle he draws.

Tractive Power of a Horse.—The *tractive power* exerted by the horse at any moment is approximately measured by the tensile stress in the traces. The *working tractive power* is that which he can exert throughout a working day when he moves at a walk on a level road. This is now generally assumed as 120 pounds for a horse weighing 1000 pounds, or is about one-eighth his weight. His maximum tractive power is that which he can exert for a few moments, as in starting a load; this is at least twice his working tractive power and is equal to one-fourth to one-third his weight.

The amount of useful work which can be performed by a horse is the product of the tractive power by the distance over which he can exert this power daily. It is generally assumed that a horse can, without undue fatigue, walk 20 miles a day on a good level road, and at the same time pull a wagon whose constant resistance is 120 pounds. This is equivalent to 12,672,000 foot-pounds of work. The normal horse-power employed in mechanics is based on a horse whose tractive power is 150 pounds moving at the rate of $2\frac{1}{2}$ miles per hour or 220 feet per minute; this gives 33,000 foot-pounds per minute or 15,840,000 per day of eight hours.

If required to exert a greater tractive force than 120 pounds, the horse must have occasional rests, which will decrease his daily rate, the distance he travels, and his daily useful work. The limit is reached when his load is so heavy that he can no longer start it.

If the horse is required to move more rapidly than a walk, his tractive power and his daily useful work are both diminished. The limit is reached in the race-horse who pulls a vehicle whose resistance is made as small as possible.

The maximum tractive power of a horse on a slope is less than on a level; the action line of his weight is inclined to the surface, which reduces the friction of his shoes and also reduces his pulling moment. The reduction in his tractive power increases with the smoothness and hardness of the surface. Since part of his muscular effort is expended in raising his own weight, his daily useful work will be less and his non-available work will be greater than on a level road.

Weight of Load.—In this country the vehicle in common use is the two-horse farm wagon whose weight is about 1200 pounds and which carries a load of 2000 pounds. The weight pulled by each horse is therefore about 1600 pounds. Our army wagon is a four-horse or six-mule wagon whose weight is 2000 pounds and its maximum load 4000 pounds; the total weight is therefore 1500 pounds per horse. It will be observed that the weight of the vehicle in each case is about one-third the total weight. On city pavements the loads are usually much heavier.

Tractive Resistance of Vehicles.—The tractive resistance of a vehicle on a *level surface* is measured by the force which must be applied to the vehicle, parallel to the surface upon which it rests, to move it at a uniform rate of speed. This resistance is the sum of the frictional resistance between the axles and hubs, and the resistance at the surface of the wheels caused by irregularities in the surface of the road. For *each wheel* the total resistance may be expressed by the formula

$$R = fW + \frac{aW}{\sqrt{r}},$$

in which R = total resistance;
f = coefficient of friction of surfaces of axle and hub;
W = weight on wheel;
r = radius of wheel;
a = a coefficient dependent on character of road surface.

The term fW is the resistance at the axle, and $\frac{aW}{\sqrt{r}}$ is the resistance due to irregularities in the surface of the road determined by experiment.

If a dynamometer be placed between the traces and the vehicle, and the vehicle be drawn at different rates of speed over level roads with different surfaces, the total resistance of the vehicle will be in the form

$$R' = f'W',$$

in which R' = total resistance in pounds;
W' = total weight of vehicle and load in pounds;
f' = coefficient dependent on the character of the road surface and speed.

Many experiments have been made to determine the value of f' corresponding to the different classes of roads and pavements. While there is a general agreement in the results obtained by the different investigators, there is also a considerable variation, due to the differences in the vehicles used, to the difficulty of driving horses at a uniform rate, and to the fluctuations in the dynamometer readings caused by slight irregularities in the surface. The values given below can therefore be considered approximate only, and for roads in fair condition when the vehicle moves at the rate of $2\frac{1}{2}$ miles per hour.

Surface.	Pounds per Ton of 2000 lbs.	Tractive Resistance Value of f'.
Steel railway	8	1/250
Steel tramway	15	1/133
Hard sheet asphalt	25	1/80
Soft sheet asphalt	70	1/30
Brick	25	1/80
Stone blocks	40	1/50
Macadam	60	1/33
Gravel road	90	1/22
Earth road	125	1/16
Loose gravel	300	1/7
Loose sand	400	1/5

Assuming the working tractive power of the horse at 120 pounds, if he can secure a good foothold, he can, on a level, pull with equal ease

15 tons on a railway.
8 tons on a tramway.
$4\frac{4}{5}$ tons on hard sheet asphalt or brick.
3 tons on a stone-block pavement.
2 tons on a macadam road.
$1\frac{1}{3}$ tons on a gravel road.
1 ton on an earth road.
$\frac{2}{5}$ ton on loose gravel.
$\frac{3}{10}$ ton on loose sand.

If the surface of a carriageway is smooth, the resistance is independent of the speed; if it is rough, the resistance increases with the speed.

In pulling a vehicle up a grade, the tractive force applied must overcome the same resistances as on a level, and in addition must also overcome the component of the weight of the vehicle parallel to the slope. The total tractive force must therefore be

$$T = f'W \cos\phi + W \sin\phi,$$

in which ϕ is the angle of inclination of the slope. Since this angle is never very large, no great error is introduced if we assume $\cos\phi$ equal to unity, and $\sin\phi$ equal to $\tan\phi$ or equal to $\frac{h}{l}$, in which h is the rise and l is the horizontal length of the slope. Making these substitutions we have

$$T = f'W + \frac{Wh}{l}.$$

The value of $\frac{Wh}{l}$, the tractive resistance due to the slope alone, is the same for all roads and varies directly with the weight of the vehicle and the inclination $\frac{h}{l}$. We may therefore form the following table:

RESISTANCE PER TON (2000 LBS.) OF LOAD DUE TO GRADE.

1% grade........	20 lbs.	6% grade.......	120 lbs.
2% "	40 "	7% "	140 "
3% "	60 "	8% "	160 "
4% "	80 "	9% "	180 "
5% "	100 "	10% "	200 "

These resistances must be added to those given in the previous table to determine the total tractive resistance of a vehicle which is being pulled up a slope. It will be seen that the *relative* effect of a slope is greater on a good road than on a poor one. Thus on a hard asphalt pavement the total resistance on a 1 per cent grade is nearly double the resistance on the level, while on an earth road the total resistance on a 1 per cent grade is only one-sixth greater than on the level. The better the road surface, therefore, the more important it is to reduce the grades.

In moving down the slope the force $\frac{Wh}{l}$ acts in the same direction as T, and the expression for T becomes

$$T = f'W - \frac{Wh}{l}.$$

If $f'W = \frac{Wh}{l}$, then T reduces to zero. Therefore in moving down a slope whose inclination, $\frac{h}{l}$, is equal to the coefficient f', the horse will not be required to exert any tractive force. In moving up the same slope his tractive force must be $2f'W$, or double the amount necessary to move the load on a level surface.

If the road has a macadam surface, for which f is equal to $\frac{1}{33}$, no tractive force is required to pull a load down a slope of $\frac{1}{33}$ or $\frac{3}{100}$, and a tractive force twice that required to pull a load on the level will be required to pull the load up this slope. For these reasons a slope of $\frac{3}{100}$, called a 3 per cent grade, is considered the ideal maximum grade of a macadam road. The angle f' is called the *angle of repose*, and, as will be seen from the table, its value depends on the character of the surface.

Maximum and Minimum Grades.—In road construction it is necessary to fix the maximum and minimum grades in the profile. On account of the expense of construction it is impossible to adopt the angle of repose as the *maximum grade* on all roads. In France, where the roads are all macadamized or paved, the macadamized road is the standard, and its angle of repose, 3 per cent, is adopted for all roads of the *first class.* For roads of the *second class* the maximum grade is 4 per cent, and for those of the *third class* 5 per cent. The allowable loads on first-class roads are about twice those adopted in this country.

In this country a road may be classed as a *first-class* road when its maximum grade does not exceed 5 per cent; a *second-class* road when this grade does not exceed 7 per cent; and a *third-class* road when its maximum grade does not exceed 10 per cent. A grade of 5 per cent corresponds to the angle of repose of a smooth earth surface.

The *minimum grade* in the profile is that which will permit of satisfactory drainage. This is usually fixed at about 0.8 to 1 per cent or $\frac{1}{125}$ to $\frac{1}{100}$.

Cross-section.—In discussing the cross-section of a road a distinction must be made between the country roads, where there are few pedestrians, and the village and city streets, where provision must be made for them.

Roads.—The total width of a road is the width of the right of way, or the strip of land over which the public has the right to pass. This width is fixed by law, and in this country varies between 2 and rods, but is commonly 3 or 4 rods, or $49\frac{1}{2}$ to 66 feet. The width of the right of way of every important road should be sufficient to permit of the construction of a carriageway 26 to 30 feet wide in cuts and on embankments. On unimportant roads its width should be sufficient to allow of the construction of a carriageway 20 feet wide. A width of 3 rods should therefore be the minimum for the former, and 2 rods for the latter. An unimproved road must be made wider than a macadamized road, since in wet weather the former is soon cut up and becomes impassable if the travel is confined to a narrow track. An unimproved road should also be freely exposed to the heat of the sun and should not be shaded by overhanging trees.

In France the width of the carriageway and not the right of way is fixed by law; an order of the royal council in 1776 divided the public highways into four classes and fixed the width of roadway of each.

The first class comprised the great roads which traversed the kingdom and connected the capital with the principal cities, ports, and industrial centers. These are now the *national highways* (*routes nationales*).

The second, the roads which connect the principal provinces, cities, ports, etc., with each other. These are now the *departmental roads* (*routes départmentales*). The third and fourth classes were the roads which connect the towns of one province with those of an adjacent one, and also the roads which connected the towns and villages of each province. These are now the *local roads* (*chemins vicinaux* and are divided into three classes.

Exclusive of the width of ditches and side slopes, the width of first-class roads was fixed at 45 feet, the second at 38 feet, the third at 32 feet, and the fourth at 25 feet.

The general form of the cross-section of these French roads is shown in Fig. 160.

FIG. 160.

The normal width of the different parts, as actually constructed, is given in the following table:

	Width in Feet.		Total.
	Macadam.	Each Unpaved Strip.	
National highways	16.4 to 22.9	8.2 to 11.5	32.8 to 45.9
Departmental highways	13.1 to 16.4	4.9 to 8.2	26.2 to 32.8
Local roads	9.8 to 16.4	4.9 to 6.5	19.6 to 26.2

The paved portion of the carriageway is designed for the use of heavy vehicles at all seasons of the year, and for light vehicles when the unpaved portions are in bad condition. The least width of the paved strip is 10 feet, which is sufficient for a single vehicle. A width of 16 feet is sufficient for two vehicles. Since the introduction of railroads the national highways have lost their importance, and a paved strip 16 feet wide is considered sufficient for any road.

The unpaved strips are used by light vehicles, serve as storing places for road materials, and if the road is of sufficient width are planted with rows of trees. The trees should leave an unobstructed carriageway at least 30 feet wide.

The curve of cross-section is either a circle or ellipse whose height at the center is about $\frac{1}{40}$ of the width of the carriageway. If the road is always kept in thorough repair, the crowning is reduced.

The side ditches have sufficient capacity to carry off the rain-water which falls on the road and that which flows from the adjacent lands toward it. If the road is raised above the surface of the country, the ditches may be omitted. If the road follows the surface of a slope and is partly in fill and partly in cut, only the ditch on the side of the cut is retained (Fig. 161).

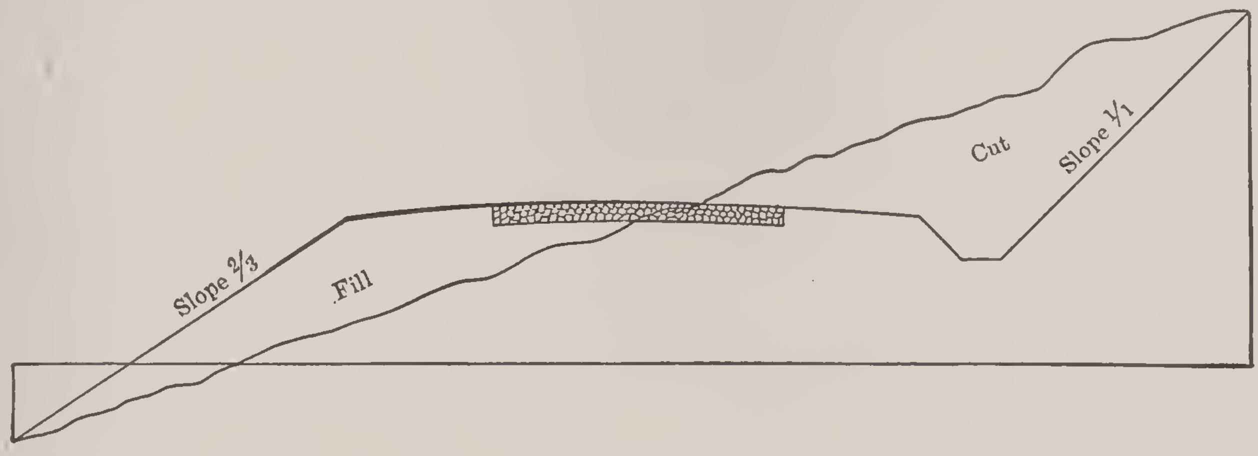


FIG. 161.

The cross-section of a side-hill road may also be made a plane surface sloping towards the center of curvature; this prevents erosion of the exterior slope by the drainage from the road, and also lessens the danger of accidents when vehicles pass over the road at high speed.

The sides of cuts and fills are given the natural slope of the earth of which they are formed, or a less inclination. They are sodded or planted with some protecting growth. The highways of other European countries differ in no essential features from those of France.

Streets.—The cross-section of a street differs from that of a road in having sidewalks; this necessitates the replacing of the ditches by shallow gutters. When a street is macadamized or paved the pavement is not ordinarily confined to a strip in the center, but extends to the gutter or curb on each side. The form of cross-section of a street is shown in Fig. 162.*

* This is a section of a street in Washington, D. C.

The width of the carriageway is about one-half that of the right of way on streets 60 feet or less in width; the ratio then increases and is about three-fifths on a street 100 feet in width. These ratios may be varied also with the importance of the street

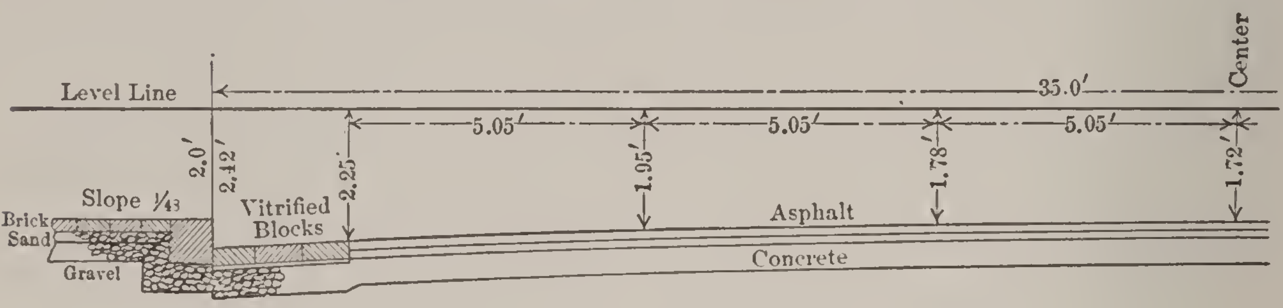


FIG. 162.

as a thoroughfare. A carriageway over 30 feet wide is uselessly expensive on a street which is subjected to moderate travel.

LOCATION AND CONSTRUCTION OF ROADS.

General Considerations.—To determine the best position of a road which is to connect any two points, it is essential to study the topography of the country between them either with or without the aid of topographical maps.

If no maps are available, the routes which seem to be the most favorable from observation or inquiry must be carefully reconnoitered. This may be done with a sketching-case, watch, and aneroid barometer. The selection of the routes to be examined are governed by the following considerations:

1. The streams of a country are its lines of most uniform declivity. The valleys formed by its streams are therefore the natural locations for the network of roads by which any country is traversed.

2. A wide valley is a better location for a road than a narrow one, since the road can be constructed at a greater distance from the banks of the stream which drains it. The road therefore requires less expensive bridges over the tributaries and is less liable to be injured in time of floods.

3. The divide between valleys is usually lowest where the source of a tributary of the stream draining the one is nearest the source of a tributary of the stream draining the other. These gaps or passes are the natural locations for the roads connecting the network of roads of one river valley with those of an adjacent one.

4. In a mountainous country the ridges separating river valleys are narrow and rugged; the roads are therefore confined almost wholly to the valleys.

5. In a rolling country the ridges are rounded and regular. The roads may therefore follow both the ridges and the valleys. The ridge roads have as a rule steeper grades, but have fewer bridges and culverts and are more easily drained.

6. In a flat country the roads are generally straight, since the expense of construction is dependent principally on the length of the road.

Trial Lines.—In reconnoitering the country to locate a new road it is well to begin first at the highest point, so as to have as extended a view as possible of the country to be traversed. In moving down slopes the angle of declivity of the road should be tested with a clinometer wherever the natural slope of the ground seems to be equal to or greater than the maximum desirable slope of the finished road. The elevations of the salient and re-entrant angles of the profile may be determined by the aneroid barometer, and the distances by pacing, by odometer readings, or by any other convenient method.

From the reconnaissance maps and the plotted profile it is possible to determine the best of a number of trial lines which have been examined.

If a contoured map of the country is available, one or more trial lines may be drawn on the map, and the profiles of these lines constructed for further study. The vertical distance between the contours being known, it is easy to draw on the map lines which have a less inclination than the maximum adopted grade. Thus if the vertical distance between contours is 10 feet, and the maximum grade is 5 per cent or $\frac{1}{20}$, any straight line whose intercept between consecutive contours measured by the scale of the map is more than ten times twenty, or 200 feet, will represent a line of the surface of the ground whose inclination to the horizontal plane is less than 5 per cent, or $\frac{1}{20}$. The inclination of the proposed road at any point may be determined by dividing the length of its trace between consecutive contours by the vertical distance between contours.

Marking Adopted Line.—Having selected the best of the trial lines, the next step is to stake it out on the ground. With a transit, and chain or stadia, the general line of the axis of the road is laid out upon the ground as a broken line, and a stake is driven at every 100 feet and at every angle. The elevations of the ground at these stakes and at the salient and re-entrant points in the profile, above an assumed datum plane, are determined with a level. At every stake, and at intermediate points if the surface is very irregular, transverse profiles are taken of the entire right of way.

Plotting.—The approximate axis of the road is now carefully plotted to a convenient scale, and the straight lines or tangents are connected by curves, usually circular.

The rules governing curves on the French highways are:

1. Make the radius of curvature at least 100 feet if possible.
2. Avoid sharp curves on steep grades.
3. Separate curves of opposite curvature by a straight track at least 60 feet long.

On mountain roads, where sharp curves are unavoidable, the road is widened at the curve as shown in Fig. 163, and the curve itself is made a horizontal platform.

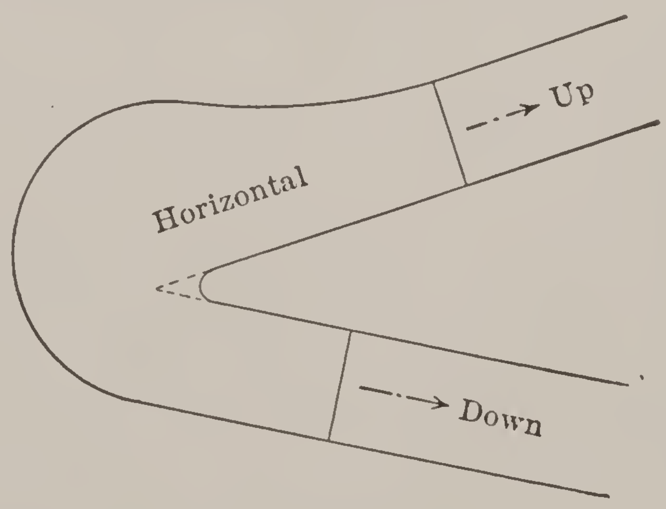


FIG. 163.

The profile is plotted on profile or cross-section paper to convenient scales. In order to magnify the irregularities the vertical scale, or scale of heights, is about ten times the horizontal scale, or scale of distances. The transverse profiles are plotted either above or below their actual positions on the profile as shown in Fig. 164. From a study of the plan and profiles it is possible to determine what slight changes, if any, should be made in the position of the axis of the road in plan. If no change is made, the axis of the road is fixed in the vertical plane by drawing on the profile a line, usually in red, which shall make the sum of the areas above it which must be excavated, equal to the sum of the areas below it which must be filled, to bring the road to

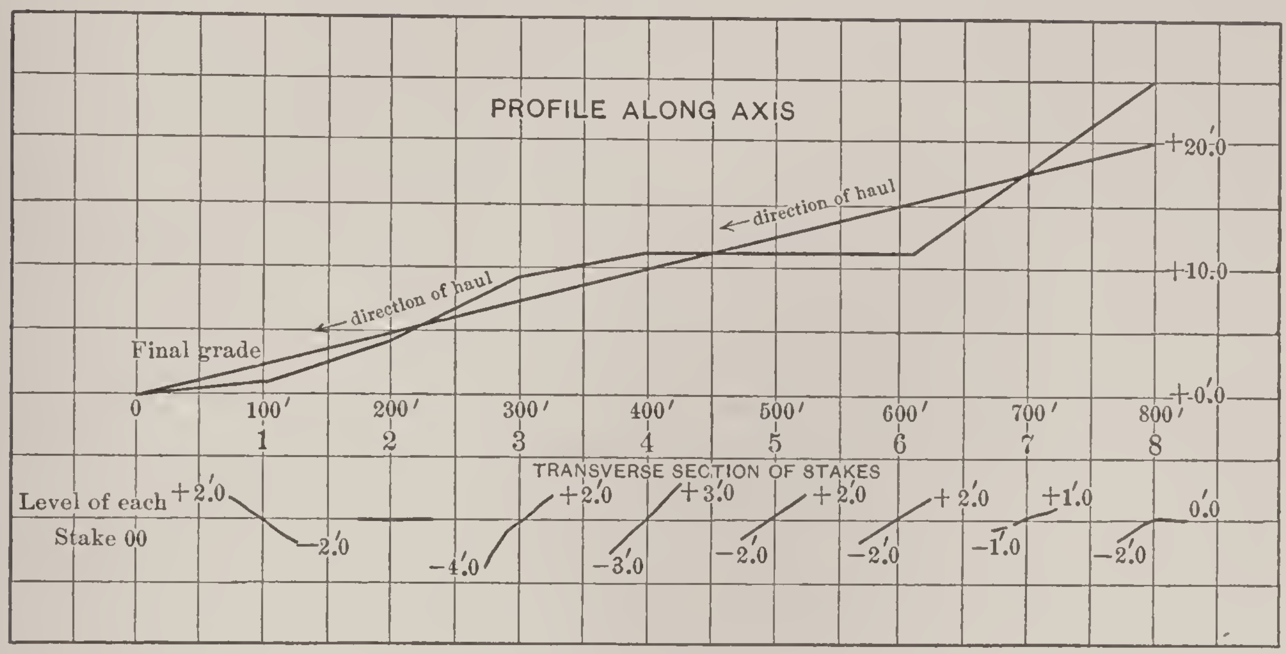


FIG. 164.

proper grade. In the profile of this axis the straight lines are connected by curves. This balancing of cuts and fills for the entire road and for short lengths reduces the cost of construction.

In making this change care must be taken not to increase the grade at any point above the maximum adopted.

Cuts and Fills.—To determine the volume of each cut and fill, each transverse profile is carefully plotted to scale on cross-section paper, and on it are drawn the surface and side slopes of the road. In Fig. 165 the assumed width of the right of way is 60

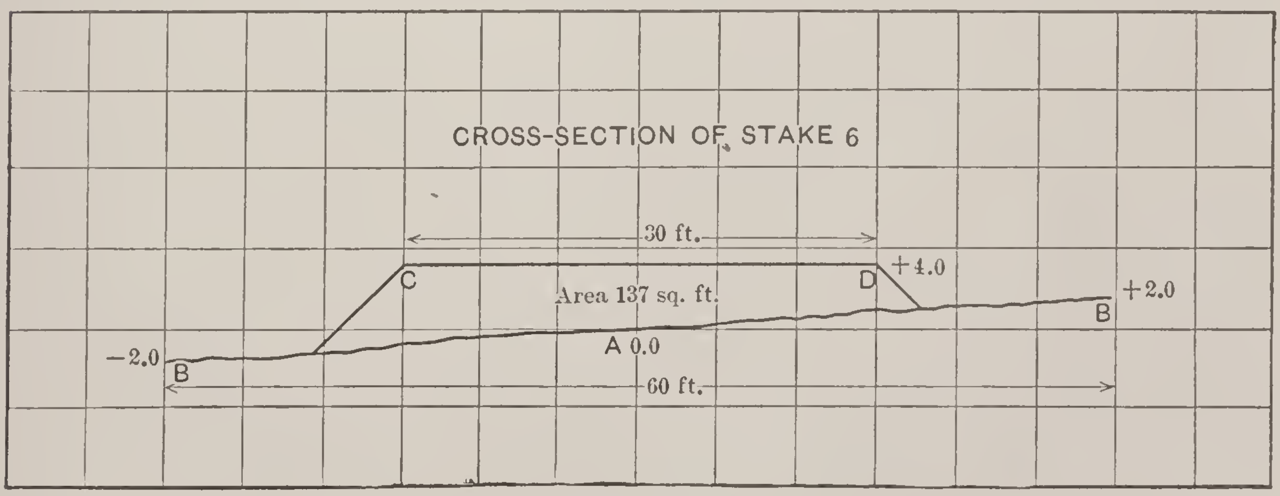


FIG. 165.

feet; let it be required to determine the area of the fill at stake 6 if the road is graded to a width of 30 feet. Through the point A of Fig. 165, which is a point of the profile, draw the line BB, or the line of transverse slope. At A erect a perpendicular and lay off on it 4 feet, the elevation of the finished grade above the surface of the ground. Draw the horizontal line *CD*, making it 30 feet in length, and draw the side slopes through *C* and *D*. The area thus inclosed will be the area of the fill. In this diagram the same scale is employed for vertical and horizontal distances. The area of the fill may be computed in several ways. It may be done by dividing the total area into a number of triangles or other figures whose areas are readily computed; it may be done by dividing the surface into a large number of small trapezoids by equidistant parallel vertical lines. If the distance between the lines is represented by d, and the length of the lines by l, l', l'', etc., the area will be $d(l+l'+l''+\text{etc.})$. The total length $l+l'+l''$, etc., may be measured with a map measure. The accuracy of the result will increase as d decreases. The area may be ascertained by counting the number of squares which it covers on the cross-section paper and then multiplying this number by the area of a square. The area of a profile may be determined by means of a planimeter.

The volume of the cut or fill between any two consecutive cross-sections is obtained in one of three ways: by the use of the prismoidal formula in which the sum of the areas of the end sections plus four times the mid-section is multiplied by one-sixth of the distance between end sections; by multiplying the half-sum of the end areas by the distance between them; and by multiplying the s ction midway between the end sections by the distance between the end sections. The first gives a correct result if the volume is regular, the second an approximate result larger than the first, and the third an approximate result smaller than the first. If by the three method we compute the volume of a frustum of a pyramid with square bases 2×2 and 4×4 feet, and a height of 10 feet, the volumes given by the three methods will be $93\frac{1}{3}$, 100, and 90 cubic feet, respectively. The second method is used because of its simplicity, unless the work is to be done by contract and the ayment is to be based on the computation; then the first method is employed.

Knowing the volume of each cut and fill, the number of square yards may be written on the profile below each cut and fill, and from the resulting diagram the most economical direction of haul of the material can be determined. A fill which is too far from the adjacent cuts to be economically made from these cuts is made from a *borrow-pit* which is secured in its vicinity; a cut which cannot be economically placed in one of the adjacent cuts is deposited in a *spoil-bank* on ground secured in its vicinity.

Earth which is excavated from a cut will shrink in volume about 10 per cent when placed in an embankment which is thoroughly compressed. Rock which is excavated from a solid ledge will expand in volume about 50 per cent when placed in an embankment. These volumes of the earth-fills must therefore be increased 10 per cent, and those of rock must be decreased 50 per cent, in the diagram above described. In contract work the amount of compression or expansion which is to be considered in computing volumes should be stated.

If the cuts and fills thus computed balance as closely as seems desirable, the location of the road in plan and profile may be considered fixed, and it only remains to complete the work on the site by staking out the curves, marking the height of the fill or the depth of the cut on each axial stake, and marking the limits of the excavations.

The curves are usually put in by eye; where accuracy is desired a curve of small radius may be described from its center by using the chain as a radius, and one of large radius by angles of deflection as described in manuals for railroad work. The height of fill and depth of cut, as well as the horizontal distance from the axial stake to the limit of the excavation, are taken from the transverse profiles or cross-sections.

Execution of the Earthwork.

Having cleared the right of way of trees and brush, which should *not* be thrown into the depressions as part of the fills, the material of the cuts is loosened by plows, excavated by hand or by machinery, and moved to the fills in barrows, drag-scrapers, carts, wagons, wheeled scrapers, or dump-cars. Steam excavators are employed only when the cut is very deep. Barrows and drag-scrapers are employed when the material is to be moved less than 200 feet; carts, wagons, and wheeled scrapers when the material is to be moved less than 600 feet; cars are employed for moving material more than 600 feet, especially if the amount is large.

Excavations.—In making deep excavations it is frequently advisable to vary the normal inclination of the side slopes of cuts to meet local conditions. If very hard material is encountered, as gravelly soil, the inclination may be increased; if, on the contrary, loose sand or soft clayey earth, which retains the water and becomes greasy, is encountered, the inclination must be reduced. The last material has a natural slope, when wet, of about one-third. If it is necessary either on account of the cost or on account of encroachment on private property to maintain the material at a steeper slope than its natural slope, the side slope of a deep cut may be terraced by leaving horizontal benches a few feet wide at vertical intervals of 10 or 12 feet; this will usually localize the sliding of the material of the slopes. The slopes may also be revetted or supported by retaining-walls on either side of the carriageway.

If the material of the cuts is very hard, it is loosened with explosives.

In making excavations care is taken not to disturb the line and grade stakes; these are usually left on small pyramids of earth which are not removed until the cut is brought to grade and tested.

Embankments.—An embankment is usually formed by hauling the material from the nearest excavation or cut and dumping it so that the beginning of the embankment is of full height and depth. The embankment is then prolonged by dumping the material at the end of the completed part. As an embankment thus made will settle, it will be made higher than the grade given on the profile sheet. The height of the fill at each stake is indicated to the workman by erecting poles with cross-pieces at the proper height.

If the embankment is made over marshy ground, it is supported by a floor made of rows of *sleepers* or longitudinal pieces, which break joints and are covered with a platform of boards, or with layers of fascines covered with brush. The platform prevents the unequal settling of the roadway, and the brush prevents the earth from sifting through the fascines. The boards and fascines may also be covered with straw or similar material.

Road embankments over marshes must be made high enough to keep the road surface about a foot or two above the surface of the water when the latter is at its highest stage.

Drainage.

The system of drainage of a road comprises the *culverts* which carry underneath the roadway the continuous and intermittent streams which intersect the line of the road; the *side ditches* which carry the water which falls on the road, as well as that which flows towards it from the adjoining lands, to the culverts or to natural lines of drainage which will carry it away from the road; the *subsoil drains* which carry off the ground-water and keep the road-bed dry; the *water-breaks* which on slopes turn the water from the carriageway into the gutter.

Culverts.—The term culvert is limited to spans of about 12 feet; when the span exceeds 12 feet the structure is a bridge. Theoretically the area of cross-section of a culvert should be just sufficient to carry off the water which reaches it in the heaviest rain and when its drainage area is in the best condition to discharge this water rapidly; no damage will then be done to the road or to the adjoining property.

The area of the culvert opening will depend, therefore, on the maximum rate of rainfall, the area of drainage basin above the culvert, the slope of the basin, the condition of the soil as affecting the amount of water which it will absorb in a unit of time, and the condition of its surface as interfering with the free flow of the water.

It is evident that the accurate solution of the problem is a very difficult one; for all ordinary cases reliance must be placed upon empirical formulas modified by good judgment. The one commonly employed (Myer's) is

$$\text{Area of cross-section in square feet} = c\sqrt{\text{drainage area in acres}},$$

in which c is a coefficient whose value depends on the character of the surface; it is 1 for a flat country, 1.6 for a rolling country, and as high as 4 for a mountainous country. The flood height and the area of cross-section of a stream in time of flood can often be best determined by making inquiries of people living near it, or by a careful inspection of its banks and the culverts or bridges along its course.

For a very important culvert, whose failure to carry off the water promptly might cause serious damage, the amount of water which will probably reach the culvert is determined by some formula of the following form:

$$D = cr\sqrt[4]{\frac{s}{a}},$$

in which D = cubic feet per second per acre reaching the culvert;

c = coefficient varying with amount of obstruction to flow from 0.75 to 0.30; (The first value corresponds to the drainage of paved city streets, and the latter to the drainage of village gardens and macadam streets. Frozen meadow-land covered with a thin film of ice would have a coefficient of about 0.60.)

r = maximum rainfall in cubic feet per second per acre, which is approximately equivalent to the rainfall in inches per hour;

s = slope of ground in feet per thousand feet;

a = number of acres in the drainage area.

To apply this formula (Bürkli-Ziegler) a contoured map of the drainage area is necessary.

Culverts should always be made in solid ground at the base or sides of a fill, and never in the made ground of the fill. If constructed in made ground, they will be broken when the embankment settles.

Small culverts up to 2 feet span are usually made of glazed sewer- or cast-iron water-pipe, 12 to 24 inches in diameter. The iron pipe is best when the top of the culvert must be close to the surface of an unpaved road. To increase the capacity of the culvert two or more of the pipes may be laid side by side.

To construct a pipe culvert a trench is dug in the solid ground, and in this the pipe is laid with a uniform fall at least sufficient to drain the pipe. The joints are carefully calked with cement or clay, and the trench is filled with earth carefully rammed. The ends of the culvert should if possible be supported by imbedding them in low retaining-walls built parallel to the roadway. If the foundation is not solid ground, the soft material should be excavated and filled with good earth well tamped, so that the water will not undermine the pipe. If there is any danger of settlement of the middle part of the culvert, this should be provided for by increasing the fall near the outlet.

Medium-sized culverts are usually rectangular or box culverts or arch culverts. The box culvert is made of dry or cemented stone walls, and is covered with slabs of stone. As the resistance of stone to a bending force is small, the cover is also sometimes made of two or more layers of slabs in which each layer projects slightly beyond the one beneath it, thus gradually decreasing the span. The bottom of the culvert is paved with stone, preferably laid in mortar.

When more durable material is not av ilable box culverts are also made of beams and planks or of square timbers. When the span exceeds 4 feet there is usually a central vertical partition which divides the culvert into two equal channels, and reduces the thickness of the covering timbers.

Arch culverts of stone, brick, and concrete are employed for spans of from 2 to 12 feet. The form of the arch is either the full center or the segmental. The depth of the arch at the crown may be determined by the formula $d = 1 \text{ foot} + \frac{1}{20}s$, in which d = depth and s = span in feet. In arches whose span does not exceed 3 feet the thickness of the arch may be uniform throughout; as the span increases the thickness of the arch at the springing-lines increases more rapidly than at the crown, and in arches whose span exceeds 6 feet the thickness at the springing-lines is double the crown thickness. The thickness of the abutments for spans up to 12 feet may be deduced from the empirical formula $t = 3 \text{ feet} + \frac{1}{10}h + \frac{1}{10}(s - 3 \text{ feet})$, in which t is the thickness and h is the height of the abutment, and s is the span of a full-center arch in feet. The value of t is increased $\frac{1}{5}$ for a segmental arch.

The box and arch culverts may extend the full width of the carriageway and their side slopes, and terminate in a vertical wall, or they may extend only the width of the carriageway. In the latter case the abutment-walls are prolonged as wing-walls through the side slopes and gradually contract the waterway.

Spandrel- or head-walls confine the earth filling on the top of the culvert proper. The paving of the bottom of the culvert is extended to include the space between the wing-walls.

If the culvert is a wide one, the pavement of the bottom may be made slightly concave upwards.

Side Ditches.—The side ditches are usually trapezoidal in cross-section and should have sufficient area of cross-section to carry the drainage during the heaviest rainfall when the surface is in the best condition to allow its rapid drainage. The ditches are paved only when the grade of the road is such that they would otherwise be rapidly eroded. When the road is constructed at or near the foot of a long slope one or more additional ditches should be dug along the face of the slope, parallel to the road, to catch the surface drainage before it reaches the side slopes of the road. This will prevent the washing of these slopes in heavy rains.

Deep side ditches assist in draining the subsoil of the road, and are especially valuable when no subsoil system of drainage is provided.

Subsoil Drains.—A subsoil drain is a covered channel designed to remove the water which penetrates the road-bed, and thus secure a dry foundation for the wearing surface. The drain must have an unobstructed channel inclosed by an envelope through which the water can percolate to reach the channel. A drain made of porous terra-cotta pipes called *drain-tiles*, or a small box culvert constructed of dry stones, will fulfill these requirements.

If the carriageway is not covered by an impervious coating, some of the water which falls on the surface or results from the melting snow will penetrate the road-bed. The percentage absorbed will depend on the form of cross-section and the ruts and depressions in the surface.

If the carriageway is lower than the adjoining land, some of the water falling on this land will also reach the road-bed if an impermeable stratum underlies the road-bed. The amount which reaches it in this manner depends on the position and dip of the impermeable strata.

If the road-bed is sandy, the water which reaches it will be rapidly carried away through the soil and will do no injury; if, however, the road-bed is clayey, the water is retained and the road-bed becomes very soft.

Subsoil drains are therefore most necessary in deep cuts in clayey soil, on roads which have a natural earth surface. They are usually unnecessary in embankments and in sandy soil.

The best form of drain is a line of porous-tile pipe, usually about 4 or 5 inches in diameter, laid on a firm bottom, true to grade, and covered with a layer of broken stone or gravel a foot or more in thickness. As the gravel or broken stone will retain the silt, the pipes may be laid without collars over the joints. If no layer of broken stone is employed, the joints must be covered with tile or clay collars to prevent their obstruction by silt.

The tiles are usually laid in a trench about 3 feet deep, but this depth is varied to meet local conditions.

The position of the drains in the road depends on the direction from which the water comes. For surface drainage there should be one deep line bisecting the carriageway longitudinally, or two shallower lines trisecting it. When the water comes from one side only, as on a macadamized road along a hillside,

FIG. 166.

the drain should be on that side, with branches if necessary leading to it from the carriageway. If the water comes from the two sides, as on a macadamized road in a cut, a drain should be constructed along each side of the road under the side ditches, as these are liable to be filled by the caving of the slopes.

Water-breaks.—Water-breaks are shallow ditches or low embankments which are constructed diagonally across the carriageway on steep grades, to prevent the water from running in the wheel-ruts and thus eroding the surface. They are unnecessary on paved carriageways or on macadamized carriageways whose surfaces are properly crowned. Small cross-drains, covered by a grating and connected with the sewers, are sometimes constructed across paved roadways on long grades to prevent the accumulation of too much water at the foot of the slope.

Catch-basins.—A catch-basin is a shallow well constructed near the curb of a street; it is designed to catch the water which flows from the gutters, and remove its silt before discharging it into the sewers. The details of construction are given under "Sewerage."

Earth, Gravel, and Macadam Roads.

The top or wearing surface of the carriageway should be *impervious to water, incompressible, offer little resistance to the movement of a vehicle,* and *furnish a good foothold to the horse.*

It should be added that the surface should not become dusty. If the highway is an unpaved one, this can be prevented in hot, dry weather only by sprinkling daily with water, or occasionally with crude oil. For country roads the latter is the only practicable method.

Earth Roads.—An earth road is one in which the wearing surface of the carriageway is the earth excavated from the sides of the carriageway or from the ditches. It fulfills none of the requisites of a good wearing surface, unless well drained and kept in thorough repair. Even then its tractive resistance is at least 100 pounds per ton. The quality of the wearing surface depends upon its composition; it is best when composed of sand or gravel with just sufficient clay or loam to bind it thoroughly; it is worst when the sand or the clay alone is used.

If the cross-section is of the general form shown in Fig. 160, the carriageway proper should have a crown of $\frac{1}{24}$, and the side ditches should be deep enough to assist in draining the road-bed. As this form of cross-section necessitates the excavation of the ditches by hand, a more common form is an arc of a circle connecting the bottoms of the side ditches. A high crown is necessary to prevent the surface-water from sinking into the road-bed, when the road is worn by travel.

The road covering being compressible is easily worn into ruts which hold the water, and in clayey soil render the road impassable in wet weather unless the road is provided with subsoil drains. The compressibility of the surface may be reduced by rolling it with a heavy roller and by making the road covering of a mixture of clay with sand or gravel. The surface drainage can be kept intact only by filling the ruts with good material as soon as made. Large stones should never be used in making these repairs.

As dependence must be placed largely upon heat and free circulation of air to dry the road-bed, a road in clayey soil should never be shaded by overhanging trees; a sandy road, being better in a moist than in a dry condition, should have as much shade as possible.

Besides the grading machinery employed in the construction of the road-bed as above described, the principal machines employed in its improvement are the *simple* and *elevating scraping graders*. The simple scraping grader is a four-wheeled vehicle which supports a heavy steel blade, either plane or curved, which may be set so that its cutting edge is oblique both to the axis of the road and to the horizontal plane. As it is pulled along it cuts the surface to an inclined plane and moves it from the sides toward the center. The carriageway may thus be given a rounded surface. The elevating scraping grader has, in addition, a carrier by means of which the earth excavated from the side ditches is conveyed directly to the middle of the carriageway. Both machines may be employed in constructing new roads and in repairing old ones.

Gravel Roads.—In a gravel road the wearing surface is a bed of compressed gravel usually from 8 to 12 inches in thickness. When in thorough repair the wearing surface is all that can be desired, but it wears away rapidly under heavy travel and requires a thoroughly drained road-bed. The quality of the wearing surface depends on the character of the gravel and binder. The gravel should vary in size from 2 inches downwards so as to reduce the volume of voids to a minimum, and should be united by a binding material which will not absorb water. The cement formed from the crushed gravel is the best binder, but it will take some time before a surface of loose gravel, such as is dredged from a river bottom, can be rendered compact. Clay loam, just sufficient to fill the voids, makes a good binder, but if in excess, will cause the carriageway to become soft and muddy in wet weather. A small quantity of lime or oxide of iron mixed with the clay improves its binding qualities.

The crown of a gravel road may be made somewhat less than an earth road, or about $\frac{1}{30}$ to $\frac{1}{35}$. The form of cross-section is like that of a macadam road.

The gravel may be taken from pits or river-beds. The first has usually an excess of sand or clay and must be screened; of the two materials in excess the sand is least objectionable. River-gravel packs slowly and is usually mixed with clay or loam to make it bind more readily; the mixture is, however, less durable than the pure gravel.

In making gravel roads the carriageway is excavated to subgrade and rolled; if the ground is clayey, the road-bed is drained or the gravel is placed on a bed of coarse broken stone. A layer of gravel about 4 inches thick is spread on the subgrade, moistened, and rolled until it is packed. The other layers are distributed and rolled in the same manner. In contract work the amount of gravel is determined by running lines or levels over the rolled subgrade and the finished work. The wearing surface may be made of uniform thickness, or the thickness of the sides of the carriageway may be made 2 or 3 inches less than the thickness at the center.

Macadam.—The term *macadam* is applied to stone broken into small fragments to be used in making broken-stone roads, and the terms *macadam road* and *macadam pavement*, applied to highways thus improved, are derived from John Macadam and his son Sir James Macadam, who were employed in improving the highways of Great Britain during the first half of the nineteenth century.

The proper way to construct a macadam road was described by Sir James Macadam about 1850 as follows:

"The road-bed is thoroughly drained and all large stones are carefully removed. The surface of the road is then graded to a lateral slope of 1/36. As sand, sandy earth, or any other soft and dry material makes the best foundation for a road, the broken stone is placed directly on the ground. On first-class roads there are three layers of stone. The first consists of any sound stone which is broken into fragments weighing not over 6 ounces and screened to remove all dirt. This layer is 4 inches thick and is compressed by travel or with a roller. The second layer, 2 or 3 inches thick, is of stone similar to the first, and is compressed in the same manner. The third layer is of granite or other hard stone, and its fragments are of the same size as those of the other layers. This is the wearing surface proper and is 3 inches thick. No sand or binding material should be spread over the surface to fill the interstices, but these should be filled by the fragments and dust produced by the travel on the road. The repairs should consist in picking the road surface to a depth of 1 inch and then placing on a new layer. A second-class road needs only a 4-inch base and a 3-inch wearing surface, and a third-class road only a 3-inch base and a 2-inch surface."

Modern practice differs from the method prescribed in only two essential particulars.

It is customary to excavate a trench to hold the broken stone. The bottom of the trench or the *subgrade* is rolled with the heaviest roller obtainable and all its soft places filled with sandy earth or ashes. The subgrade when thoroughly rolled should be parallel to the finished surface of the carriageway, and at the proper distance below it. The thickness of the macadam depends upon the character of the soil, the amount of travel, and the frequency of repairs. The standard depth under favorable conditions is *6 inches;* this may be decreased to *4 inches* under very favorable conditions, and is increased to as much as *12 inches* if the conditions are more or less unfavorable.

Binder.—The Macadams insisted that the surface of the road must be impervious, but at the same time insisted that no binding material should be used to close the crevices between the stones. In the course of time these crevices will naturally be filled by the fragments and dust produced by travel; in the meantime, however, the surface is rough and permeable. It is now customary to fill the crevices at once by placing some suitable binding material on the surface layer after it is thoroughly rolled, and to force it into the crevices by additional rolling. This gives a smooth, impermeable surface at once.

The natural tendency of inexperienced road-constructors is to employ clay or some form of clayey earth for the binder because of its adhesive properties. It is, however, the worst material which can be used for this purpose, because it holds the water, which settles in the depressions of the road, and becomes soft and greasy. Sand or sandy gravel is often used and forms a good binder, especially if the wearing layer is limestone. Stone screenings from the crusher make the best binder; those from trap-rock give the best results. As a rule, a silicious binder gives the best results on a limestone macadam, and a calcareous one on a silicious rock macadam. If circumstances require the use of clay, it should be used sparingly; one containing iron is the best.

The binder is usually placed only on the wearing layer, though some engineers place it on the foundation layers as well. This last method probably gives a result similar to that obtained by Macadam when he had each layer compressed separately by travel. With a perfect binder the result is approximately a concrete, and the pavement is ideal; if, however, the binder retains the water and softens, it will assist in destroying the pavement. If only a surface binder is employed, the water which passes through it will at once reach the subgrade. If this complies with Macadam's requirements, no damage will be done, as the water will be at once drained away; if, however, the soil retains the water, it will soften, be pressed up into the macadam and assist in the destruction of the pavement. When the soil is dry and the binder is good, a layer of binder worked into each layer of macadam will improve the pavement, otherwise it is inadvisable and uselessly expensive.

Stone.—As to its characteristics, the stone in the surface layer should be tough, hard, of uniform strength in all directions, its dust should have cementitious qualities, and it should resist the destructive action of the weather. The igneous or unstratified rocks, such as porphyry, trap, basalt, granite, syenite, and diorite have the best average qualities. The limestones, the metamorphic rocks, quartzite, gneiss, schist, and millstone grit, rank second. In the igneous and metamorphic rocks sodium is an element of weakness, as it usually mak the stone more perishable. In the second class the difference between the best and poorest of any variety is greater than in the first class; greater care is therefore necessary in selecting the stone. In the foundation layers any sound stone may be used. Sandstones are often used in these layers.

Rolling.—The subgrade, the successive layers of stone, and the binder are all rolled separately with a heavy roller. If a horse-roller is used, its weight should be about 5 tons, and if a steam-roller is used, its weight should be about 15 tons. A heavy roller will make the road-bed more unyielding and the pavement more compact than a light one. The layers of stone and the binder are sprinkled with water before they are rolled.

Telford Macadam.—Telford-macadam pavement is one in which flat stones placed on edge replace the foundation-layers of ordinary macadam. It is named from Thomas Telford, a

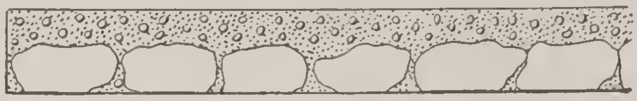


FIG. 167.

celebrated Scotch engineer who was a contemporary of the elder Macadam, and was also employed on highway improvement in Great Britain (Fig. 167).

This method of construction was first employed in France. The highways in France were, before 1764, repaired only twice a year, and then by unskilled laborers; they were therefore made in a very substantial manner. At the base were two or three layers of flat stones laid on their sides; these were covered with broken stone to such a depth that the whole pavement was 18 inches thick in the center and 12 inches on the side.

After 1764 the highways were regularly maintained, and in a memoir published in 1775 the French engineer Tresaguet described a method of construction by which the depth of the pavement could be greatly reduced. He laid the foundation-stones on edge, in rows, across the carriageway; the height of this layer was 7 inches. Its crevices were filled by fragments of stone driven in with a hammer. The top layer consisted of 3 inches of broken stone of ordinary dimensions. The road-bed was usually excavated to a subgrade parallel to the finished surface, and a row of stones on edge, 10 inches high, was placed on either side to hold the broken stone in place. These stones formed a curb which did not project above the surface of the carriageway. He made his carriageways both convex and concave in cross-section.

Telford varied this method of construction by making the subgrade horizontal and securing the crown by varying the depth of the foundation-stones; this was, however, not an essential detail. The foundation-stones were from 3 to 5 inches thick on top, and from 8 to 14 inches long. He omitted the border-stones and usually covered the shoulders of the pavement with a layer of gravel. As he was engaged only with the improvement of important roads, he made the surface layer of broken stone 6 inches thick. In a road 30 feet wide he first adopted a crown of 9 inches, but later considered 6 inches sufficient. His roads were always well drained and carefully constructed. Like macadam pavements, the foundation-course of a telford pavement may be made of sound stones which are either too soft or too brittle to be used in the surface layer

Comparison of Telford and Macadam Methods.—The telford foundation is more open than the macadam; the water, which reaches the road bed either from above or below, can thus escape if the bed has any longitudinal slope. The interstices of this foundation are open, and hence the ground-water cannot reach the binder of the top surface to soften it. In the macadam foundation the interstices are usually filled with binder or dust, which prevents the escape of the water and allows it to be drawn up and distributed through the binder by capillary attraction. This softens the pavement. Macadam pavement, being the cheaper, should therefore be laid when the soil is sandy and allows the escape of water through it; if laid in wet places, the foundation-layer should be of stones 3 or 4 inches in diameter, and the binder should be confined to the top layer. The foundation of the telford pavement must be laid by hand; it is therefore more expensive. The telford pavement is less liable to be affected by ground-water, and is therefore best when a pavement must be laid on clayey soil, especially in cuts where the water is liable to accumulate.

Block and Sheet Pavements.

The macadam and telford pavements are not suitable for city streets because of the dust and mud, the necessity of constant repairs, and the impossibility of keeping the surface clean without injuring it. For these reasons sheet and block pavements are laid on these streets.

An ideal city pavement is *smooth without being slippery*, *elastic without losing its shape*, *impervious to liquids*, and *contains no decaying substances*. On such a pavement the tractive resistance of a vehicle and the noise produced by the travel is reduced to a minimum, and the pavement contributes to the health of the community. In addition to the above qualities, *the cost of construction and maintenance should not be excessive.*

A city pavement usually consists of a *wearing surface* which is exposed to the concussion and friction of travel and must be renewed from time to time, and a *foundation* which supports the wearing surface and prevents it from losing its shape.

Wearing Surfaces.—The ordinary wearing surfaces are made of *blocks* of wood, natural and artificial stone, and asphalt concrete, and of *sheets* of asphalt mortar.

All block pavements are laid in a similar manner, with the blocks in rows as nearly as possible perpendicular to the line of travel. Between street intersections the rows are perpendicular to the axis of the street; at the street intersections the rows may be parallel to the diagonals which bisect the four angles made by the two axes. The blocks break joints in the direction of the travel. The blocks are placed by hand on a cushion of sand, and are brought to a firm bearing by tamping the blocks with a heavy hammer or by rolling the pavement with a light roller. A steel shoe covering several blocks is placed on the pavement to receive the blows of the hammer; if the blocks are brittle, a wooden shoe is employed instead of a steel one. Tamping, although more expensive, is usually preferred to rolling because the blocks are not displaced laterally. The thickness of the sand cushion varies from $\frac{1}{2}$ inch to 2 inches, depending on the allowable variation in the depth of the blocks and the smoothness of the surface of the foundation.

Foundations.—The best foundation for all forms of pavement is a bed of concrete whose upper surface is fairly smooth and parallel to the finished surface of the pavement. The bed is of uniform thickness and is laid on the subgrade, which has been graded so as to be parallel to the finished surface and then thoroughly rolled with a heavy roller. The form of cross-section of the carriageway is determined by making the height of the crown equal to $\frac{1}{50}$ or $\frac{1}{60}$ of the width of the roadway and the height of the quarter-points $\frac{3}{4}$ the height of the middle point. The subgrade and the surface of the concrete are tested by measuring down from a horizontal cord connecting the curbs or attached to stakes driven on either side of the carriageway.

A concrete foundation is absolutely rigid, and will hold the wearing surface in position at every point, and thus prevent the formation of local depressions which increase the concussion and friction at those points and hasten the destruction of the wearing surface. If the foundation is removed for the purpose of laying water-mains or sewers, or making house connections, it is easily restored, and the pavement will show no depression.

The normal thickness of the concrete foundation is *6 inches.* This may be reduced to 4 or 5 inches if the concrete is rich, carefully prepared, and laid upon a thoroughly rolled subgrade, and the foundation is not liable to be disturbed for the purpose of laying sewers, mains, etc. The proportion of the ingredients is usually 1 natural cement, 2 sand, and 4 stone, or 1 portland cement, 3 sand, and 6 stone. If the cement is carefully tested and the mixing carefully inspected, the proportion of cement may be reduced; in the city of Washington the proportions are: 1 portland cement, 4 sand, 5 gravel and 5 stone. In place of gravel the run of the crusher may be substituted; it differs from the other stone in containing material smaller than $\frac{1}{2}$-inch pieces.

Where concrete is considered too expensive a bed of gravel or cinders of about the same thickness and compressed with a heavy roller may be employed. As this form of bed will nearly always show depressions over cuts made for the purposes above described, it will probably be more expensive in the end than a concrete base, because of the more rapid destruction of the wearing surface.

Wood Pavements. — A pavement of untreated soft-wood blocks on a rigid foundation is the most noiseless and elastic and the least slippery form of pavement, but is unsanitary and perishable on account of its absorption of liquids, and wears rapidly under heavy travel. As the blocks will decay in three to five years, depending on the care taken to keep them clean, and will, except under the heaviest city travel, stand the wear of travel for a much longer time, it is customary to preserve the blocks by the creosote process. This makes the blocks impervious and protects them from rot, but makes them less elastic and more slippery. On the great thoroughfares of London a pavement of this kind is worn out in five to seven years, but under light travel it would probably last fifteen years or more. A first-class pavement of creosoted Georgia pine costs in this country about \$3.25 per square yard.

If hard-wood blocks are used the pavement is more slippery but less perishable than one of soft wood. In Australia there are pavements of very hard, dense, untreated wood that have stood ordinary wear for fifteen years; these pavements are, however, extremely slippery.

The best wood pavements consist of soft creosoted or dense hard-wood blocks whose surface is $3'' \times 9''$. The maximum depth of the blocks is 6 inches, and the minimum 4 inches.

The best creosoted blocks are fir and yellow pine; the best natural hard-wood blocks found in the market are the Jarri and Karri woods of Australia.

Hard or creosoted soft-wood blocks are extensively used in paving the streets of London and Paris, because they are less noisy and less slippery than the stone-block and rock-asphalt pavements, which are the other principal forms; in these cities wood pavements are laid on the streets which are subjected to very heavy travel. In this country soft untreated wood blocks were laid in nearly all the principal cities between 1870 and 1875. This pavement, called the Nicholson, was usually laid with wide transversal joints filled with gravel, and on a wooden platform consisting of two layers of pine boards coated with tar. The surface was covered with tar and gravel. These pavements were short-lived and led to a distrust of wood as a paving material. The Nicholson pavement was followed by the cylindrical cedar-block pavement laid extensively in the northern central States. This was an improvement, but was also perishable and unsatisfactory. The introduction and development of the Trinidad asphalt pavements between 1875 and 1880 furnished a suitable substitute which was generally adopted in cities of the United States. It is only since about 1890 that good wood pavements have been laid in this country.

A first-class wood pavement has a foundation of 6 inches of concrete. If the concrete is finished off with a smooth surface by covering it with a thin layer of mortar, the blocks may be set on this mortar; as a rule, however, the concrete is covered by an inch of sand or a layer of asphalt.

The blocks are rectangular prisms with the grain of the wood vertical. If thoroughly creosoted, so that they will not absorb moisture and expand, they may be laid with close joints in both directions; as a rule the blocks are laid in close contact in each row, and the rows are separated by a space $\frac{1}{4}$ to $\frac{3}{8}$ inch by the insertion of thin strips which are afterwards removed and the joints are filled with cement grout, or by asphalt cement covered by an inch of grout, as the latter stands the wear of travel better than the asphalt. An expansion-joint is left at the gutter ends of each row, and the joints between the rows are filled after the blocks have expanded. Close joints are found to resist heavy travel better than open ones. A wood pavement must be kept in repair by the removal of the blocks which are found to be defective; otherwise depressions are formed which hasten its destruction.

Plank and corduroy pavements are crude forms of wood pavements; they are employed to carry a road across a swamp or over dry sand into which the wheels would sink to a considerable depth were the wagon not supported by some form of platform. The foundation of the platform is two or more rows of longitudinal sleepers, usually logs. In the plank pavement these are covered with a floor of 2- or 3-inch plank, and in the corduroy by saplings placed in close contact and held in place by wheel-guards. The upper surfaces of the saplings may be roughly hewn, or they may be covered with brush or grass and a layer of earth.

Stone-block Pavement.—A pavement of granite blocks on a rigid foundation is the most durable of all pavements, and requires no attention for many years after it has been laid; it may be made impervious by filling the joints with hot gravel and asphalt cement pitch. As the surface is unelastic and rough, the noise made by the passing travel is so great that in many cities they are being gradually replaced by the less noisy forms.

The blocks are made of granite, trap, and sandstone. The granite blocks are best, since they are more durable than sandstone and less slippery than trap. The sandstone must be one with silicious or ferruginous binding material.

The size of the blocks varies considerably in different cities. In Liverpool, which has probably the best stone-block pavements, the blocks are about $3\frac{1}{2}$ inches square and 6 inches deep; they must be culled so that the maximum variation shall not exceed $\frac{1}{4}$ inch. These blocks are laid on a half-inch cushion of gravel on a 6-inch concrete base. In Washington the blocks are 6 to 8 inches long, 3 to 4 inches wide, and $5\frac{1}{2}$ to 6 inches deep; these blocks are laid with close joints filled with pitch on a 2-inch sand cushion. The foundation is 4 inches of rolled gravel. In other large cities the blocks are 8 to 12 inches long, $3\frac{1}{2}$ to $4\frac{1}{2}$ inches wide, and 7 to 8 inches deep; as a rule these are not laid on concrete. If laid on a concrete foundation, small blocks are as unyielding as large ones and are less easily displaced; if the foundation is not rigid, the blocks must have a larger bearing surface. The size of the blocks is less important than the uniformity in length and in breadth, since the latter governs the dimensions of the joints, which should be as narrow as possible.

The joints in the best pavements are filled with hot gravel and asphalt cement, tar, or pitch; those of inferior pavements are filled with gravel only.

The price of stone blocks at the quarry is about $1.15 per square yard, and the cost of laying the blocks on a 6-inch concrete base after they have been delivered in the city is about $1.10 per square yard. To the total cost of $2.25 per square yard must be added the freight charges and the contractor's profits.

Pavements with a stone wearing surface were introduced into Europe by the Romans in the construction of their military roads and in the paving of their city streets. These pavements consisted of irregular-shaped slabs, usually of large area, which were supported by a firm foundation of flat stones covered by a layer of concrete. In many Italian cities the influence of the Roman method of construction is seen in pavements whose surface is made of large flat stones. In the principal street of Genoa the blocks are nearly $2\frac{1}{2}$ feet long and 1 foot wide; grooves are cut in the stones dividing them into narrow strips.

The *cobblestone pavement* is a rough form of block pavement which is made of the spherical or egg-shaped stones found in the beds of streams and in fields. It is very rough, very slippery, hard to keep clean, and destructive to vehicles. The noise of travel is much greater than over stone blocks. The best of these pavements is made of stones 4 to 6 inches long, 2 to 4 inches in diameter. The foundation is a bed of compressed gravel 4 or 5 inches thick and a cushion of sand. The stones are laid with close joints, covered with sand, and then tamped with a heavy rammer until they are firmly bedded.

Brick Pavements.—A brick pavement is simply a stone-block pavement in which blocks of natural stone are replaced by blocks of artificial stone. Bricks are somewhat less durable than stone blocks, but they can be made of uniform size and with smooth faces; the resulting pavement has therefore narrower joints and a smoother surface, both of which greatly reduce the noise of travel and the resistance to traction. The pavement is also cheaper than stone, since the cost of the brick at the kiln is about one-half the cost of stone blocks at the quarry.

The bricks used for pavements are made of selected clay, usually shale, and are burned at a high heat just short of vitrifaction. They are called *vitrified bricks* if of the same size as ordinary bricks, and *vitrified blocks* if of larger size. Blocks are more commonly used than bricks.

The blocks are usually 3 inches wide, 9 inches long, and 4 inches deep. If made of shale, they are reddish brown in color, and if of fire-clay, they are mottled yellow. To make a durable pavement the blocks must be tough and thoroughly burned; to make a smooth pavement they must be of regular form and of uniform size. The edges of the blocks are often rounded, and the vertical faces are grooved or studded. The rounded edges reduce the loss of weight in the rattler test to be described, but add nothing to the value of the blocks; the studs and grooves are to make space for the grout or asphalt cement with which the joints are filled, but this is unnecessary. A brick with square edges and plane faces is probably the best form.

When bricks are compared with a standard or with each other they are usually subjected to two tests, the *absorption* and the *rattler* test. The absorption test consists in determining the percentage of water by weight which will be absorbed in forty-eight hours by a block which has been thoroughly dried in an oven. As the surface can be easily made impervious by a suitable coating, the blocks are broken into two nearly equal pieces. A good paving-block should absorb very little water. The rattler test is made by placing the paving-blocks and small cast-iron blocks in a rattler and rotating the rattler at a fixed speed for a definite time. The toughness and durability of any specimen is assumed to vary inversely with its percentage loss of weight in the rattler. A rattler is a metal barrel placed in a horizontal position and rotated by machinery about its longer axis. It is employed in cleaning castings. In the rattler the edges of the bricks are worn away by concussion and friction, and the bricks, if too soft, are broken. Standard testing rattlers are made for this, which is considered the most valuable of the paving-brick tests. The blocks are also sometimes tested for compressive strength, but this is not an important test.

In laying the pavement the bricks are placed by hand on edge on a cushion of sand 1 or 2 inches thick. This thickness depends on the smoothness of the surface of the foundation. The blocks are then forced to a firm bearing by dropping a heavy tamp on a short thick plank, or the pavement is rolled with a light roller. The former displaces the bricks less than the latter, but is more expensive.

If the joints are filled with cement grout, it is necessary to make expansion-joints parallel to the axis at the curb, and joints perpendicular to the axis at intervals of 200 feet; these joints are filled with asphalt cement or pitch. In inferior pavements all joints are filled with sand and are not impervious.

While a concrete foundation 4 to 6 inches thick is better than any other form, the pavement may be laid on rolled macadam, gravel, or cinder. When these pavements were first laid in this country the foundation-course was a layer of bricks placed on their sides.

Brick pavements were introduced into this country about 1870, and because of their cheapness and other qualities have become the principal pavements of our interior cities.

The price of re-pressed vitrified paving-blocks at the place of manufacture is about $0.60 per square yard; the common are about $0.10 less. The cost of lay ng the blocks on a 6-inch concrete base is about $0.85 per square yard, and is less than the cost of laying stone blocks because of the regularity of the bricks, which makes it easier to break joints, and because of the smaller amount of cementing material required in the joints. To the total cost of $1.45 per square yard must be added the freight charges and contractor's profits.

Asphalt Pavements. — Sheet-asphalt pavements are of two general classes: those made of rock asphalt or a natural mixture of limestone and asphalt, and those made of asphalt mortar or a mixture of asphalt cement and sand. The former having been found too slippery, only the latter class is laid in this country.

The standard sheet pavement consists of a cement-concrete foundation 6 inches thick, a cushion or binder course of asphalt concrete $1\frac{1}{2}$ inches thick, and a surface coat of asphalt mortar $2\frac{1}{2}$ inches thick. The binder is made by mixing asphalt cement with clean broken stone which will pass an inch and a half screen. The stone is heated to a high temperature in drums and then mixed by machinery with hot asphalt cement until every particle of stone is thoroughly coated. It is hauled to the street, spread, and rolled while still hot. The mortar is made in a similar manner by mixing hot sand and asphalt cement, and is also laid and rolled while still hot. The mortar is first compressed with hand-rollers, then covered with a thin layer of hydraulic cement or limestone dust, and finally compressed with a heavy steam-roller. Under the roller the original thickness of $2\frac{1}{2}$ inches is reduced to about $1\frac{1}{2}$ inches. The cement closes the surface pores and gives it a gray color.

A sheet-asphalt pavement is smooth, impervious to liquids, and, if of pure asphalt and sand, will not become brittle if exposed to continued moisture; in hot weather it is soft and noiseless, but offers considerable resistance to the passage of vehicles; in cold weather it is incompressible, hard, offers slight resistance to the passage of vehicles, but is slippery, especially if covered with mud or with a thin film of ice. It is more easily kept clean than any other form of pavement.

The cost of sheet-asphalt pavement on a 6-inch concrete base in the city of Washington was, in 1903, $1.51 per square yard. This did not include the preparation of the street for the pavement, but included the cost of maintenance for five years. An allowance of about 10 cents per yard per year covers the cost of annual repairs and the renewal of the surface and binder when completely worn out.

Asphalt binder and mortar may also be laid on a macadam, stone-block, or cobblestone base.

The rock-asphalt pavement is made by reducing to a powder the natural rock, which is impregnated with bitumen, heating the powder to a high heat, and then spreading it over the concrete base and rolling it in the same manner as the asphalt mortar. This pavement has no binder or cushion.

Asphalt-block pavements are made of asphalt blocks laid in the same manner as other block pavements. The blocks are made by compressing in molds a hot mixture of asphalt cement, mineral dust, and crushed stone. In the city of Washington, if laid on a 5-inch concrete foundation, the blocks are made 3 inches deep and either 4×12 or 5×12 inches; if laid on a gravel base the blocks are 5 inches deep and 4×12 inches. The cost of the pavement, including repairs for five years, is about $1.80 per square yard.

An inferior form of sheet pavement is made of asphalt or tar concrete; it is not so smooth nor so durable as the sheet asphalt.

Comparison of Pavements.

Sheet asphalt made of asphalt cement and sand is the most satisfactory form of pavement, as its tractive resistance is small, vehicles move over it without noise, it is easily swept and kept clean, it is impervious to liquids, and can be easily and thoroughly repaired.

The ordinary sheet-asphalt pavement must, however, be kept *dry*, *clean*, and *in constant repair*. If these three conditions cannot be fulfilled, it should not be laid. If kept moist, the ordinary Trinidad asphalt pavement becomes brittle and porous. For this reason street-gutters are usually paved with vitrified blocks. If the pavement is not kept clean, it will be slippery in wet weather and the moist dirt will cause its deterioration. Imperfections in sheet-asphalt pavement are certain to be developed from time to time; unless the bad places are at once repaired they will rapidly enlarge. Bermudez asphalt is more durable in moist places than Trinidad. In the summer the sheet asphalt is soft and its tractive resistance is great, but the horse secures a good foothold; in winter the pavement is hard and its tractive resistance is small, but the horse cannot get a good foothold.

Wood pavement of creosoted soft or hard wood is also as smooth as sheet asphalt, less noisy in cold weather, and does not require such constant repairs. It is, however, not so impervious, nor can it be so thoroughly cleaned. If kept clean, it is less slippery than sheet asphalt and gives a better foothold to the horse in cold weather, but the contrary is true in warm weather.

Asphalt-block pavement is neither so smooth nor so noiseless as sheet asphalt or wood, but not much inferior to them. Like sheet pavement it must be kept clean, otherwise the cementing material deteriorates. It affords a better foothold for the horse than sheet asphalt, and is therefore often laid on grades considered too steep for sheet or wood pavement. It is not so durable as wood pavement, but does not require the constant repair of sheet-asphalt pavement.

Brick pavement is inferior to the pavements above mentioned only because the noise of travel over the pavement is very much greater. It is unaffected by moisture or dirt, and, although it can be easily cleaned, does not need to be kept clean. It is therefore an ideal pavement for a small town or village where only a few streets are improved. It is less slippery in cold or wet weather than the asphalt or wood pavements.

Stone-block pavements are rough, noisy, and hard to keep clean. For these reasons very few of these pavements are being laid. As the stones are very durable, and this was the only form of durable pavement in the country before 1875, it is probable that it will be long before they are wholly replaced.

For further information consult Baker's "Roads and Pavements"; Byrne's "Highway Construction"; Herschel's "Science of Road-making"; Aitken's "Road-making and Maintenance."

CHAPTER XXVI.

WATER-SUPPLY.

A COMPLETE system of water-works, designed to supply a town or city with water for domestic use, fire protection, and manufacturing purposes, consists of a *collecting*, a *purifying*, and a *distributing system*.

The *collecting system* comprises the *intakes* where the water is taken from the source of supply, the *receiving-reservoirs* in which it is stored, the *conduit* through which it flows, and the *pumping-machinery* which may be necessary to raise it from one level to another.

The *purifying system* comprises the *settling-basins* in which it is clarified, and the *filters* in which it is rendered pure and fit for use.

The *distributing system* comprises the *distributing-reservoirs* in which the filtered water is received, and the *network of pipes* by means of which it is conveyed to the points of consumption.

As will be shown later, all these elements are not found in every system of water-supply.

A system of water-supply is either a *gravity* or a *pumping* system.

A simple *gravity system* is one in which the intake is the most elevated point in the system, and no other force than that of gravity is required to move the water to and through the distributing system. The elevation of the intake should if possible be such as to produce a pressure at every point of the distributing system of about 45 to 120 pounds per square inch. If the pressure is too small, the water will not be raised to the upper floors of the buildings; if too great, it will cause leakage in the fixtures. The limits above given may be passed to meet local conditions.

Pumping-machinery is introduced whenever the intake is not high enough to produce the required pressure in the pipes, or when it is necessary to convey water over elevations over which it cannot be siphoned. The pumping-machinery may be located at the intake, or at a reservoir supplied by gravity from the intake. A pumping system is *direct* when the pumps force the water directly into the distributing system; it is *indirect* when they force the water into reservoirs from which it flows by gravity into the distributing system.

The gravity system is the most economical to operate and the least liable to be interrupted by accidents. The direct pumping system is the most expensive to operate and the most liable to be interrupted by breakages. The indirect pumping system is more economical than the direct, especially for small plants, since the reservoir may be filled by day and the pumps stopped at night; it is also less liable to be interrupted, since the distributing system can draw on the reservoir whenever the pumps need repair.

Collecting System.

Character of Supply.—The ultimate source of every supply is the rain. The water which falls on the ground either flows along the surface until it reaches the ocean, or it sinks into the soil to form subterranean streams or reservoirs of ground-water, which usually have their outlets in surface-waters at a lower level. Much of the surface-water is lost on the way to its final destination by evaporation, or by sinking into the soil and becoming ground-water. Much of the ground-water is absorbed by vegetation.

The immediate sources of supply are the *ground-water of springs* and *wells*, or the *ground-* and *surface-waters mixed*, of *rivers*, *lakes*, *streams*, and *ponds*.

Springs are the natural outlets of an inclined porous saturated stratum of soil which rests on an impervious stratum or is enclosed between two such strata. Springs are found on hillsides where a porous stratum terminates, and at points where the upper impervious layer is broken by a geological fault and the head is sufficient to force the water to the surface.

An *artesian well* is a spring formed by forcing a tube through an upper impervious stratum into a saturated porous stratum and thus making a path for the water to reach the surface.

A *deep well* is an artesian well in which the head is not sufficient to force the water to the surface when the upper impervious layer is pierced. If no very hard material is encountered, artesian and deep wells may be sunk by the water-jet process described under pile-driving. The water is forced to the bottom of the tube through an interior pipe, and not only softens the material there, but also washes out the earth between the two pipes. Through rock the hole is bored with a diamond or churn drill.

A *shallow well* is a pocket made in a porous surface layer to collect the water for removal by pumps or other machinery. This well is usually either a large masonry shaft, like the common well, or a small metal tube sunk below the surface of the ground-water. To allow the water to enter freely, the bottom of the masonry shaft is open; the metal tube, which is 2 to 12 inches in diameter, is terminated by a perforated section or *strainer* which allows the water to collect in the tube. The tube is usually sunk by fitting its base with a conical cap and then driving it like an ordinary pile; the well is therefore called a *driven well.*

An infiltration-gallery is a long chamber of masonry or crib-work which is constructed across the line of flow of a water-bearing stratum which lies a short distance below the surface. It intercepts the flow of a large area of cross-section. It may also be constructed underneath the bed or in the banks of a running stream.

Amount Required.—The amount of water consumed by any community depends primarily upon its population, and secondly upon the care which is taken to prevent waste. The amount used by factories is usually a small fraction of the total consumption.

In England, where great care is taken to prevent waste, the average daily consumption is 33 gallons per capita. The average consumption in the principal cities varies from 21 to 61 gallons per capita. In this country, in cities where meters have been introduced to measure the amount supplied to each building, the average daily consumption is about 65 gallons per capita; this varies in the different cities from 30 to 100 gallons. If no attempt is made to control the wastage, the demand will usually be limited only by the capacity of the entire system. In many cities the daily consumption exceeds 200 gallons per capita, and in a few small communities the consumption exceeds even 300 gallons per capita. In this country, however, unless the conditions are exceptional, an allowance of 100 gallons per capita daily should be sufficient for any community in which the water furnished to each consumer is measured.

The daily, weekly, and monthly consumption of water are by no means uniform. In warm weather much water is consumed in watering gardens and lawns; in cold weather much is allowed to run to waste to prevent the freezing of the pipes in the buildings. In this latitude, due to these causes, the maximum monthly consumption may exceed the mean monthly consumption for the year by 33 per cent; the maximum weekly consumption may exceed the mean weekly for the year by 50 per cent; the maximum daily consumption may exceed the mean daily consumption for the year by 100 per cent. If, therefore, 100 gallons is considered a sufficient average daily allowance per capita throughout the year, provision must be made in the system to allow this to be increased to 133 gallons in a single month, to 150 gallons in a single week, and to 200 gallons in a single day. The hourly consumption is also variable; the consumption for a single hour is sometimes 40 per cent more than the average hourly consumption for the day.

The variations from the mean consumption are very much greater in small systems than in large ones; and larger in unmetered systems than in metered ones. The volume required by a steam fire-engine during a conflagration depends upon the character of the engine; it usually varies between 400 and 1200 gallons per minute; the amount thus drawn may be a heavy tax on a small system.

Measuring the Supply.—If the source of supply is a spring, well, or small stream, the volume of the supply can be ascertained at any time by noting the time required to fill a reservoir of known capacity; or by noting the time required to restore, in a pool, a volume which has been removed by buckets of known capacity.

The volume of discharge of a stream too large to be measured in this manner may be gauged by causing the discharge to take place through an orifice or over a weir.

A large river is gauged by measuring its mean velocity and its area of cross-section.

Variation in Flow of Streams.—Because of the irregularity of the rainfall, the *discharge or run-off* of any drainage-basin or *catchment area* is a variable and not a constant quantity. In order that there may be no water-famine even in the period of extreme drought, the system of supply must be based on the *minimum discharge* of the stream which drains the basin.

The most reliable method of determining the minimum discharge is to measure the daily discharge for a long series of years. The *approximate* methods which are ordinarily resorted to are either to estimate the discharge from the measured discharge of another basin subject to similar climatic conditions, or to estimate the discharge from the rainfall, employing the ratio of discharge to rainfall determined in some basin having similar climatic conditions.

The Sudbury watershed near Boston, on which careful observations have been made since 1875, is the standard employed for estimating water-supplies from watersheds east of the Mississippi River. This basin has an area of about 75 square miles, of which $6\frac{1}{2}$ per cent is water surface; the basin is generally hilly. The evaporation from the water surfaces in the basin is about equal to the rainfall.

From the records of this basin it appears that—

1. The *run-off* from the basin which may be utilized for water-supply is about one-half the measured rainfall; or approximately 1,000,000 gallons per day from each square mile of the basin.

2. In a year of extreme drought the rainfall is two-thirds of its mean, and the discharge or run-off one-half of its mean.

3. For five years in succession the rainfall and run-off may be less than the mean; for three successive years the rainfall may average only 80 per cent of its mean, and the run-off 70 per cent of its mean.

4. The mean monthly rainfall is fairly uniform in this basin, being a minimum of 2.98 inches in June, and a maximum of 4.57 inches in March. The mean annual rainfall is 45.83 inches.

5. The average monthly run-off, on the contrary, is variable. It is equivalent to a depth of 5.17 inches over the basin in March,

and only 0.35 inch in July. The run-off during February, March, and April is 50 per cent of the total annual run-off, and that from November 1st to May 31st is 85 per cent of the annual run-off.

6. The minimum rainfall and the minimum run-off in a month may be only 10 per cent of their mean values.

From observations on other watersheds it appears that the ratio of the run-off to the rainfall decreases as the amount of annual rainfall decreases. Hence in a basin in which the rainfall is less than in the Sudbury basin the run-off will be less than one-half the measured rainfall.

The difference between the run-off and the rainfall is due principally to evaporation and absorption by vegetation; water may also be lost by escaping from the watershed by subterranean channels.

The annual evaporation from water surfaces depends on the local hygrometric conditions. In the United States the annual evaporation for places having an annual rainfall between 30 and 50 inches may be either somewhat greater or somewhat less than the rainfall, depending on local conditions; if the rainfall is very great, the evaporation is relatively small, and vice versa. At Astoria, on the Pacific coast, the rainfall is 77 inches and the evaporation is 25 inches; at Cheyenne, Wyoming, the rainfall is 13 inches and the evaporation 76 inches.

In calculating the run-off of a watershed where the rainfall and the evaporation are equal to each other, the area of all ponds, lakes, streams, etc., must be subtracted from the total area.

The amount of water absorbed by the vegetation will vary from 10 to 15 inches if fed by the rainfall; some irrigated fields can absorb at least ten times as much.

Variation in Flow of Springs and Wells.—The irregularity of the rainfall, especially its inequality over long periods of time, as seasons and years, must affect the underground streams and reservoirs in the same way that it affects the run-off of watersheds. The underground storage will be a maximum when the greater part of the annual rainfall occurs in the months when the soil is porous and the loss by evaporation is least. A mild wet winter is favorable to the undergound flow and storage of water, and a dry cold winter is unfavorable to it.

The flow of a spring or well is not alone dependent on the amount of rainfall or the drainage area from which it receives its supply; it is also dependent on the *extent* of the underground area drained by the spring or well, and by the *velocity* of flow toward the outlet.

Any difference of level between two points of a continuous water surface, whether above or below the surface of the ground, will cause a flow of water from the higher to the lower point; the velocity of the flow will vary directly with the difference of level between the two points, and will vary inversely with the resistance to the flow.

The flow of a spring or well may be increased by increasing the depth of its water-level below the general surface of the ground-water. This will increase both the area drained and the velocity of flow towards the outlet. In an open porous water-bearing stratum the effect will be greater than in a compact one, since the resistance to the flow of the water through the soil is less. As a slight difference of head will increase the flow, the supply from springs and wells, in which the water-level may be lowered by pumping, is practically constant except in seasons of extreme drought.

Receiving-reservoirs.—A reservoir is an artificial basin in which water is temporarily stored. Those belonging to the collecting system are called *receiving-reservoirs;* those to the distributing system are called *distributing-reservoirs.* All are also called *storage-reservoirs.*

In a gravity system the purpose of a receiving-reservoir is to store the water flowing from an irregular source of supply and secure uniform daily flow into the distributing system greater than the minimum daily flow from the source of supply. In a pumping system it may also be employed to store the water which is pumped by an intermittent-acting pumping-plant, and secure a uniform flow into the distributing system from the receiving-reservoir.

A receiving-reservoir is unnecessary only when the source of supply is a great river or lake whose minimum daily capacity is much greater than the maximum daily consumption; or when the source of supply is practically constant, as in the case of wells and springs.

If the source of supply is the flow from a drainage-basin in any of the States bordering the Atlantic Ocean, the average daily supply which can be collected from a square mile of area is about 1,000,000 gallons. In a year of extreme drought, however, the average daily yield may be reduced one-half, or be only 500,000 gallons. It is evident that if the distributing system requires more than 500,000 gallons daily from each square mile of the basin, there will be a water-famine during this year of drought, unless sufficient water has been stored in reservoirs to provide for this deficiency. It is also apparent that the nearer the daily consumption is to 1,000,000 gallons the greater must be the volume of water stored. If this year of extreme drought is preceded by one or more years in which the average daily yield is also less than the average daily consumption, an additional supply will be needed for these years.

As it is not desirable to have reservoirs partially empty for too long a period, and as several years of drought may follow each other, it is usual to limit the maximum daily consumption to about 60 to 75 per cent of the average daily capacity, or to 600,000 to 750,000 gallons daily per square mile of the drainage-basin. Reservoirs having a capacity of 200 to 250 days' supply will then be sufficient to supply the deficiency of three successive years of drought. Even under favorable conditions it is difficult to secure sites for reservoirs to store greater volumes.

If the reservoirs are made by damming one or more of the streams of a basin, as is usually the case when reservoirs of large capacity are required, they are also called *impounding-reservoirs.*

If the average daily consumption is less than the average daily supply during the year of extreme drought, it will still be necessary to have receiving-reservoirs, if the maximum daily consumption is more than the minimum daily flow which has been recorded or may be expected in any summer or autumn month. From the Sudbury basin the average daily flow in July is only about one-fifth of the average daily flow for the year, or 200,000 gallons daily per square mile. In a year of extreme drought this may be reduced to one-tenth of this amount, or 20,000 gallons daily. If, therefore, the daily consumption exceeds 20,000 gallons per square mile of drainage area, some provision must be made for storage. To provide for an average daily consumption approaching 500,000 gallons, the total excess yield of the winter and spring months must be stored for use in the summer and autumn. The maximum storage required to equalize the flow for a single year is about 75 days' supply.

If pumping-machinery is employed to raise the water from a constant supply to a receiving-reservoir, the latter need hold only a few days' supply to allow the engines to work intermittently, and as a safeguard against the temporary stoppage of the pumps due to accidents and breakages. Reservoirs of this type are made on some convenient elevation near the pumping-plant or purifying system.

Dams.—The dams for impounding-reservoirs are made of earth or masonry.

An earthen dam is usually of the type shown in Fig. 168.

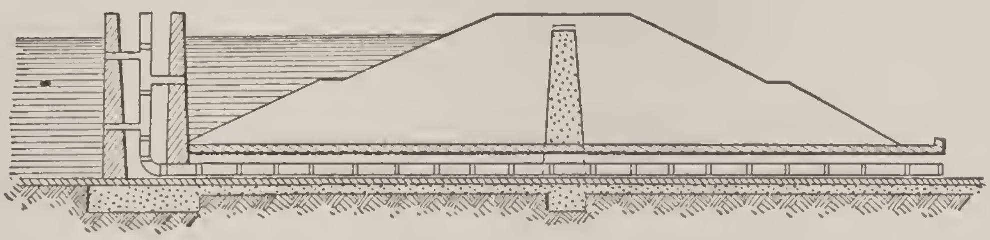


Fig. 168.

In the construction of an earthen dam care must be taken to prevent leakage through the dam itself, to prevent the water of the reservoir from rising to the height of its crest, and to prevent leakage underneath the dam or around its ends.

Leakage through the dam is prevented by making a wide embankment with gentle slopes of carefully selected, thoroughly compacted material; or by constructing a thin impervious *core-wall* of puddle or masonry to resist leakage, and supporting this wall by an embankment of less carefully selected material (Fig. 168).

The water is prevented from rising to the level of the crest of the dam by constructing a spillway of sufficient capacity to discharge the water which flows into the reservoir at the time of maximum flood.

Leakage under the dam and around its ends is prevented by removing all porous material and making a firm bond between the non-porous material of the site and that of the dam.

A dam without a core is from 10 to 30 feet wide on top, depending on its height. The inner slope is 1/2, or if the material is soft, 1/3; this slope is usually paved or riprapped with stone. The outer slope is 1/2, and is sodded to prevent its destruction by the rains. Both slopes, if long, are broken by berms. The material of the dam is of the same character as that employed in the coffer-dam, and is laid in thin layers which are moistened and then compressed by driving animals or moving grooved or studded rollers over them.

A *masonry core* is a concrete or stone wall about 2 to 4 feet wide at the water surface, with a batter of 24/1; it is firmly imbedded in the impervious stratum upon which the dam rests. It must be water-tight. A *puddle core* is a similar wall of puddle, but is made twice as thick. The puddle wall is inferior to the masonry one in that it is more liable to crack and is not proof against burrowing animals. For these reasons the layers of the embankment immediately in contact with the faces of the puddle core are usually of selected material. In a dam with a core the main function of the embankment is simply to support the core; it may therefore be of less carefully selected material than in the dam without a core. When random rock is employed to support the core the dam is called a *rock-fill dam*.

Cores have been made of steel plate supported by a framework, and they have been replaced by an impervious apron of timber, concrete, or metal placed over the inner slope of the dam.

To prevent the leakage which would be liable to occur at the discharge-pipe which passes through the dam, especially if it settles, this pipe is laid on a bed of concrete resting on the impervious stratum beneath the dam. The pipe itself is then imbedded in concrete, and this covering of concrete has projecting rings to increase the resistance to leakage along its surface. The pipe may also be laid in a small culvert which is constructed on the impervious layer, passes underneath the dam, and terminates in a vertical masonry tower, called a *valve-tower* or *gate-house*. A pipe thus laid is subject to constant inspection. (Fig. 168.)

Spillway.—One of the most important features of an impounding-reservoir having an earthen dam is the *spillway* or *waste-weir* through which the surplus water is discharged when the reservoir is full. If this outlet is too small, in time of flood the water will rise above the crest of the dam, and flowing over it will soon cause its total destruction. If placed in the dam itself and not properly constructed, the spillway itself may be washed out and thus destroy the dam.

The length of the crest of the spillway, determined by the formula on page 302, should be such that it can discharge the maximum flood without causing the water in the reservoir to rise to a plane above the top of the core-wall or 3 feet below the crest of the dam.

Various empirical formulas have been devised for the maximum flood discharge; the most common is Fanning's formula,

$$D = 200A^{\frac{5}{6}},$$

in which D = the discharge in cubic feet per second,
A = the area of the basin in square miles.

In this formula no factors are introduced which depend on the shape, slope, or character of the surface of the basin, although all these must affect the maximum discharge. It is thought to give too small a discharge for rapidly discharging basins in which the area is less than 10 square miles. The Burkli-Ziegler formula previously given is also used.

The spillway should if possible be constructed in the solid ground beyond one end of the dam, and the water carried in a separate channel so far away from the dam that the backwater will not come in contact with the earth embankment. Unless this channel passes over a ledge of rock it should be paved to prevent scouring. If the spillway must be made in the dam itself, the part of the dam containing the spillway should be of masonry; the channel for the water should be so inclosed that the water cannot reach the earthen dam. Spillways, in unimportant dams, are made of cribwork filled with stones and covered by a plank sluiceway through which the water flows. The length of the weir of the spillway may also be determined less accurately by the Gould formula

$$l = 20\sqrt{A},$$

in which l = length in feet,
A = area of basin in square miles.

Masonry and Rock-fill Dams.—A rock-fill dam is one whose stability depends on a wall of random rock. The faces of the wall are given the natural slope of random rock dumped in place. Leakage may be prevented by covering the upper slope with an apron of well-calked planks or a sheet of steel plate, by a facing of concrete or masonry, by a thick layer of earth either above or below the rock dam, or by a core of masonry, reinforced concrete, or steel plate. Leakage under the dam is usually prevented by the construction of a wall connecting the apron or core with the bed-rock. This type is employed in the construction of irrigation-reservoirs in our own country.

Very high impounding-reservoir walls are usually made of masonry, resting on a natural rock surface. The principles governing their construction have already been discussed. The entire wall or a portion thereof may be used as a spillway. If the water wastes over the dam itself, the dam is usually made of the form shown in Fig. 169; this is called an *ogee-faced* dam. The water follows the contour of the face and is discharged at the base parallel to the river-bed. This reduces to a minimum the tendency to erode the bed.

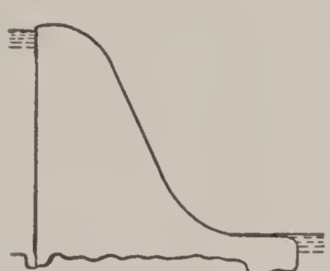


FIG. 169.

Valve-tower, Gate-house, and By-pass.—A *valve-tower*, such as shown in Fig. 168, allows the water to be drawn from different levels of the reservoir; the water can be screened, however, only at the inlets. The screens or sieves cannot therefore be readily examined.

A *gate-house*, whose exterior is similar to a tower, is usually divided into two vertical chambers. In the separating partition is an opening in which are placed a pair of movable vertical screens. The water enters the up-stream chamber through sluice-gates in its walls at different elevations, and after passing through the screens escapes through the discharge-pipe. Either screen can be removed at any time for cleansing.

In both tower and gate-house there is usually a separate waste-pipe near the bottom for emptying the reservoir.

The by-pass is a conduit connecting the inlet-pipe with the discharge-pipe so that the water may pass around the reservoir when it is emptied for any purpose.

Intake.—If the source of supply is a stream, a dam is usually constructed at the intake so that the level of the water will be constant, and the inlet may be placed well below the level of the ice; the end of the conduit is covered by a suitable netting to prevent ingress of materials which might obstruct the flow through it. If the pool at the intake is a large one so as to be only slightly affected by the suction of the conduit, there will be little danger of the obstruction of the conduit by the needles of ice called *anchor-ice.*

In the great lakes on our Canadian boundary the intakes are located in strong cribwork shafts which are far enough from the shore, if possible, to secure water unaffected by the city sewage.

Conduits.—The main conduits of a system of water-supply may connect the intake directly with the purifying or the distributing system, or the impounding-reservoirs with either of the above systems. These may be constructed of masonry, steel, cast iron, or wood. Large conduits not exposed to internal pressure are usually made of masonry; they are simply covered canals, and are constructed with a uniform grade of slight inclination. The Croton Aqueduct of New York, which connects the impounding-reservoir at Croton with the receiving-reservoir in New York City, is a masonry conduit whose cross-section is either a circle whose diameter is $12\frac{1}{4}$ to 14 feet, or an equivalent horseshoe section; the uniform slope of the larger section is seven-tenths of a foot per mile. Large conduits which are exposed to considerable internal pressure are made in the form of riveted steel pipes; conduits from 4 to 6 feet in diameter have been thus constructed. Cast-iron pipes are more commonly employed than any other form, and are manufactured in sizes varying from 4 inches to 5 feet in diameter. Conduits made of wooden staves with steel bands are often constructed where suitable lumber is cheap and the pressure in the conduit is not excessive; conduits 6 feet in diameter have been made of wood.

Pumping-machinery.—Pumps are of a great variety of forms, depending upon the volume of water to be raised in a unit of time, the head or back pressure against which they must work, and the depth of the water-level below the surface of the ground. The back pressure is constant if the water is pumped into a reservoir, and variable if pumped directly into the distributing-pipes. One or more reserve pumps are usually installed to provide for emergencies; these are absolutely essential in a direct pumping system.

Water is sometimes lifted from wells to the surface by forcing compressed air to the bottom of the vertical collecting-pipe through a small interior pipe. The mixed air and water rise to the surface in the outer pipe.

Purifying System.

The object of the purifying system is to remove the inorganic material in suspension which makes the water turbid, the mineral matter in solution which gives it undesirable qualities, the organic substances in suspension which are the food of pathogenic bacteria or disease organisms, these organisms themselves, and finally the organic matter which colors the water.

The first is principally the mud found in streams derived from surface drainage; the turbidity of such streams is exceedingly variable, being very great after severe rain-storms and very slight in times of drought. Mud may be removed by allowing the water to remain quiescent in settling-basins for a long period without the assistance of coagulants, or to remain a short time with coagulants. The process of clarifying may also be partially made in the settling-basins, and completed in the filters.

The inorganic properties in water, such as lime and iron, which give it undesirable qualities, are removed by special processes as described hereafter.

The organic substances which furnish food for pathogenic bacteria and the bacteria themselves are to a great extent removed in a settling-reservoir if the water is allowed to remain in it long enough, perhaps for a month or more; as a rule, however, to remove bacteria the process of sedimentation is followed by that of filtration. As it is impossible at present to destroy the pathogenic bacteria without at the same time destroying the harmless varieties, the purity of the water is measured by the total number of bacteria in it If the number of bacteria in a cubic centimeter does not exceed one hundred, the water, while not absolutely pure, is considered healthful within reasonable limits. This result is obtained in ordinary river-water only by the destruction of over 99 per cent of the bacteria in it.

The organic coloring-matter is usually removed in the processes of sedimentation and filtration.

Selection of Sources of Supply.—As it is impossible to remove all the bacteria from water, a source of supply should be selected which is not liable to contain pathogenic bacteria. Waters have been classified as follows:

1. Water from springs, deep wells, and the surface drainage of uncultivated and unpopulated lands are classed as *wholesome.*

2. The surface drainage of cultivated and sparsely populated land, and rain-water drained from roofs, are classed as *suspicious.*

3. Waters from rivers into which sewers empty, from shallow wells, and from fields enriched by night-soil are classed as *dangerous.*

Ground-waters, except those of the last class, are usually wholesome as far as bacteria are concerned, but are liable to contain inorganic substances which may be objectionable or even dangerous. Ground-waters are also more liable than surface-waters to nourish vegetable growths which give the water an offensive odor or an objectionable taste. These may be prevented by keeping the water under cover from the time it reaches the surface of the ground until it passes into the distributing mains.

Settling-basins.—The term settling-basin is ordinarily applied to a reservoir which contains from one to four days' supply of water; in such a reservoir the water derived from surface drainage is made clear enough to prevent it from clogging the filter-beds. The bed of the basin is of concrete, so that it can be thoroughly cleaned when necessary. The depth of the water in the basin is usually from 8 to 15 feet. A less depth does not prevent the growth of vegetation, and a greater depth makes the time required for sedimentation too great. Sedimentation covering such a short period has no important effect in reducing the number of bacteria.

The flow of the water through the basins may be either continuous or intermittent. In a continuous system, if there is but a single small basin, the water enters near the bottom of one end, moves across the basin with a very slight velocity, and leaves the basin near the top of the opposite end. The reservoir can be cleaned only by conducting the water through a by-pass or pipe which connects the conduit above with the conduit below. The water may be compelled to pursue a serpentine course through the reservoir by inserting vertical plank partitions called *baffles*.

To clean the basin a day is selected upon which the water received from the source of supply is unusually clear. The inlet into the reservoir is closed, and the water is allowed to flow around it through the by-pass. The basin is then emptied by means of an outlet constructed for this purpose. If there are two or more basins, the water flows through them all except when it is desired to isolate and empty one of them. The water then flows around it through its by-pass.

In an intermittent system the basins are arranged in series of three. One of the three is the receiving-basin. Another is the settling-basin, in which the water remains quiescent while the first is being filled. The third is the basin from which the settled water flows into the distributing system.

If there is a large receiving-reservoir in the system, it will perform all the functions of a settling-basin better than a special basin, as the water remains in it a longer time and its depth is usually greater than that of a basin. Such a reservoir has also the effect of materially reducing the number of bacteria.

The turbidity of water is tested by noting the distance below the surface at which some standard object can be seen. This object is usually a fine platinum wire fastened at right angles to the axis of a rod graduated to millimeters.

Filters.—The object of filtration is to complete the purification of the water begun in the settling-basin or the receiving-reservoir. Filters are often divided into two classes; one is the *slow*, *sand*, or *English* filter, and the other is the *rapid*, *mechanical*, or *American* filter.

Slow Filters.—The ordinary slow filter is a basin into which the water flows through an inlet above the filter-bed, and from which it escapes through an outlet below the filter-bed. The basin may be open or covered. In a cold climate it is usually considered advisable, though not absolutely necessary, to cover the basin and thus prevent the water from freezing; the removal of the ice is expensive and the filter works less thoroughly when the water is very cold. In a warm climate some authorities prefer a covered basin because there is less vegetable growth in it, while others prefer the action of the sun-rays on the water because of its bacteria-destroying effect.

The basin is made in two or more divisions, so that a single division may be emptied and cleaned without interfering with the filtration process.

The slow filter as designed by Mr. Hazen, C.E., for Albany, Washington, and other places, consists of a basin with a bed and vertical side walls of concrete. The roof consists of groined arches resting on pillars of brick or concrete. The pillars are about 9 feet high and 20 inches square, and are placed in rows which are 14 feet apart in each direction. A section through two

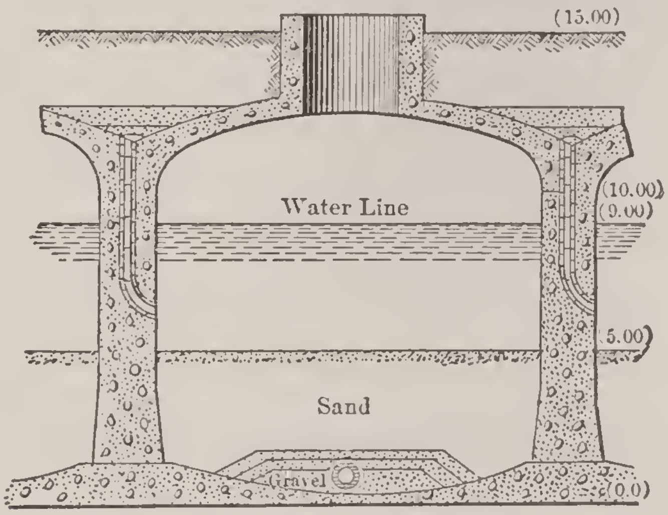


FIG. 170.

pillars is shown in Fig. 170. By substituting a continuous wall for a row of pillars, the basin is divided into two or more separate parts. The roof of each groined arch is pierced by a manhole for light and for conveying material; the concrete roof is covered with earth.

The filter-bed proper consists of a bed of sand about $3\frac{1}{2}$ feet thick. The specifications require that the sand shall be fine; no particle must be over 5 mm. in diameter, and 70 per cent must not exceed 1 mm. in diameter. Around the collecting-pipe is a bed of gravel varying in size; the finest gravel is near the sand. The gravel prevents the sand from being washed away.

Each division of the filter has a large collecting-drain which is laid below the level of its floor and bisects its area. Upon the floor of the filter and at right angles to the main drain are the branch drains which collect the filtered water. The main and branch drains are stoneware pipes; the latter are laid with slightly open joints. From the main drain the water passes by gravity or is pumped to a covered *clear-water basin.* Fig. 171 shows one chamber of a filter, in which the heavy black line is the main drain and the lighter cross lines are the tributary drains. The roof is covered by a layer of earth about two feet thick. The top of the covering is usually sodded.

To prevent the disturbance of the bed of sand by the force of the current from the inlet, the inlet is in a small chamber separated from the main floor-area by a low wall over which the water flows; the wall checks its velocity. To prevent disturbance when the filter is started after being cleaned, provision

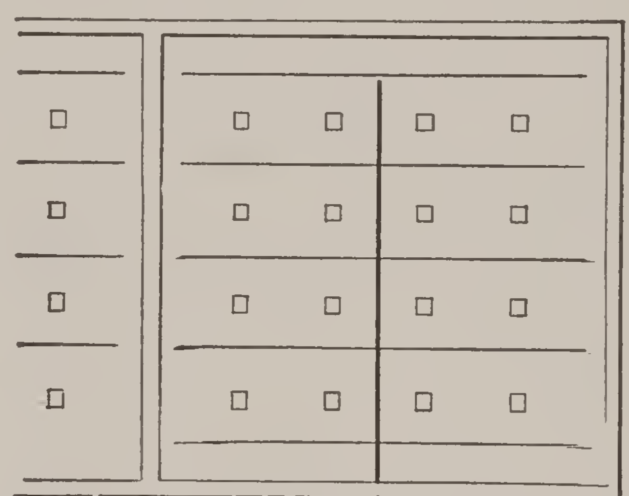


FIG. 171.

is made for allowing filtered water to enter from the drains. Regulators are also provided for controlling at all times the head or depth of water over the filter-bed.

The filter is started by admitting water from below until the bed is flooded; the water is then allowed to enter through the regular inlet. The head is regulated to make the rate of filtration such as will produce an effluent of the desired purity. The rate which is usually considered safe is a velocity of flow vertically through the filter-bed of 4 inches per hour, or 8 feet per day. This is equivalent to about 2,600,000 gallons per acre per day. Experiments have shown that this rate may be increased for clear and slightly polluted waters to 6,000,000 gallons per acre per day without producing an effluent which is bacterially impure.

After a time the surface of the sand becomes covered by a gelatinous coat which reduces the velocity of flow and requires the head to be increased. When the limiting head has been reached, and the rate is still further reduced, the filter is emptied and scraped. In this operation the surface layer to a depth of an inch, more or less, is removed and the filter is again ready for work. The scrapings are carefully washed, and when the thickness of the sand is reduced to about 2 feet the washed sand is again placed in the filter.

The process of the filter is both mechanical and chemical. The large organic and inorganic bodies are removed as by a sieve; the minute organic substances are converted into their elements, principally at the gelatinous coat, by the aid of bacteria; the bacteria disappear with the substances upon which they thrive. The gelatinous coat is therefore considered the most valuable feature of the filter, and care is taken not to rupture it by a too rapid change of velocity of flow. The bacteria engaged in the reduction are dependent on oxygen. If the supply of oxygen in the water is sufficient, the process of filtration may be continuous; if the supply is not sufficient, the process is intermittent, and the filter is emptied at regular intervals to allow the air to penetrate the pores of the bed.

To prevent the rapid clogging of the sand-bed, the water is sometimes passed through a preliminary coarse filter or screen to remove the coarse particles. This is called a *scrubber*.

Rapid Filters.—Filtration as above described is necessarily a slow process, and a large area is required for the filter-beds. To increase the rapidity of filtration, filters have been devised in which a coagulant is employed to remove the inorganic materials, and the sand is aerated by mechanical means. The coagulant, usually sulphate of alumina, forms a gelatinous coat over the sand, which performs a similar function to that in the slow filter. The water is forced through the filter by its natural head if sufficiently high, otherwise by pressure. When the flow is checked by the sediment on the bed, the sand is washed by reversing its current and at the same time stirring it by a current of air or by mechanical means. When thoroughly washed the process of filtration is repeated. In the rapid filter, therefore, the process is intermittent. The rate of filtration is about 125,000,000 gallons per acre per day, or many times that of the slow filter. Many of the mechanical filters are in the form of wooden or metal tanks having a filter-bed whose area is about 10 square feet. The sand is coarser than that employed in the slow filters and rests upon some form of sieve. If the water is very turbid, it is clarified to some extent in a clarifying-tank before it is allowed to enter the filter proper. Care must be taken to regulate the supply of alum so that there shall be no free sulphuric acid in the water; sometimes lime is added to precipitate the acid.

Iron may be removed from water by first oxidizing it by aeration and then passing it through a sand filter. This process is employed to purify water from wells. If the water does not contain organic matter, the process may be a rapid one.

Temporary hardness in water is due to the presence of calcium carbonate in water which contains free carbonic acid. The carbonate is soluble only when the carbonic acid is also present, and may therefore be precipitated by any process that removes the acid. The carbonic acid may be removed from small quantities by boiling the water; from large quantities, by adding lime-water, with which the carbonic acid can unite and form additional carbonate and thus remove the free carbonic acid. The carbonate will then be deposited. Filters have been designed for expediting this process, which is usually a very slow one.

Distributing System.

Distributing-reservoir.—A distributing-reservoir usually serves one or more of the following objects:

1. It is a source of supply for the distributing system to prevent a water-famine in case of accident to any part of the system of supply between the intake and the distributing-reservoir. In a pumping system it is perhaps more necessary in this sense than in a gravity system. A reservoir of three or four days' supply is usually considered of sufficient capacity for this purpose.

2. It is a basin for the storage of the filtered water which flows from the purifying system. A reservoir of this type is also called a storage-reservoir. To reduce the probability of contamination of the water in a storage-reservoir to a minimum, the reservoir is usually limited to a day's supply or less, and the reservoir is wholly inclosed and covered.

3. It allows the hourly consumption to be variable, although the hourly supply which flows from the receiving-reservoir or from the purifying system is constant. As stated before, this hourly consumption may be nearly 40 per cent in excess of the mean hourly consumption. To furnish a reservoir for this pur-

pose the capacity of the reservoir is based on the hourly variation on the day of maximum daily consumption. In small plants the maximum hourly consumption is at the time of a large fire, and the reservoir is constructed to meet the emergency. A reservoir to meet the hourly variation in the consumption need never contain a maximum day's consumption.

Water-towers and Stand-pipes.—In pumping systems the distributing-reservoir may be a small tank placed on a skeleton steel tower, or it may be a vertical cylinder of riveted steel plate, called a stand-pipe. The object of the former is simply to regulate the back pressure, while the latter is in addition a small reservoir. Both the filling and discharging pipes enter the cylinder at its base. If the water-supply belongs to the class designated as wholesome and the receiving-reservoir is a large one, settling-basins and filters are often omitted and the receiving-reservoir serves also as a distributing one.

Water-mains.—The system of distributing-mains should fulfill the following requirements:

1. Every part of the system should be strong enough to resist the static head of the distributing-reservoir.

2. The hydraulic head at every point of the system should at all times be sufficient to raise the water to the desired height in the adjoining buildings.

3. The water in every part of the system should be in constant motion, and should not be allowed to become stagnant.

4. If a break or leak occurs in any main, it should be possible to stop the flow through a short section in the immediate vicinity of the break without interfering with the supply in other parts of the system.

5. It should furnish an abundant supply to fire-plugs placed at intervals along the mains.

These requirements are fulfilled in general by laying, in the streets of a city, a network of connecting-mains, thus forming an underground reservoir of considerable capacity, Fig. 172. The smallest of these mains has a diameter of at least 4 inches. The aggregate area of cross-section decreases with the decrease of the area to be supplied.

If in the low part of the city the hydraulic head is too great, it is decreased by partially closing a valve in the main which supplies it, or a special valve may be introduced to cut off the supply when the hydrostatic head reaches a certain amount.

If in a high part of the city the head is too small, it may be increased within limits by supplying that section by a special main of large cross-section. If this does not give the desired head, the section is provided with a special pumping system.

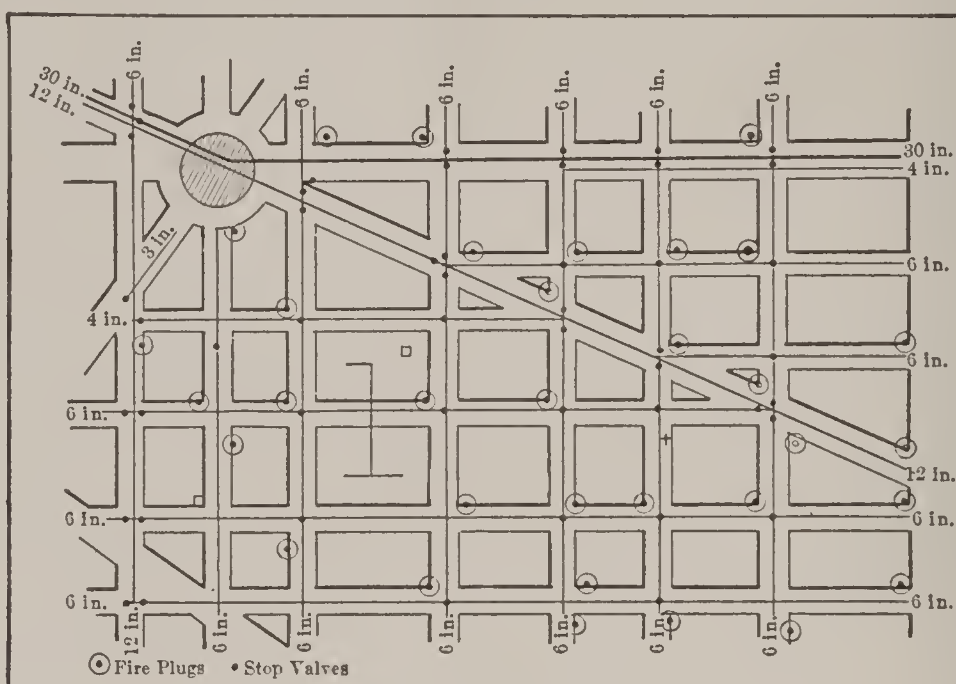


FIG. 172.*

This is called the *high service*, in contradistinction to the *low* or *gravity service*.

By connecting the ends of the mains, the stagnant water which is always found in dead ends is avoided. By connecting the mains at intervals the head is made more uniform, and it is possible to stop the flow in a short section of a main without materially affecting the supply along the other main. It also increases the volume which may be drawn in a given time from any point in the system, as at a fire-plug.

The water-mains of cities are usually cast-iron pipes with bell and spigot joints which are closed with hemp and lead. Valves are inserted at intervals so that any part of a main can be temporarily closed at both ends in order to make necessary repairs. The valves are vertical leaves which are lowered by means of a screw; this prevents their being closed rapidly. If the flow of water through a pipe is suddenly checked, the pipe is subjected to a severe blow from the *water-hammer* thus formed. The

* A plan of a part of water-main system in Washington, D. C.

intensity of the blow varies directly with the volume and velocity of the flowing water.

Small house connections are generally lead or galvanized-iron pipes, usually about $\frac{3}{4}$ inch in diameter, which can, by a special device, be attached to the main without stopping the flow of water.

Fire-plugs are attached to short branches connected with the street-mains at the street-corners and in the interior of long blocks, and also for flushing purposes at the dead ends of the mains. These have studs to which the fire-hose can be attached.

Blow-off.—A blow-off is a short branch with a suitable valve, through which the water in the main can be discharged. Blow-offs are placed at low points and at the dead ends of conduits and mains to force out deposits which may collect there.

Air-escape Valves.—These valves are placed at the high points of conduits to remove the air which may collect there and interfere with the flow.

Water-meters.—Meters of different patterns are employed for measuring the house consumption; they are attached to the service-pipes. The *Venturi* meter is one which may be employed for measuring the flow under pressure through a main or conduit of any size.

For further information see Merriman's "Hydraulics," Bovey's "Hydraulics," Folwell's "Water-supply Engineering," Burton's "Water-supply of Towns," Turneaure and Russell's "Public Water-supplies," Goodell's "Water-works for Small Cities and Towns."

CHAPTER XXVII.

SEWERAGE.

THE term *sewerage* is applied to the removal of the liquid and soft solid waste products of a community, through a system of conduits, by means of its water-supply. The conduits are the *sewers*, and the waste material which flows through them is the *sewage*.

The principal sources of the sewage are the household and factory wastes, and the surface drainage after rainfalls. If the household and factory wastes are carried in one system of conduits and the surface drainage in another, the sewerage system is a *separate* one; if the wastes and drainage are carried in the same conduits, the system is a *combined* one.

The principal advantages of the separate system are:

1. Sewers for carrying the household and factory wastes may be constructed while the surface drainage is still carried in surface drains; the storm-water sewers may be added at a later period. This reduces the first cost of the system.

2. If the sewage must be treated before it is finally discharged into streams and lakes, the separate system is the more economical, since only the sewage of the house-sewers must be treated; that of the storm-sewers can be discharged without treatment. In the combined system the entire sewage must be treated until the dry-weather flow is sufficiently diluted by storm-water to allow it to be discharged without danger of creating a nuisance.

3. In the separate system only the house-sewers need be deep enough to drain the basements of houses; the storm-sewers need only be deep enough to remove the surface drainage. In the combined system all the sewers must be deep enough to drain the basements.

The principal advantages of the combined system are:

1. Only a single system of conduits is in each street. This prevents confusion and possible errors in connecting houses with sewers.

2. The periodical flushing of the sewers by the storms cleans them thoroughly. In the separate system reliance must be placed on automatic flushing-tanks, assisted by connecting small drainage areas, as roofs and yards, with the house-sewers.

The separate system is usually better than the combined one for small communities whose surface drainage can be carried largely in open drains, and for all communities whose sewage must be treated before it can be discharged into the natural drainage streams of the country. It may also be advantageous when the sewage must be pumped to its final destination.

The combined system is usually best for a large city, whose sewage may be discharged without treatment into the ocean or into a large river.

Household Wastes.—The household wastes constitute the most important element of the sewage because of their offensive character and rapid decomposition. The aim of a sewerage system is:

1. To remove in closed conduits from each house, and from the limits of the community itself, all household wastes before their decomposition begins.

2. To seal every opening in the house-pipes and often, in addition, the connection between the house-pipes and sewer, so that no gaseous products of decomposition generated in the pipes or sewer can enter the house.

3. To make the sewers water-tight to prevent both the pollution of the surrounding ground by sewage and the infiltration of ground-water.

4. To ventilate the sewers or otherwise remove offensive odors.

5. To finally dispose of the sewage in a manner which will not produce a nuisance, or a menace to public health.

House-drainage.

The general system of house-drainage (Fig. 173) consists of one or more vertical soil- and waste-pipes which receive the wastes from the fixtures and which empty into the house-drain.

The *house-drain* is usually an extra-heavy cast-iron pipe with leaded joints. For ordinary houses it is 5 inches in diameter and has a fall of at least 1 inch in 4 feet. If the basement is to be drained, the drain is laid on a bed of concrete, true to grade, below the floor of the basement. If the basement is not to be drained, the house-drain may be supported on masonry pillars, on brackets attached to the walls, or by stirrups attached to the floor-beams. Steel and wrought-iron drain-pipes are allowed when the drains are above the basement floor. The house-drain is provided with cleaning-holes at intervals, and empties into a stoneware pipe called the *house-sewer*, a short distance beyond

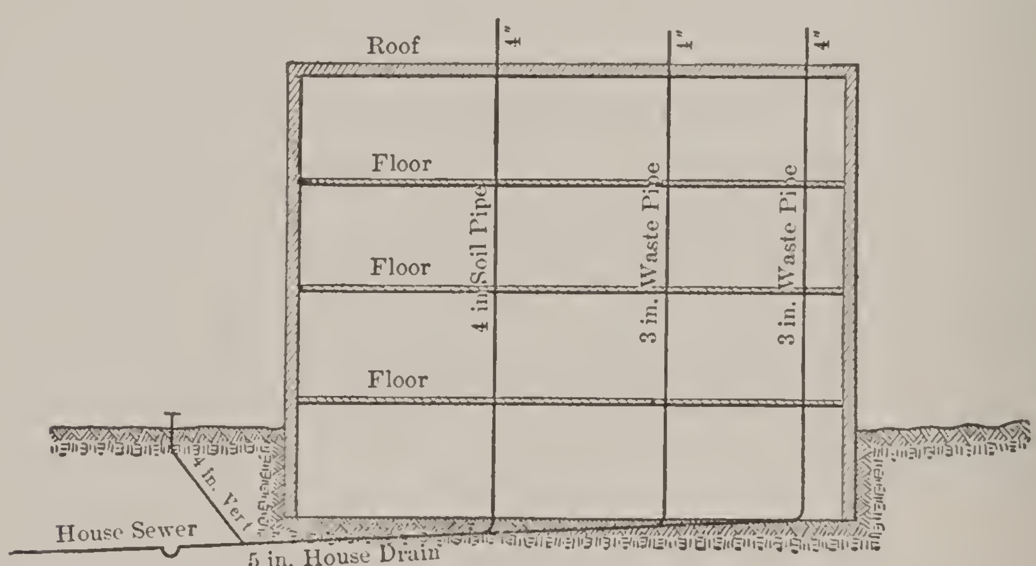


FIG. 173.

the exterior walls of the house. A *soil-pipe* is a vertical pipe which receives the discharge of a water-closet. It is a heavy metal pipe, usually cast iron, which extends from the house-drain to a point about 2 feet above the roof. At its base it is supported on a masonry pier; its upper end either is left open or is closed merely by a wire cage to prevent the ingress of materials which might obstruct it. For ordinary houses the soil-pipe is 4 inches in diameter. In warm climates the soil-pipe is often placed on the outer face of one of the exterior walls of the building, to prevent air, escaping from a leak, from entering the house.

A *waste-pipe* is a vertical pipe which receives wastes from other sources than water-closets. It is also a metal pipe supported below and terminated above like the soil-pipe. The diameter of a waste-pipe is usually 3 inches, but the length above the roof is enlarged to 4 inches in diameter to prevent its being closed by frost.

The drainage of the roof is frequently conducted to the drain-pipe by a vertical iron pipe, called a *leader*, which is placed against the outer or the inner face of an exterior wall. For the ordinary house this is also 4 inches in diameter.

The pipes which carry the wastes from the fixtures to the soil- and waste-pipes are called *branches*. These are smaller than the verticals, and are usually made of lead, as lead pipe can be readily bent to the proper shape; branches are also made of nickel and brass.

The general principles governing house-drainage are, *to make the system of pipes simple and short, to avoid concealing the pipes, to make them readily accessible, and to provide for their ventilation and for the removal of obstructions.*

Traps.—A trap is any contrivance which, inserted in a pipe, will automatically prevent the passage of air or gas. The ordinary form of trap is a U-shaped pipe filled with liquid (Fig. 174). The depth of the seal is the difference of level of *A* and *B*, which does not ordinarily exceed 4 inches. It is evident that no air can pass the trap so long as the U-shaped pipe is filled above the level of *B*.

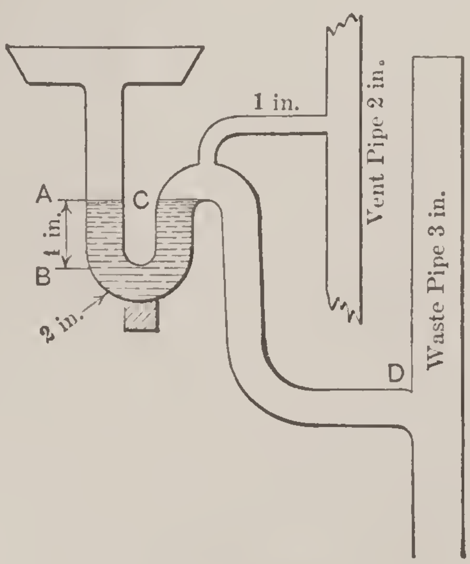


FIG. 174.

This water-seal may be removed in one of the following ways:

1. *By evaporation.* If the liquid in the trap is not renewed from time to time, it will gradually evaporate. This occurs in houses which are unoccupied for some time. To prevent this, the water may be replaced by some oily liquid like glycerin, which is not easily evaporated, or the inlet may be plugged if the fixture is not to be frequently flushed.

2. *By velocity.* The velocity of the flush may be so great that the friction in the trap may not be sufficient to retain the water required for the seal when the flow is stopped. The nearer the trap to the inlet, the less is the probability of its being unsealed in this way.

3. *By capillary attraction.* If a piece of cloth or waste is caught in the water of the trap and the other end extends into

the branch CD, the water will be slowly drawn into the branch CD by capillary attraction.

4. *By difference of pressure.* If the pressure due to a current of air is greater in CD than at A, the water will be forced up the tube AB towards the inlet, and the air will escape as soon as the lower surface of the water reaches B.

5. *By siphonage.* If the pipe from C to D runs full, a siphon will be formed which will drain the trap. This is most liable to occur when the branch is of uniform diameter throughout and the length CD is great. It can be prevented by making an air-inlet above C as shown in the figure.

The *vent-pipe* to which the air-inlet is connected is carried to a point above the roof, or is connected with a soil- or waste-pipe above the highest branch. As vent-pipes have the disadvantage of complicating the system, non-siphoning traps are now generally employed. In these (Fig. 175) the trap is suddenly enlarged or so obstructed as to check the velocity of flow through it.

Fig. 175.

There are many forms of traps. Those in common use on waste branches are of lead, and if of the general form shown in Fig. 174 are called S traps. The best are of drawn lead, which makes them perfectly smooth inside; at the base is a cleaning-cap.

A *running* or U-shaped trap is usually placed in the house-drain near its junction with the house-sewer (Fig. 173). This is omitted when it is desired to ventilate the sewer by means of the soil- and waste-pipes. A trap is also placed on every branch pipe as close as possible to its inlet.

The house-drainage system is ventilated by carrying the soil- and waste-pipes above the roof and by making a fresh-air inlet into the house-drain in rear of the running trap near its outlet, as shown in Fig. 173. The draft through the vertical pipes may be increased by placing them near a chimney in constant use. The fresh-air inlet must not be placed near the cold-air shaft of a furnace, nor should this inlet or the upper end of a soil- or waste-pipe be near a window.

Sewers.

The term *sewer* is usually applied to a common conduit which carries the drainage of several houses. However, in some city regulations the stoneware pipe which connects the house-drain with the common sewer is called the house-sewer, and in others the term sewer is applied to all conduits which carry household and factory wastes, in contradistinction to those which carry surface drainage only; the latter are then called *drains*.

General Principles of Construction.—The sewer which removes the drainage of a block of houses differs from the water-main which supplies it in the following particulars:

1. The water-main is always under sufficient head to raise the water to a considerable elevation above the street. The sewer should never be under a head sufficient to force the traps in the house-drains.

2. From the above it follows that while the water-pipe may be laid in any convenient manner, providing no point is higher than the hydraulic grade-line, the sewer must be laid with great care so that the slope of its bed shall give the required velocity of flow without producing pressure on its crown.

3. The depth of the water-main is immaterial so long as it is great enough to protect the service-pipes from frost. The sewer, however, must ordinarily be low enough to drain the basement of every house, and also to receive the sewage from all tributary sewers which are nearer the head of the system. Its slope will usually be approximately that of the street, unless this slope is less or greater than the limits for sewer grades.

4. If all the water-taps in the block are opened and allowed to discharge into the sewer, the velocity of flow in the water-main will be much greater than in the sewer, due to its greater head. Since the water-main which supplies and the sewer which discharges must provide for the same volume, the diameter of the sewer must be much greater than that of the water-main.

5. While the water-main carries a practically perfect liquid, the sewer always carries a liquid charged with solids which impede its flow. In sewers carrying surface drainage the sewage contains sand, gravel, sticks, etc. The sewer is therefore much more liable to be obstructed, and special provision must be made for the removal of obstructions.

6. The character of the sewage demands also its removal to the place of final disposal in as short a time as possible, and the ventilation of the conduit in which it is carried.

Material of Construction.—The materials of which a sewer is constructed are subjected to external rather than internal pressure, and only need be strong enough to resist this pressure. The inner surface of a sewer should be smooth, so as to offer little resistance to flow, and it should not be acted upon by the acids of the sewage. There should be as few joints as possible in the sewer, and the material should lend itself to the construction of curves at the points where the sewer changes direction, so as to reduce the resistance at these points to a minimum.

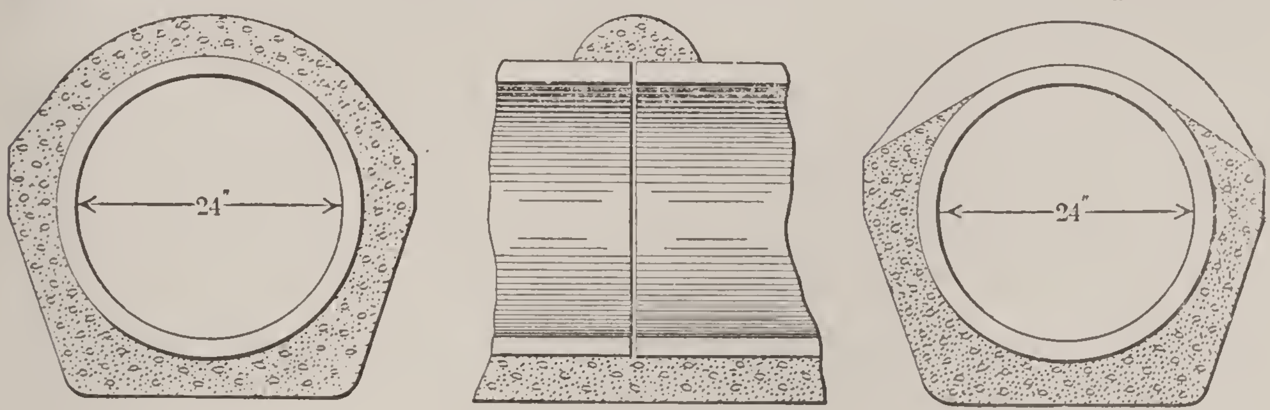
The materials which best fulfill these conditions are stoneware pipes, glazed on the interior, concrete pipes, and brick conduits lined with some smooth surface. Iron pipe is employed if the sewer is under a head, as an inverted siphon sewer under the bed of a river which connects the sewers on the opposite banks; iron pipe is also sometimes employed in very compressible soil where the stoneware pipe would be liable to break.

Glazed Stoneware or Vitrified Pipe.—The standard sizes of vitrified pipe vary from 2 to 24 inches in diameter; special pipes are made of larger diameters. Except for house-sewers, which may be 4 or 5 inches in diameter, the smallest pipe employed in sewer construction is the 6-inch pipe. In a combined system the smallest sewer is usually the 8-inch pipe. Stoneware pipes of a diameter exceeding 24 inches are not often employed.

Fig. 176.

Sewer-pipes are made of two general forms; one is called the *socket*, *spigot-and-bell*, or *spigot-and-hub pipe*, and the other is the *ring pipe* (Fig. 176). The latter is a simple cylinder, and is provided with rings or bands to cover the joints. A *channel* or *split pipe* is a semi-cylindrical pipe. *Increasers* and *reducers* are conical-shaped pipes employed to unite sewers of different radii; the former have the socket at the small end, and the latter at the large end.

In making a joint in a socket-pipe, the spigot of one pipe is inserted in the socket of the other and surrounded by a gasket of waste. The socket is then filled with neat cement or mortar well compressed. Experiments indicate that neat portland



A B C

Fig. 177.

makes the tightest joint; one-to-one portland is next in order, and one-to-one natural cement is third. The waste prevents the mortar from escaping into the pipe and making a rough surface. The ring pipe is usually laid in a bed of concrete; the joints are covered with concrete or with bands, and the bands themselves are buried in concrete. Fig. 177 shows the method of laying ring sewers in Washington, D. C. Section *A* is at a joint and section *C* between joints.

Brick and Concrete Sewers.—Sewers whose area of cross-section exceeds that of the 24-inch pipe are commonly made of

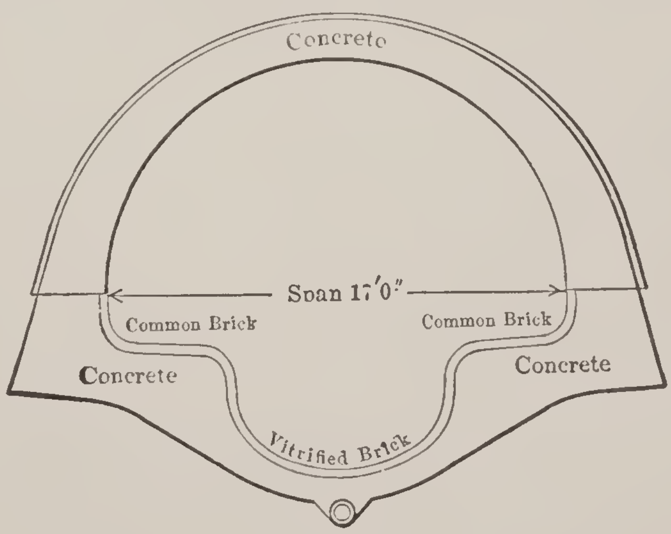


Fig. 178.

brick or concrete. The usual forms of cross-section are the *circular*, the *egg-shaped*, and the *horseshoe*. The circle gives the greatest area of cross-section for the same amount of material and is therefore the cheapest; the egg-shape is employed to increase the velocity of flow in a combined sewer when the dry-weather flow is small as compared with the storm flow. The hydraulic mean depth, and hence the velocity, is greater in the egg-shaped sewer when only slightly filled than in the circular sewer of the same area of cross-section. The horseshoe-shaped sewer is employed when it is desired to secure a large area of cross-section without an excessive height. Fig. 178 shows a horseshoe-shaped sewer constructed in Washington, D. C. To reduce the amount of friction, the lower part of a brick sewer is often made of blocks of glazed vitrified stoneware, called *sewer-* or *invert-blocks*. To reduce the wear on the beds of sewers with steep slopes, the beds are sometimes lined with vitrified bricks or stone paving-blocks.

Manholes.—A manhole is a vertical masonry shaft which gives access to a sewer from the surface of the site, either for inspecting or for cleaning the sewer. The top of the shaft is usually circular and 2 feet in diameter; the bottom of the shaft is either circular, oval, or square, and is large enough to allow the sewer to be inspected and cleaned with ease. For this purpose the larger dimension need not exceed 5 feet. It is desirable to have a manhole at every point where a sewer changes its direction or slope, so that every stretch can be readily inspected. In the straight stretches it is also desirable to have a manhole every 100 yards, but this distance may be increased or diminished. The top of the manhole is usually closed with a perforated iron cover, and some form of ladder is attached to its walls to enable the workman to pass up and down. The shaft need not be directly over the sewer; sometimes it is placed on one side and connected with the sewer by a horizontal passage. Fig. 179 shows a manhole on an egg-shaped

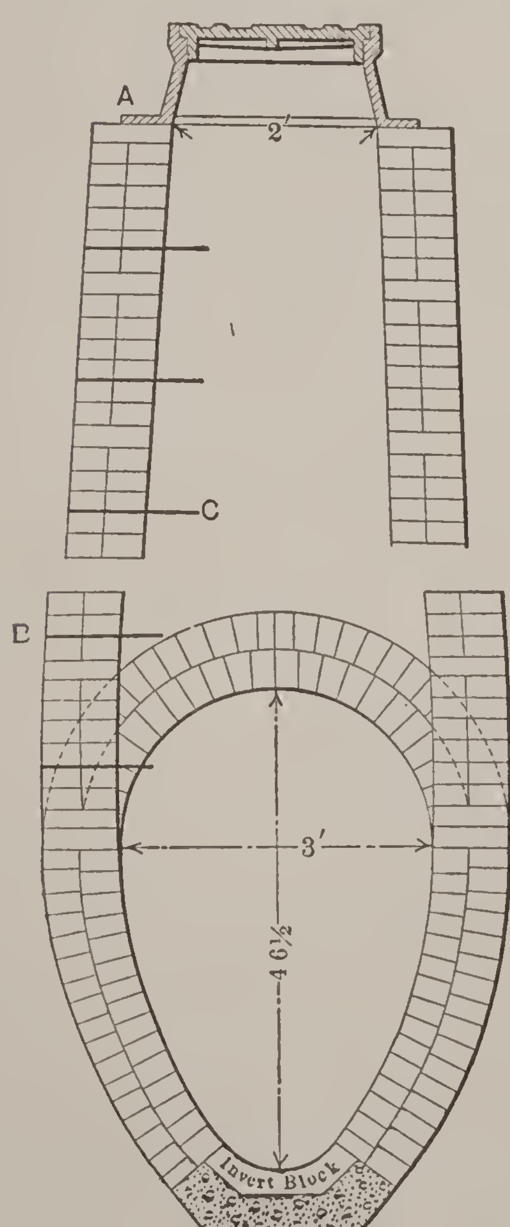


FIG. 179.

sewer as constructed in Washington, D. C. The ladder is made of U-shaped iron bars set in the walls of the manhole (*C*, Fig. 179). The horizontal section at *A* is circular, and at *B* rectangular.

When the manholes of small sewers are of necessity at long intervals, small shafts, large enough to admit of the lowering of a light of some description, are sometimes placed between manholes. These are called *lamp-holes*.

Catch-basin.—As heretofore described, a catch-basin is a shallow well constructed near the curb of a street to catch the water which flows in the gutters, and remove matters in suspension before discharging the water into the sewers. A vertical section of such a basin through the inlet and outlet is shown in Fig. 180. The basin is constructed under the sidewalk adjacent to the curb. The water enters at *C* and flows into the sewer from *A*. The silt which collects at the bottom of the basin is removed through the manhole *B*. The partition between *A* and *B* holds the floating material and also forms a trap.

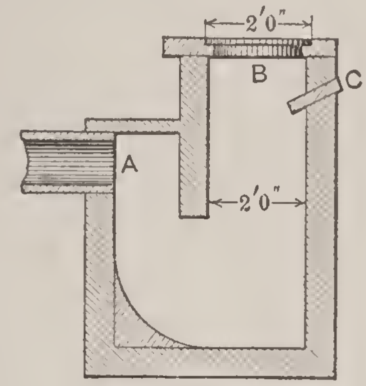


FIG. 180.

Flow in Sewers.—The velocity of flow in a sewer should not be so small that deposits will be formed in the sewer, nor should the velocity be so great that stones will be swept along swiftly and chip the bottom of the sewer. The velocities recommended in Moore are:

	Minimum.	Maximum.
Sewers 6 to 9 inches. . . .	3 feet per second	4 feet per second
" 12 to 24 "	$2\frac{1}{2}$ " " "	4 " " "
" large.	2 " " "	4 " " "

These velocities are recommended for sewers either full or half-full.

The corresponding slopes can be determined by solving the formula $v=c\sqrt{RS}$ for S, substituting for v, c, and R their proper values.

Variation of Velocity with Depth.—For any given circular sewer the values of c and S in the equation $v=c\sqrt{RS}$ are constant, since they depend only on the material of the walls and the slope of the sewer. In that case v will vary directly with $\sqrt{R}$, the square root of the hydraulic mean depth, or the square root of the ratio of the area of cross-section to its wetted perimeter.

Let A = area of cross-section of stream in a circular sewer in square feet;

P = wetted perimeter or wetted arc of cross-section in feet;

D = diameter of sewer in feet;

ϕ = angle subtended at the center of the circle by P in radius.

Then we shall have

$$A = \frac{PD}{4} - \frac{D^2 \sin \phi}{4},$$

$$R = \frac{A}{P} = \frac{D}{4} - \frac{D^2 \sin \phi}{4P}$$

$$= \frac{D}{4}\left(1 - \frac{\sin \phi}{\phi}\right).$$

If we substitute for ϕ different values and deduce the corresponding values of R, we may construct the following table:

VALUES OF R.

ϕ	R	ϕ	R	ϕ	R
00°	$.00\frac{D}{4}$	140°	$.74\frac{D}{4}$	260°	$1.21\frac{D}{4}$
20	.02"	160	.88"	280	1.20"
40	.08"	180	1.00"	300	1.16"
60	.18"	200	1.09"	320	1.11"
80	.30"	220	1.16"	340	1.06"
100	.44"	240	1.20"	360	1.00"
120	.59"				

The maximum value of R will be where the sin ϕ is negative and the ratio $\frac{\sin \phi}{\phi}$ is a numerical maximum.

Differentiating $\frac{\sin \phi}{\phi}$ with respect to ϕ we have

$$\partial\left(\frac{\sin \phi}{\phi}\right) / \partial\phi = \frac{\phi \cos \phi - \sin \phi}{\phi^2}.$$

Placing first differential coefficient equal to zero,

$$\frac{\phi \cos \phi - \sin \phi}{\phi^2} = 0, \quad \text{or} \quad \phi = \frac{\sin \phi}{\cos \phi} = \tan \phi,$$

will give a maximum. R will therefore be a maximum when ϕ equals about 257 degrees.

From the table it is seen that the velocity, which varies directly with $\sqrt{R}$, increases as the depth increases until the wetted perimeter is 257 degrees or the depth is about $0.8D$; the velocity will then decrease as the depth is still further increased. The variation in the velocity as the wetted perimeter increases from zero to 180 degrees is very great, but the variation in the velocity as the wetted perimeter increases from 180 to 360 degrees is very slight. The velocity in a sewer one-fourth full is nearly 0.8 of the velocity in a half-full sewer.

Since the discharge is the product of the area of cross-section of a stream and its velocity, it follows from the table that the maximum discharge will take place when the sewer is almost but not quite full.

Tables and curves of velocity and discharge for circular sewers running full, corresponding to different values of S and to different diameters, are given in works on sewerage.

The variation in the value of R for egg-shaped sewers differs somewhat from that of the circular sewer; R and v are both greater for slight depths than in the circular sewer. Tables and curves of velocity for standard shapes of egg-shaped sewers are also given in works on sewerage.

Area of Cross-section.—The required area of cross-section of a sewer is determined by the volume of sewage which it must carry, and by the maximum slope that may be given it. The volume of house-sewage is approximately equal to and never exceeds the amount of water introduced into the houses which are tributary to it. This volume will depend on the per-capita consumption and the density of the population. In all sewer systems allowance must be made for an increase in this density due to the growth of the community.

The amount of factory sewage may be approximately determined in the same manner.

The volume of storm-water which must be provided for in the construction of a sewer depends on the extent, slope, and character of the area tributary to the sewer, and on the maximum rainfall. In a separate system the area tributary to the house-sewer may be nothing, it may be only the roof area, or it may be the roof and yard area. In a combined system it is the entire area whose surface-waters flow into the sewer, and comprises not only the roof and yard areas, but also the streets, parks, etc.

Numerous attempts have been made to determine an empirical formula that will give the volume of water reaching the sewer in a unit of time.

Among those most commonly employed is the Burkli-Ziegler formula given on page 498. In this country the maximum rainfall employed in that formula is often assumed as great as 2.75 inches per hour.

Flushing.—The velocity of flow in a sewer may not be sufficient to prevent the formation of deposits. Under such circumstances it is necessary to flush the sewer at regular intervals by introducing into it a volume of water; if the slope is slight, a sufficient head must be given. Automatic flushing-tanks are usually basins which are gradually filled by a small pipe attached to a water-main, and discharged quickly by means of a large siphon-pipe. They are placed at the dead-ends of main sewers and branches.

Ventilation.—Sewers are ventilated as much as possible to dilute the sewer-air and thus render it less offensive and noxious. The ordinary method is to provide perforated covers for the manholes and thus give entrance to the outer air. They may be ventilated through the soil-, waste-, and conductor-pipes of the houses by omitting the trap in the house-drain. They may also be ventilated by the hollow posts of street lights, either gas or electric; if a gas-jet burns at the top of these poles, it will produce an artificial draft and aid in the ventilation. The sewer-gas may also be purified by burning disinfectants in the sewer itself.

Sewage Disposal.

Raw sewage, or the sewage as found in sewers, is a turbid liquid containing animal and vegetable organic matter, as well as inorganic matter, in suspension and solution. These matters are in varying proportions; their exact quantity and character can be determined only by chemical analysis.

The organic matter in the presence of air and moisture is subject to putrefaction and reduction to simpler inorganic compounds and elements. During the process of reduction foul odors are given off which, if not positively dangerous to human life, are at least sufficiently objectionable to constitute a nuisance unless the process is properly regulated.

The primary aim of every disposal scheme is to avoid the creation of such a nuisance.

Raw sewage is distinguished from purified sewage by the absence of nitrates, by its large amount of ammonia, by the large amount of oxygen it will absorb, and by the large amount of matter in suspension. Purified sewage is distinguished from pure water by its large amount of chlorine; the chlorine in raw sewage is not altered in amount by any reduction process.

Bacteria.—Raw sewage also contains myriads of microscopic organisms called bacteria; several millions are often found in a cubic centimeter. Their function seems to be to break down the complex organic matters in the sewage and with the aid of oxygen to form simple stable compounds and elements. Pasteur, who established the fact that fermentation and putrefaction took place only in the presence of living organisms, divided these organisms into two classes, *aerobes* and *anaerobes*. The former live and work only in a medium well supplied with oxygen, while the latter live and work in a medium in which there is no oxygen. To these has since been added a third class, *facultative* bacteria, which live and work in a medium in which oxygen is present, but in a small ratio to the other elements. As there are many varieties of each class and as each variety has its own mode of action, the successive steps in the reduction of the organic matter are extremely complicated.

In general it may be said if sewage is spread intermittently in a thin sheet over a porous soil, such as coarse sand, whose pores are filled with air, the conditions are favorable for aerobic bacteria, and the work will be done principally by them. The same is true if the sewage is discharged into highly aerated water. The effect of these bacteria being to oxidize the nitrogen of the organic matter and produce nitrates, the process of reduction by aerobic bacteria is called *nitrification*. It is a process of reduction unaccompanied by foul odors. To prepare sewage for aerobic action it is usually screened to remove sticks, paper, rags, etc., not easily reduced, and is also allowed to stand for a few hours in settling-tanks to remove the coarser materials in suspension not removed by the screen, which would be liable to close the pores of the filtering material.

In general it may be said that if sewage stands in tanks or pools for any length of time, its oxygen is soon exhausted by the aerobes, and the conditions are then favorable for anaerobic action. The anaerobes break down the organic matter and form ammonia, nitrites, and release such gases as nitrogen, hydrogen, carbon dioxide, marsh-gas, and sulphuretted hydrogen. As nearly all the vegetable and animal organic matter is liquefied by the anaerobes, the process is called *liquefaction* or *hydrolysis*. During the process of liquefaction foul odors are given off, and the sewage itself is usually in a foul condition and requires the action of the aerobic bacteria to purify it.

The process of reduction or purification to prevent a nuisance is therefore usually effected by bacteria of all three varieties. If the sewage is screened and settled, the reduction, as explained hereafter, may be effected almost wholly by aerobic bacteria, but the other varieties will be present in some stage of the process and will assist. If the sewage is kept in an open or closed tank for twenty-four hours, the aerobic bacteria will almost disappear, and the process in the tank will be almost wholly anaerobic. In preparing the sewage for anaerobic action it is not so necessary to strain it, nor is it necessary to first pass it through a settling-tank. The anaerobic action must, however, always be followed by aerobic action.

Bacteria may also be divided into the disease-producing bacteria and those that are not disease-producing. The principal varieties of the first class, or *pathogenic bacteria*, which may be found in sewage, are those of *cholera*, *diarrhœa*, *dysentery*, and *typhoid fever*. They are found in the discharges of patients who have these diseases. If sewage containing such bacteria is discharged into any stream used as a water- or ice-supply, they at once become a menace to public health.

Disposal.—Sewage, either in a raw state or after more or less screening and purification, may be disposed of by discharging it into a body of water, upon natural land, upon prepared land, or upon specially constructed filter-beds. If the discharge is into fresh water, as is usually the case in inland cities, care must be taken not only to avoid a nuisance caused by too great a concentration of sewage or its deposition on the shores, but also to reduce the number of bacteria in it as much as possible, so that the pathogenic bacteria which may find access to it are destroyed.

Dilution.—If raw sewage is discharged into a body of water simply to weaken the sewage and thus render it inoffensive, it is said to be disposed of by dilution.

No nuisance will be created if raw sewage is discharged into a stream of moderate velocity whose dilution ratio, or the ratio of the volume of discharge of the stream to the sewage, is about forty. If the stream is a rapid-flowing mountain torrent, the factor may be reduced to twenty. These dilution ratios will vary somewhat with the character of the sewage. When sewage is discharged into still water, such as a lake, the discharge should not be concentrated at a single point, as it will then putrefy. Its distribution may be effected by discharging the sewage through a long conduit, laid in the bed of the lake, with outlets at intervals along it.

In still water the mixed water and sewage is so rapidly purified by the process of sedimentation that the pollution of the water does not extend to a great distance from the sewer-outlet. At Burlington, Vt., a city of 15,000 inhabitants, which discharges its sewage at a single point into Lake Champlain, the presence of sewage cannot be detected by chemical processes at a distance of half a mile from the sewer-outlet. In running water the sedimentation is less rapid, and the pollution of the water is observed at a greater distance from the sewer-outlet. The reduction of the organic matter is effected by bacteria and other organisms which are found in the sewage, in the water, and in the mud of the bed. In the process of reduction oxygen is extracted from the water to form the new compounds. As the organisms which effect the final change are aerobic, this process is stopped, and offensive decomposition begins whenever the oxygen in the water is too greatly reduced. It is for this reason that the discharge into still water should be distributed over a wide area, and that a stream which can purify a given volume of sewage becomes foul when charged with a greater one. It explains why a torrent which is constantly aerated can have a smaller dilution ratio than a slow stream of equal volume.

The bacteria of the sewage are reduced largely by the process of sedimentation which carries them to the bottom with the suspended material. Other causes of purification are the lack of food and the unfavorable surroundings in the clarified water. The reduction of the number of these organisms in still water is very great. At Burlington, Vt., they are said to be reduced from one million to one thousand per cubic centimeter in one hundred feet. Judging from the water-borne diseases among the consumers of the water-supply, it was assumed, however, that the bacteria were not wholly removed at a distance of a half-mile, the original distance between the outlet-sewer and the inlet of the water-supply in Lake Champlain.

In running streams bacteria must be carried to considerable distances, since it has been shown by experiments that the bacteria of typhoid fever may live twenty-four days even in ice-cold water.

From the above it would appear that raw sewage may be disposed of by dilution without creating a nuisance if the volume of running water or the area of distribution in still water is sufficiently great. It will, however, always be more or less dangerous to discharge it into waters which may be used for water- and ice-supplies on account of the pathogenic bacteria it may contain. As the volume of discharge of most inland streams is small, and as they are in addition liable to be used as water- and ice-supplies, efforts are being made everywhere to prevent the discharge of raw sewage into small fresh-water streams or ponds without previous treatment to reduce the organic matter and the number of bacteria.

If the discharge is into salt water, which cannot be employed for domestic use unless it is distilled, no possible harm can be done except to marine life. The sewage, however, should be screened of all floating materials, and then be discharged, below the surface into a current which will carry it away from the shores and prevent deposits which will be exposed at low tides. To prevent the latter it is often necessary to construct reservoirs to hold the sewage which reaches the outlet during flood-tide, and discharge it only when the tide is at ebb. Such a reservoir is an element of the Boston sewerage system. The principal danger resulting from the discharge of raw sewage into salt water is the infection of oysters by pathogenic bacteria. Cases of infection thus propagated are of record.

Broad Irrigation.—Disposal by irrigation consists in applying screened sewage to the growing vegetation of a sewage farm. The greater part of the liquid is absorbed by the vegetation, and the remainder after filtration through the soil may be caught in subsoil drains and conveyed to the natural drainage streams of the country. The solids not removed by previous screening are absorbed by the soil in the same manner as manures and fertilizers applied to land. The main objection to irrigation is the great extent of land required; about an acre is required for every 25 to 100 people who contribute to the sewage, depending on the character of the soil, and the irregularity of the sewage-supply due to rainfall. This method of disposal is, however, extensively employed, especially in Europe.

In level country the sewage may be conveyed in parallel troughs about 40 feet apart, raised slightly above the surface of the soil. From these it overflows the ground, properly sloped, on either side, and the surplus is carried off in surface or subsoil drains midway between the troughs. Another method is to shape the ground in alternate ridges and furrows and let the sewage flow in the furrows. It thus reaches only the roots of the plants which grow on the ridges; subsoil drains may be placed beneath the ridges. As the nitrogen of the organic material must be reduced to nitrates before it can be absorbed by plants, the bacterial action in irrigation must be wholly aerobic if it is desired to avoid foul odors. This necessitates a porous soil and the even distribution of sewage; the formation of pools must be avoided. The sewage must be applied intermittently if the pores of the soil show any signs of becoming clogged.

Intermittent Filtration.—This is a modified form of irrigation in which the sewage is applied to the land, not for the purpose of utilizing the sewage for plants, but rather to purify as much sewage as possible per acre of land. The sewage is applied intermittently with the view of supporting as large a number of aerobic bacteria as possible and thus avoid the expense of large irrigating farms. The land used for intermittent filtration requires more thorough underdrainage than that used for irrigation. The underdrains assist in ventilating the soil.

The sewage is usually prepared by screening and then by standing for a few hours in settling-basins. The land is divided into a number of beds, some of which are settling-basins and the others are purifying-beds. The sewage is first conveyed to the settling-basins, where the sludge is precipitated by gravity, and from them to the purifiers. From time to time the settling-basins are emptied and the sludge dried by evaporation. When the sludge dries it is raked up and carted away to be burned, buried, or otherwise disposed of. There are always sufficient beds to allow each purifying-bed to rest dry some time after it is emptied. This process is extensively employed in this country in regions having a sandy or gravelly soil.

The natural soil may be replaced by specially constructed filter-beds of sand and gravel or other material thoroughly underdrained. The area of such beds will naturally be less than the area of natural soil required for purification.

Chemical Precipitation.—The suspended solids which remain after screening are often removed by precipitating them by some chemical, as lime, copperas, or alum. The substances may be employed alone or with each other. The kind and the amount of the precipitate best suited to the sewage must be determined by analysis of the effluent.

The chemicals are usually dissolved in water in a separate tank and then allowed to mix well with the sewage before the latter is admitted into the precipitation-tank. The process of precipitation may be intermittent or continuous. In the intermittent process there are three tanks, one being filled, one standing full, and the third being emptied. In the continuous tank the sewage charged with the precipitate flows slowly through the tank either in a horizontal or a vertical direction, depositing its sludge on the bottom. One of the favorite forms is the vertical, or Dortmund, tank. This is composed of two cylindrical concentric tubes of equal length, the diameter of the outer being about five times that of the inner. To the bottom of the outer tube is riveted an extension in the form of an inverted cone whose smaller end is equal to the diameter of the inner tube. The sewage charged with precipitate is admitted at the top of the inner tube, flows down to its bottom, where it is distributed by radial arms through the cross-section of the larger one. It then flows up through the outer tube and is discharged at its top. The sludge sinks to the bottom and is collected at the bottom of the conical extension of the outer tube; from there it is discharged through a discharge-pipe by gravity or by pumping.

If water transportation is available, the sludge is usually received in closed boats and carried out to sea. Otherwise it must be discharged into settling-basins, where it is allowed to evaporate. When nearly dry it is molded into cakes and burned.

The effluent of precipitation-tanks is clear liquid, but unless the process also removes the dissolved organic substances it is subject to further decomposition and cannot be discharged into streams without creating a nuisance. The effluent also usually contains a larger number of bacteria than is considered safe for its discharge into streams which are employed as water- or ice-supplies.

Land Required.—The amount of land required for the above processes has been determined in England to be as follows:

	Soil.	Number of Acres.	Number of Persons.
Simple irrigation	Stiff clay	1	25
	Loamy gravel	1	100
Simple intermittent filtration	Sandy gravel	1	100 to 300
Irrigation preceded by precipitation	Clay	1	200
	Loamy gravel	1	400
Intermittent filtration preceded by precipitation	Sandy gravel	1	500 to 600
Intermittent filtration in specially prepared filters, preceded by precipitation and followed by irrigation		1	2000

Contact-beds.—Contact-beds are specially prepared filter-beds arranged in series. They were first installed in Sutton, England, to replace a system of disposal by precipitation and irrigation which had proved unsatisfactory.

The settling-tanks, which were about 30 by 50 feet in area, were converted into *coarse-contact* beds by filling them to a depth of $3\frac{1}{2}$ feet with burned-clay ballast, or clay burned and reduced to 1-inch fragments. It was drained by a 6-inch underdrain to which were attached, at intervals of 3 feet, parallel branches 3 inches in diameter. The sewage was fed to the bed through a trough supported above it.

The *fine-contact* beds were each about 20 by 40 feet, and were filled to a depth of 3 feet by various combinations of material screened to pass through a ⅜-inch screen. These materials were burned clay, coke breeze, gravel, and different varieties of sand. The sewage was fed to these beds in the same manner as to the coarse ones.

In operating the beds, *screened sewage* was first allowed to flow on a coarse bed, whose outlet was closed, until the bed was filled to within 6 inches of the surface. The bed was then allowed to remain full for two hours. At the end of that time it was emptied, and the effluent was distributed slowly over one or more of the fine beds; this operation took about an hour. The effluent of the fine beds was sufficiently purified to admit its discharge into a stream which flowed to the sewage farm. The beds were allowed to stand empty for at least two hours before being again used. As a rule the intervals between the filling of each coarse bed was eight hours.

While porous material like coke is the best for contact-beds, especially for the fine ones, beds have been satisfactorily worked with broken stone, slag, coal, etc.

In some of the later beds a series of three beds instead of two have been employed, coarse, medium, and fine; the effluent of such a series is purer than from a series of two.

To more thoroughly aerate the sewage and the filter-beds, a revolving sprinkler has been devised like that employed in sprinkling lawns. Where this sprinkler is employed the beds are circular.

In the contact-beds as above described the action is principally aerobic, although there must be some anaerobic action in the coarse bed; it is, however, not sufficient to produce disagreeable odors. The aerobic action in the coarse bed may be increased by employing a revolving sprinkler to distribute the sewage over it.

Most of the sludge is destroyed by bacterial action, and this is one of the advantages of the system. This requires the cleansing of the beds only at long intervals.

Septic Tanks.—The septic tank is an open or closed tank in which raw sewage is allowed to stand for a period of about twenty-four hours for the purpose of liquefying its organic matter by anaerobic action. It differs from a settling-tank only in the length of time the sewage remains in it. In this tank the sewage putrefies, such gases as hydrogen, nitrogen, marsh-gas, carbon dioxide, and hydrogen sulphide are given off, and ammonia is formed. A thick scum forms over the top of the tank, and the sludge is deposited in a fine powder. On account of its odors a septic tank usually creates a nuisance if placed near dwellings.

Although the amount of sludge is much less than in the simple settling-tank and the tank therefore requires less frequent cleaning, the sludge produced is of a more offensive character.

The effluent of a septic tank usually requires further purification by passing through fine filter-beds like those of the contact system. Before it reaches the fine filters it is aerated by passing in a thin sheet over an aerating-weir. The weir and beds produce nitrification, which is the last stage of every system of sewage treatment.

Septic tanks have been introduced in this country where the factory wastes can be reduced only by anaerobic action, and where the soil is clayey and not suited to intermittent filtration. The tanks are either open or closed; the latter are best in very cold climates. The sewage is admitted some distance below the level of the liquid in the tank, and usually into a grit-chamber separated from the main tank by a low wall over which the sewage flows. It is also withdrawn from the tank by a pipe tapping the tank some distance below the surface of the liquid. The most suitable rate of flow through the tank is determined by experiment.

A modified form of septic tank is a coarse-contact bed in which the sewage is admitted at the bottom and escapes to the fine bed from the top. This is called *upward filtration.*

Destruction of Bacteria.—Bacteria are removed from sewage either by filtering through the soil, as in the methods of irrigation and intermittent filtration, or by passing through the fine beds of contact-filters and septic tanks.

For further information see Gerhard's "Sanitary Engineering of Buildings," Folwell's "Sewerage," Moore's "Sanitary Engineering," Sedgwick's "Principles of Sanitary Science," Rideal's "Sewage and the Bacterial Purification of Sewage," Baumeister's "Cleaning and Sewerage of Cities," and Rafter and Baker's "Sewage Disposal in the United States."

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